

**2017 SchweserNotes™**

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**Part I**

**FRM®**

**Financial Markets  
and Products**

**Exam Prep**

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**eBook 3**



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## Part I FRM® Exam

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Sincerely,

*Derek Burkett*

Derek Burkett, CFA, FRM, CAIA

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# FRM PART I BOOK 3:

## FINANCIAL MARKETS AND PRODUCTS

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<b>READING ASSIGNMENTS AND LEARNING OBJECTIVES</b>	<b>v</b>
<b>FINANCIAL MARKETS AND PRODUCTS</b>	
31: Banks	1
32: Insurance Companies and Pension Plans	10
33: Mutual Funds and Hedge Funds	24
34: Introduction (Options, Futures, and Other Derivatives)	40
35: Mechanics of Futures Markets	56
36: Hedging Strategies Using Futures	68
37: Interest Rates	80
38: Determination of Forward and Futures Prices	96
39: Interest Rate Futures	109
40: Swaps	123
41: Mechanics of Options Markets	140
42: Properties of Stock Options	155
43: Trading Strategies Involving Options	167
44: Exotic Options	183
45: Commodity Forwards and Futures	194
46: Exchanges, OTC Derivatives, DPCs and SPVs	215
47: Basic Principles of Central Clearing	225
48: Risks Caused by CCPs	236
49: Foreign Exchange Risk	243
50: Corporate Bonds	257
51: Mortgages and Mortgage-Backed Securities	270
<b>SELF-TEST: FINANCIAL MARKETS AND PRODUCTS</b>	<b>296</b>
<b>FORMULAS</b>	<b>305</b>
<b>INDEX</b>	<b>309</b>

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# READING ASSIGNMENTS AND LEARNING OBJECTIVES

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*The following material is a review of the Financial Markets and Products principles designed to address the learning objectives set forth by the Global Association of Risk Professionals.*

## READING ASSIGNMENTS

John C. Hull, *Risk Management and Financial Institutions, 4th Edition* (Hoboken, New Jersey: John Wiley & Sons, 2015).

- 31. "Banks," Chapter 2 (page 1)
- 32. "Insurance Companies and Pension Plans," Chapter 3 (page 10)
- 33. "Mutual Funds and Hedge Funds," Chapter 4 (page 24)

John C. Hull, *Options, Futures, and Other Derivatives, 9th Edition* (New York: Pearson Prentice Hall, 2014).

- 34. "Introduction," Chapter 1 (page 40)
- 35. "Mechanics of Futures Markets," Chapter 2 (page 56)
- 36. "Hedging Strategies Using Futures," Chapter 3 (page 68)
- 37. "Interest Rates," Chapter 4 (page 80)
- 38. "Determination of Forward and Futures Prices," Chapter 5 (page 96)
- 39. "Interest Rate Futures," Chapter 6 (page 109)
- 40. "Swaps," Chapter 7 (page 123)
- 41. "Mechanics of Options Markets," Chapter 10 (page 140)
- 42. "Properties of Stock Options," Chapter 11 (page 155)
- 43. "Trading Strategies Involving Options," Chapter 12 (page 167)
- 44. "Exotic Options," Chapter 26 (page 183)

Robert McDonald, *Derivatives Markets, 3rd Edition* (Boston: Addison-Wesley, 2013).

- 45. "Commodity Forwards and Futures," Chapter 6 (page 194)

Jon Gregory, *Central Counterparties: Mandatory Clearing and Bilateral Margin Requirements for OTC Derivatives* (New York: John Wiley & Sons, 2014).

46. “Exchanges, OTC Derivatives, DPCs and SPVs,” Chapter 2 (page 215)

47. “Basic Principles of Central Clearing,” Chapter 3 (page 225)

48. “Risks Caused by CCPs,” Chapter 14 (section 14.4 only) (page 236)

Anthony Saunders and Marcia Millon Cornett, *Financial Institutions Management: A Risk Management Approach, 8th Edition* (New York: McGraw-Hill, 2014).

49. “Foreign Exchange Risk,” Chapter 13 (page 243)

Frank Fabozzi (editor), *The Handbook of Fixed Income Securities, 8th Edition* (New York: McGraw-Hill, 2012).

50. “Corporate Bonds,” Chapter 12 (page 257)

Bruce Tuckman and Angel Serrat, *Fixed Income Securities: Tools for Today's Markets, 3rd Edition* (New York: John Wiley & Sons, 2011).

51. “Mortgages and Mortgage-Backed Securities,” Chapter 20 (page 270)

## LEARNING OBJECTIVES

### 31. Banks

1. Identify the major risks faced by a bank. (page 1)
2. Distinguish between economic capital and regulatory capital. (page 2)
3. Explain how deposit insurance gives rise to a moral hazard problem. (page 2)
4. Describe investment banking financing arrangements including private placement, public offering, best efforts, firm commitment, and Dutch auction approaches. (page 3)
5. Describe the potential conflicts of interest among commercial banking, securities services, and investment banking divisions of a bank and recommend solutions to the conflict of interest problems. (page 4)
6. Describe the distinctions between the “banking book” and the “trading book” of a bank. (page 4)
7. Explain the originate-to-distribute model of a bank and discuss its benefits and drawbacks. (page 5)

### 32. Insurance Companies and Pension Plans

1. Describe the key features of the various categories of insurance companies and identify the risks facing insurance companies. (page 10)
2. Describe the use of mortality tables and calculate the premium payment for a policy holder. (page 12)
3. Calculate and interpret loss ratio, expense ratio, combined ratio, and operating ratio for a property-casualty insurance company. (page 15)
4. Describe moral hazard and adverse selection risks facing insurance companies, provide examples of each, and describe how to overcome the problems. (page 15)
5. Distinguish between mortality risk and longevity risk and describe how to hedge these risks. (page 16)
6. Evaluate the capital requirements for life insurance and property-casualty insurance companies. (page 16)
7. Compare the guaranty system and the regulatory requirements for insurance companies with those for banks. (page 17)
8. Describe a defined benefit plan and a defined contribution plan for a pension fund and explain the differences between them. (page 18)

### 33. Mutual Funds and Hedge Funds

1. Differentiate among open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs). (page 24)
2. Calculate the net asset value (NAV) of an open-end mutual fund. (page 28)
3. Explain the key differences between hedge funds and mutual funds. (page 28)
4. Calculate the return on a hedge fund investment and explain the incentive fee structure of a hedge fund including the terms hurdle rate, high-water mark, and clawback. (page 29)
5. Describe various hedge fund strategies, including long/short equity, dedicated short, distressed securities, merger arbitrage, convertible arbitrage, fixed income arbitrage, emerging markets, global macro, and managed futures, and identify the risks faced by hedge funds. (page 31)
6. Describe hedge fund performance and explain the effect of measurement biases on performance measurement. (page 34)



**34. Introduction (Options, Futures, and Other Derivatives)**

1. Describe the over-the-counter market, distinguish it from trading on an exchange, and evaluate its advantages and disadvantages. (page 40)
2. Differentiate between options, forwards, and futures contracts. (page 41)
3. Identify and calculate option and forward contract payoffs. (page 41)
4. Calculate and compare the payoffs from hedging strategies involving forward contracts and options. (page 45)
5. Calculate and compare the payoffs from speculative strategies involving futures and options. (page 47)
6. Calculate an arbitrage payoff and describe how arbitrage opportunities are temporary. (page 50)
7. Describe some of the risks that can arise from the use of derivatives. (page 50)

**35. Mechanics of Futures Markets**

1. Define and describe the key features of a futures contract, including the asset, the contract price and size, delivery, and limits. (page 56)
2. Explain the convergence of futures and spot prices. (page 58)
3. Describe the rationale for margin requirements and explain how they work. (page 58)
4. Describe the role of a clearinghouse in futures and over-the-counter market transactions. (page 59)
5. Describe the role of collateralization in the over-the-counter market and compare it to the margining system. (page 60)
6. Identify the differences between a normal and inverted futures market. (page 61)
7. Describe the mechanics of the delivery process and contrast it with cash settlement. (page 61)
8. Evaluate the impact of different trading order types. (page 62)
9. Compare and contrast forward and futures contracts. (page 56)

**36. Hedging Strategies Using Futures**

1. Define and differentiate between short and long hedges and identify their appropriate uses. (page 68)
2. Describe the arguments for and against hedging and the potential impact of hedging on firm profitability. (page 68)
3. Define the basis and explain the various sources of basis risk, and explain how basis risks arise when hedging with futures. (page 69)
4. Define cross hedging, and compute and interpret the minimum variance hedge ratio and hedge effectiveness. (page 69)
5. Compute the optimal number of futures contracts needed to hedge an exposure, and explain and calculate the “tailing the hedge” adjustment. (page 72)
6. Explain how to use stock index futures contracts to change a stock portfolio’s beta. (page 73)
7. Explain the term “rolling the hedge forward” and describe some of the risks that arise from this strategy. (page 74)

**37. Interest Rates**

1. Describe Treasury rates, LIBOR, and repo rates, and explain what is meant by the “risk-free” rate. (page 80)
2. Calculate the value of an investment using different compounding frequencies. (page 81)



3. Convert interest rates based on different compounding frequencies. (page 81)
4. Calculate the theoretical price of a bond using spot rates. (page 82)
5. Derive forward interest rates from a set of spot rates. (page 86)
6. Derive the value of the cash flows from a forward rate agreement (FRA). (page 87)
7. Calculate the duration, modified duration and dollar duration of a bond. (page 88)
8. Evaluate the limitations of duration and explain how convexity addresses some of them. (page 89)
9. Calculate the change in a bond's price given its duration, its convexity, and a change in interest rates. (page 90)
10. Compare and contrast the major theories of the term structure of interest rates. (page 91)

### **38. Determination of Forward and Futures Prices**

1. Differentiate between investment and consumption assets. (page 96)
2. Define short-selling and calculate the net profit of a short sale of a dividend-paying stock. (page 96)
3. Describe the differences between forward and futures contracts and explain the relationship between forward and spot prices. (page 97)
4. Calculate the forward price given the underlying asset's spot price, and describe an arbitrage argument between spot and forward prices. (page 97)
5. Explain the relationship between forward and futures prices. (page 101)
6. Calculate a forward foreign exchange rate using the interest rate parity relationship. (page 100)
7. Define income, storage costs, and convenience yield. (page 102)
8. Calculate the futures price on commodities incorporating income/storage costs and/or convenience yields. (page 102)
9. Calculate, using the cost-of-carry model, forward prices where the underlying asset either does or does not have interim cash flows. (page 97)
10. Describe the various delivery options available in the futures markets and how they can influence futures prices. (page 103)
11. Explain the relationship between current futures prices and expected future spot prices, including the impact of systematic and nonsystematic risk. (page 103)
12. Define and interpret contango and backwardation, and explain how they relate to the cost-of-carry model. (page 104)

### **39. Interest Rate Futures**

1. Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation. (page 109)
2. Calculate the conversion of a discount rate to a price for a US Treasury bill. (page 111)
3. Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond. (page 110)
4. Explain and calculate a US Treasury bond futures contract conversion factor. (page 112)
5. Calculate the cost of delivering a bond into a Treasury bond futures contract. (page 112)
6. Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision. (page 112)
7. Calculate the theoretical futures price for a Treasury bond futures contract. (page 113)

8. Calculate the final contract price on a Eurodollar futures contract. (page 115)
9. Describe and compute the Eurodollar futures contract convexity adjustment. (page 115)
10. Explain how Eurodollar futures can be used to extend the LIBOR zero curve. (page 116)
11. Calculate the duration-based hedge ratio and create a duration-based hedging strategy using interest rate futures. (page 116)
12. Explain the limitations of using a duration-based hedging strategy. (page 117)

#### 40. Swaps

1. Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows. (page 123)
2. Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows. (page 124)
3. Explain the role of financial intermediaries in the swaps market. (page 124)
4. Describe the role of the confirmation in a swap transaction. (page 124)
5. Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument. (page 125)
6. Explain how the discount rates in a plain vanilla interest rate swap are computed. (page 126)
7. Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions. (page 126)
8. Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs). (page 128)
9. Explain the mechanics of a currency swap and compute its cash flows. (page 130)
10. Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows. (page 132)
11. Calculate the value of a currency swap based on two simultaneous bond positions. (page 130)
12. Calculate the value of a currency swap based on a sequence of FRAs. (page 131)
13. Describe the credit risk exposure in a swap position. (page 133)
14. Identify and describe other types of swaps, including commodity, volatility and exotic swaps. (page 133)

#### 41. Mechanics of Options Markets

1. Describe the types, position variations, and typical underlying assets of options. (page 140)
2. Explain the specification of exchange-traded stock option contracts, including that of nonstandard products. (page 146)
3. Describe how trading, commissions, margin requirements, and exercise typically work for exchange-traded options. (page 148)

#### 42. Properties of Stock Options

1. Identify the six factors that affect an option's price and describe how these six factors affect the price for both European and American options. (page 155)
2. Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks. (page 157)
3. Explain put-call parity and apply it to the valuation of European and American stock options. (page 158)
4. Explain the early exercise features of American call and put options. (page 160)

#### 43. Trading Strategies Involving Options

1. Explain the motivation to initiate a covered call or a protective put strategy. (page 167)
2. Describe the use and calculate the payoffs of various spread strategies. (page 168)
3. Describe the use and explain the payoff functions of combination strategies. (page 173)

#### 44. Exotic Options

1. Define and contrast exotic derivatives and plain vanilla derivatives. (page 183)
2. Describe some of the factors that drive the development of exotic products. (page 183)
3. Explain how any derivative can be converted into a zero-cost product. (page 184)
4. Describe how standard American options can be transformed into nonstandard American options. (page 184)
5. Identify and describe the characteristics and pay-off structure of the following exotic options: gap, forward start, compound, chooser, barrier, binary, lookback, shout, Asian, exchange, rainbow, and basket options. (page 185)
6. Describe and contrast volatility and variance swaps. (page 188)
7. Explain the basic premise of static option replication and how it can be applied to hedging exotic options. (page 189)

#### 45. Commodity Forwards and Futures

1. Apply commodity concepts such as storage costs, carry markets, lease rate, and convenience yield. (page 194)
2. Explain the basic equilibrium formula for pricing commodity forwards. (page 194)
3. Describe an arbitrage transaction in commodity forwards, and compute the potential arbitrage profit. (page 196)
4. Define the lease rate and explain how it determines the no-arbitrage values for commodity forwards and futures. (page 199)
5. Define carry markets, and illustrate the impact of storage costs and convenience yields on commodity forward prices and no-arbitrage bounds. (page 201)
6. Compute the forward price of a commodity with storage costs. (page 201)
7. Compare the lease rate with the convenience yield. (page 203)
8. Identify factors that impact gold, corn, electricity, natural gas, and oil forward prices. (page 203)
9. Compute a commodity spread. (page 206)
10. Explain how basis risk can occur when hedging commodity price exposure. (page 206)
11. Evaluate the differences between a strip hedge and a stack hedge and explain how these differences impact risk management. (page 207)
12. Provide examples of cross-hedging, specifically the process of hedging jet fuel with crude oil and using weather derivatives. (page 208)
13. Explain how to create a synthetic commodity position, and use it to explain the relationship between the forward price and the expected future spot price. (page 194)

#### 46. Exchanges, OTC Derivatives, DPCs and SPVs

1. Describe how exchanges can be used to alleviate counterparty risk. (page 215)
2. Explain the developments in clearing that reduce risk. (page 215)
3. Compare exchange-traded and OTC markets and describe their uses. (page 216)

4. Identify the classes of derivatives securities and explain the risk associated with them. (page 217)
5. Identify risks associated with OTC markets and explain how these risks can be mitigated. (page 218)

**47. Basic Principles of Central Clearing**

1. Provide examples of the mechanics of a central counterparty (CCP). (page 225)
2. Describe advantages and disadvantages of central clearing of OTC derivatives. (page 228)
3. Compare margin requirements in centrally cleared and bilateral markets, and explain how margin can mitigate risk. (page 229)
4. Compare and contrast bilateral markets to the use of novation and netting. (page 229)
5. Assess the impact of central clearing on the broader financial markets. (page 230)

**48. Risks Caused by CCPs**

1. Identify and explain the types of risks faced by CCPs. (page 236)
2. Identify and distinguish between the risks to clearing members as well as non-members. (page 238)
3. Identify and evaluate lessons learned from prior CCP failures. (page 239)

**49. Foreign Exchange Risk**

1. Calculate a financial institution's overall foreign exchange exposure. (page 243)
2. Explain how a financial institution could alter its net position exposure to reduce foreign exchange risk. (page 243)
3. Calculate a financial institution's potential dollar gain or loss exposure to a particular currency. (page 243)
4. Identify and describe the different types of foreign exchange trading activities. (page 244)
5. Identify the sources of foreign exchange trading gains and losses. (page 245)
6. Calculate the potential gain or loss from a foreign currency denominated investment. (page 245)
7. Explain balance-sheet hedging with forwards. (page 247)
8. Describe how a non-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem, and use this theorem to calculate forward foreign exchange rates. (page 250)
9. Explain why diversification in multicurrency asset-liability positions could reduce portfolio risk. (page 251)
10. Describe the relationship between nominal and real interest rates. (page 251)

**50. Corporate Bonds**

1. Describe a bond indenture and explain the role of the corporate trustee in a bond indenture. (page 257)
2. Explain a bond's maturity date and how it impacts bond retirements. (page 257)
3. Describe the main types of interest payment classifications. (page 258)
4. Describe zero-coupon bonds and explain the relationship between original-issue discount and reinvestment risk. (page 258)
5. Distinguish among the following security types relevant for corporate bonds: mortgage bonds, collateral trust bonds, equipment trust certificates, subordinated and convertible debenture bonds, and guaranteed bonds. (page 259)

6. Describe the mechanisms by which corporate bonds can be retired before maturity. (page 261)
7. Differentiate between credit default risk and credit spread risk. (page 262)
8. Describe event risk and explain what may cause it in corporate bonds. (page 263)
9. Define high-yield bonds, and describe types of high-yield bond issuers and some of the payment features unique to high yield bonds. (page 263)
10. Define and differentiate between an issuer default rate and a dollar default rate. (page 264)
11. Define recovery rates and describe the relationship between recovery rates and seniority. (page 265)

#### **51. Mortgages and Mortgage-Backed Securities**

1. Describe the various types of residential mortgage products. (page 270)
2. Calculate a fixed rate mortgage payment, and its principal and interest components. (page 273)
3. Describe the mortgage prepayment option and the factors that influence prepayments. (page 276)
4. Summarize the securitization process of mortgage backed securities (MBS), particularly formation of mortgage pools including specific pools and TBAs. (page 277)
5. Calculate weighted average coupon, weighted average maturity, and conditional prepayment rate (CPR) for a mortgage pool. (page 277)
6. Describe a dollar roll transaction and how to value a dollar roll. (page 282)
7. Explain prepayment modeling and its four components: refinancing, turnover, defaults, and curtailments. (page 285)
8. Describe the steps in valuing an MBS using Monte Carlo Simulation. (page 287)
9. Define Option Adjusted Spread (OAS), and explain its challenges and its uses. (page 290)





# BANKS

Topic 31

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## EXAM FOCUS

This topic introduces a number of concepts about banks that are developed more fully elsewhere in the FRM curriculum. For the exam, focus on understanding the major types of risk a bank faces and how they are addressed, both by banks themselves and by bank regulators. Be prepared to explain the differences between commercial banking and investment banking as well as the conflicts that exist in an organization that performs both of these services. Also, understand the distinctions between the lending and trading operations of a bank. Finally, be able to describe the implications of banks originating loans and distributing them to other parties.

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## TYPES OF BANKS

When we speak of “banks,” we include financial institutions that provide a variety of services. Banks can be categorized by the functions they perform and the customers they serve.

**Commercial banks** are those that take deposits and make loans. Commercial banks include **retail banks**, which primarily serve individuals and small businesses, and **wholesale banks**, which primarily serve corporate and institutional customers.

**Investment banks** are those that assist in raising capital for their customers (e.g., by managing the issuance of debt and equity securities) and advising them on corporate finance matters such as mergers and restructurings.

Whether a bank or bank holding company engages in both commercial banking and investment banking or must only do one or the other depends on the regulations where it does business.

## MAJOR RISKS FACED BY BANKS

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### LO 31.1: Identify the major risks faced by a bank.

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The main risks faced by a bank include credit risk, market risk, and operational risk.

- **Credit risk** refers to the risk that borrowers may default on loans or that counterparties to contracts such as derivatives may default on their obligations. One measure of credit risk is a bank's loan losses as a percentage of its assets.

- **Market risk** refers to the risk of losses from a bank's trading activities, such as declines in the value of securities the bank owns. Later in this topic, we will distinguish between the “trading book” and the “banking book” of a bank.
- **Operational risk** refers to the possibility of losses arising from external events or failures of a bank's internal controls. We will describe this risk in greater detail in Book 4, Topic 66.

Regulators in most jurisdictions require banks to hold adequate capital against these risks. Typically, they consider credit risk and operational risk with a time horizon of one year and market risk with a shorter time horizon.

## ECONOMIC CAPITAL VS. REGULATORY CAPITAL

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### LO 31.2: Distinguish between economic capital and regulatory capital.

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To mitigate the risk of bank failures caused by losses on loans or trading assets, banks must be funded by adequate sources of capital. Equity capital as a percentage of assets is a key measure of capital adequacy. Banks may also issue long-term debt to bolster their capital. This debt is subordinated to the claims of depositors if a bank faces financial distress.

Banks and their regulators may have different views about how much capital is sufficient in light of the risks a bank faces. **Regulatory capital** refers to the amount determined by bank regulators. In terms of bank regulation, equity is referred to as “Tier 1 capital” and subordinated long-term debt is referred to as “Tier 2 capital.”



*Professor's Note: Regulations concerning bank capital, such as Basel I, Basel II, and Solvency II, are described in the FRM Part II curriculum.*

**Economic capital** refers to the amount of capital that a bank believes is adequate based on its own risk models. Even if economic capital is less than regulatory capital, as is often the case, a bank must maintain its capital at the regulatory minimum or greater.

## DEPOSIT INSURANCE AND MORAL HAZARD

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### LO 31.3: Explain how deposit insurance gives rise to a moral hazard problem.

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To increase public confidence in the banking system and prevent runs on banks, most countries have established systems of **deposit insurance**. Typically, a depositor's funds are guaranteed up to some maximum amount if a bank fails. These systems are funded by insurance premiums paid by banks.

Like other forms of insurance (as we will cover in the next topic on “Insurance Companies and Pension Plans”), deposit insurance brings an element of **moral hazard**. Moral hazard is the observed phenomenon that insured parties take greater risks than they would normally take if they were not insured. In the banking context, with deposit insurance in place, the moral hazard arises when depositors pay less attention to banks' financial health than they otherwise would. This allows banks to offer higher interest rates on deposits and make



higher-risk loans with the funds they attract. Losses on such loans contributed to increased bank failures in the United States in the 1980s and 2000s.

One way of mitigating moral hazard is by making insurance premiums risk-based. For example, in recent years, poorly-capitalized banks have been required to pay higher deposit insurance premiums than well-capitalized banks.

## INVESTMENT BANKING FINANCING ARRANGEMENTS

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**LO 31.4: Describe investment banking financing arrangements including private placement, public offering, best efforts, firm commitment, and Dutch auction approaches.**

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When an investment bank arranges a securities issuance for a customer, it may try to place the entire issue with a particular buyer or group of buyers or sell the issue in the public market.

In a **private placement**, securities are sold directly to qualified investors with substantial wealth and investment knowledge. The investment bank earns fee income for arranging a private placement.

If the securities are sold to the investing public at large, the issuance is referred to as a **public offering**. Investment banks have two methods of assisting with a public offering. With a **firm commitment**, the investment bank agrees to purchase the entire issue at a price that is negotiated between the issuer and bank. The investment bank earns income by selling the issue to the public at a spread above the price it paid the issuer. An investment bank can also agree to distribute an issue on a **best efforts** basis rather than agreeing to purchase the whole issue. If only part of the issue can be sold, the bank is not obligated to buy the unsold portion. As with a private placement, the investment bank earns fee income for its services.

First-time issues of stock by firms whose shares are not currently publicly traded are called **initial public offerings** (IPOs). An investment bank can assist in determining an IPO price by analyzing the value of the issuer. An IPO price may also be discovered through a **Dutch auction** process. A Dutch auction begins with a price greater than what any bidder will pay, and this price is reduced until a bidder agrees to pay it. Each bidder may specify how many units they will purchase when accepting a price. The price continues to be reduced until bidders have accepted all the shares. The price at which the last of the shares can be sold becomes the price paid by all successful bidders.

## POTENTIAL CONFLICTS OF INTEREST

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**LO 31.5: Describe the potential conflicts of interest among commercial banking, securities services, and investment banking divisions of a bank and recommend solutions to the conflict of interest problems.**

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If a bank or a bank holding company provides commercial banking, investment banking, and securities services, several conflicts of interest may arise. For example, an investment banking division that is trying to sell newly issued stocks or bonds might want the securities division to sell these to their clients. The investment bankers may press the securities division's financial analysts to maintain "Buy" recommendations, or press its financial advisors to allocate these stocks and bonds to customer accounts. Such pressure may interfere with analysts' independence and objectivity or conflict with advisors' duties to clients.

Another clear conflict of interest among banking departments involves material nonpublic information. A commercial banking or investment banking division may acquire nonpublic information about a company when negotiating a loan or arranging a securities issuance. Other parts of the banking company, such as its trading desk, may benefit unfairly if they gain access to this information.

Because of these inherent conflicts, most bank regulators require some degree of separation among commercial banking, securities services, and investment banking. In some cases, they have prohibited firms from engaging in more than one of these activities, as was true in the United States when the Glass-Steagall Act was in force. Where banking firms are permitted to have commercial banking, securities, and investment banking units, the firms must implement **Chinese walls**, which are internal controls to prevent information from being shared among these units.

## BANKING BOOK vs. TRADING BOOK

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**LO 31.6: Describe the distinctions between the "banking book" and the "trading book" of a bank.**

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A bank's financial statements reflect accounting rules that apply to different aspects of its business. Revenue and income from its fee-based activities are recorded using the normal rules of accrual accounting, but other rules apply to its lending and trading activities.

The **banking book** refers to loans made, which are the primary assets of a commercial bank. Normally, the balance sheet value of a loan includes the principal amount to be repaid and accrued interest on the loan. However, for a **nonperforming loan** the value does not include accrued interest. A loan is typically classified as nonperforming if payments are more than 90 days overdue.

A bank will recognize a loss on a loan if it becomes likely that the borrower will not fully repay the principal. Bank financial statements reflect a reserve for loan losses that is determined by management, against which actual loan losses are charged. Increases or

decreases in the loan loss reserve are a potential tool for earnings manipulation, such as smoothing across business cycles, by a bank's management.

The **trading book** refers to assets and liabilities related to a bank's trading activities. Unlike other assets and liabilities, trading book items are marked to market daily. This is straightforward for items that trade in liquid markets and have readily available prices. For items that lack a liquid market, do not trade frequently, or are complex or custom instruments, marking to market involves estimating a price. Such items are sometimes said to be "marked to model."

## THE ORIGINATE-TO-DISTRIBUTE MODEL

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### LO 31.7: Explain the originate-to-distribute model of a bank and discuss its benefits and drawbacks.

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In contrast to a bank making loans and keeping them as assets, the **originate-to-distribute model** involves making loans and selling them to other parties. Many mortgage lenders in the United States operate on the originate-to-distribute model. Government agencies such as Ginnie Mae (GNMA), Fannie Mae (FNMA), and Freddie Mac (FHLMC) purchase mortgage loans from banks and issue securities backed by the cash flows from these mortgages.

The benefit of the originate-to-distribute model is that it increases liquidity in the sectors of the lending market where it is used. In addition to the residential mortgage market, this model has been applied in other areas such as student loans, credit card balances, and commercial loans and mortgages. For the banks that originate the loans, selling them to other parties is a way of freeing up capital with which they can meet regulatory requirements or make new loans.

A drawback of this model is that, in some cases, it has led banks to loosen lending standards. This was one of the factors that led to the credit crisis in the United States from 2007–2009.

## KEY CONCEPTS

### LO 31.1

The major risks faced by a bank include the following:

- Credit risk from defaults on loans or by counterparties.
- Market risk from declines in the value of trading book assets.
- Operational risk from external events or failure of internal controls.

### LO 31.2

Regulatory capital is the amount of capital that regulators require a bank to hold. This may include equity, or Tier 1 capital, and long-term subordinated debt, or Tier 2 capital.

Economic capital is the amount of capital a bank believes it needs to hold based on its own models. Regulatory capital is typically greater than economic capital.

### LO 31.3

Deposit insurance exists to increase public trust in the banking system. However, it gives rise to moral hazard by decreasing the attention depositors pay to a bank's financial health and increasing the level of risk a bank is willing to take when its depositors are insured.

### LO 31.4

In a private placement, securities are sold directly to qualified investors. In a public offering, securities are sold to the investing public.

When assisting a securities issuer on a best efforts basis, an investment bank sells as much of the issue to the public as it can. In a firm commitment, an investment bank buys an entire issue of securities from the issuer for one price and resells the securities to the public for a higher price. A Dutch auction process may be used to determine a price for an initial public offering.

### LO 31.5

Within a firm that provides commercial banking, investment banking, and securities services, inherent conflicts of interest exist. Information may be acquired in a commercial banking or investment banking transaction that would give the other units an unfair advantage. An investment bank's task of selling newly issued stocks and bonds may conflict with a securities unit's duties to act in the best interests of its clients and recommend trading actions independently.

Bank regulators generally require commercial banking, investment banking, and securities activities to be kept separate, either by preventing firms from engaging in more than one of these activities or by requiring Chinese walls between these units of a bank.

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**LO 31.6**

The banking book refers to loans made by a bank. The balance sheet value of a loan includes the principal amount to be repaid and accrued interest, unless the loan becomes nonperforming, in which case the value does not include accrued interest.

The trading book refers to assets and liabilities related to a bank's trading activities. Trading book items are marked to market daily based on actual market prices when they exist or on estimated prices when necessary.

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**LO 31.7**

The originate-to-distribute model involves banks making loans and selling them to other parties, many of which pool the loans and issue securities backed by their cash flows. This model frees up capital for the originating banks and may increase liquidity in sectors of the loan market. However, it has also led to decreased lending standards and lower credit quality of the loans sold.

## CONCEPT CHECKERS

1. The minimum level of capital a bank needs to maintain, according to its own estimates, models, and risk assessments, is best described as its:
  - A. equity capital.
  - B. financial capital.
  - C. economic capital.
  - D. regulatory capital.
  
2. Which of the following actions in the banking system is most likely intended to address the problem of moral hazard?
  - A. Deposit insurers charge risk-based premiums.
  - B. Banks increase loans to higher-risk borrowers.
  - C. Governments implement deposit insurance programs.
  - D. Banks increase the interest rates they offer to depositors.
  
3. An investment bank is most likely to earn a trading profit from buying and selling securities if it arranges a:
  - A. Dutch auction.
  - B. private placement.
  - C. best efforts offering.
  - D. firm commitment offering.
  
4. The purpose of a “Chinese wall” in banking is to:
  - A. prevent a bank failure from endangering other banks.
  - B. prevent a bank’s departments from sharing information.
  - C. restrict companies from offering both banking and securities services.
  - D. restrict companies from engaging in both commercial and investment banking.
  
5. A drawback of the originate-to-distribute banking model is that it has led to:
  - A. too little liquidity in certain sectors.
  - B. too much liquidity in certain sectors.
  - C. looser credit standards in certain sectors.
  - D. tighter credit standards in certain sectors.

## CONCEPT CHECKER ANSWERS

1. C Economic capital refers to a bank's own assessment of the minimum level of capital it needs to maintain. Economic capital is often less than regulatory capital, which is the minimum level a bank must maintain to comply with capital adequacy regulations.
2. A Charging risk-based premiums is a measure intended to address the problem of moral hazard, which exists when insured parties take greater risks than they would take in the absence of insurance.
3. D With a firm commitment offering, an investment bank buys an entire issue of securities from the issuer and attempts to sell them to the public at a higher price. In a private placement or a best efforts offering, an investment bank earns fee income rather than trading income. A Dutch auction is a method of price discovery for an initial public offering that does not involve buying and reselling shares.
4. B Chinese walls are internal controls to prevent a banking company's commercial banking, securities, and investment banking operations from sharing information.
5. C One drawback to the originate-to-distribute model is that it has led to looser credit standards in certain sectors, such as residential mortgages. A benefit of the model is that it has increased liquidity in certain sectors.



# INSURANCE COMPANIES AND PENSION PLANS

Topic 32

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## EXAM FOCUS

The focus of this topic is primarily on concepts related to life insurance and nonlife (property and casualty) insurance, such as moral hazard, adverse selection, mortality risk, and longevity risk. For the exam, be able to apply mortality tables to perform life expectancy computations and breakeven premium computations for life insurance companies. Also, be able to compute ratios relevant to property and casualty insurance companies. In addition, understand the risks facing insurance companies and be able to discuss specific ways to mitigate them.

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## CATEGORIES OF INSURANCE COMPANIES

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**LO 32.1: Describe the key features of the various categories of insurance companies and identify the risks facing insurance companies.**

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Insurance companies protect policyholders from specific loss events in exchange for the payment of periodic premiums. Three categories of insurance companies include life insurance, property and casualty (nonlife) insurance, and health insurance.

### Life Insurance

Life insurance companies usually provide long-term coverage and make a specified payment to the policyholder's beneficiaries upon the natural death (i.e., certain event) of the policyholder during the policy term. Coverage is also available for accidental death (i.e., uncertain event). **Term** (temporary) **life insurance** provides a specified amount of insurance coverage for a fixed period of time. No payments are made to the policyholder's beneficiaries if the policyholder survives the term of the policy; therefore, payment is not certain. **Whole** (permanent) **life insurance** provides a specified amount of insurance coverage for the life of the policyholder so payment will occur upon death, but there is uncertainty as to the timing. For both term and whole life insurance, it is most common for premiums and the amount of coverage to be fixed for the entire period in question.

In analyzing the relationship between the cost of one year of life insurance and whole life insurance premiums, assume a 30-year-old male purchases a \$2 million whole life policy with an annual premium of \$12,000. Based on mortality tables (as shown in LO 32.2), the probability of death within the year of a 30-year-old male is 0.001467, so the premium for one year of insurance should be \$2,934. The excess of \$9,066 is a surplus premium that is not required to cover the risk of a payout and is therefore invested by the insurance company for the policyholder. The process continues year after year while the cost of a one-



year policy increases as the policyholder ages. Later in the policyholder's life, the one-year policy cost will exceed the annual premium (\$12,000). From an overall perspective, the surplus in the earlier years is offset by the deficit in the later years.

## Property and Casualty (P&C) Insurance

P&C insurance companies usually provide annual and renewable coverage against loss events. The premiums may increase or decrease based on any changes in estimates of expected payout. **Property insurance** covers property losses such as fire and theft. Property insurers may be subject to catastrophic risks arising from many large claims due to natural disasters. Such risks could be managed using geographical, seismographical, and meteorological information to determine the probability and severity of catastrophic events. **Casualty (liability) insurance** covers third-party liability for injuries sustained while on a policyholder's premises or caused by the policyholder's use of a vehicle, for example. Liability insurance is subject to long-tail risk, which is the risk of legitimate claims being submitted years after the insurance coverage has ended. An example could include exposure to cancer-causing substances during the period of coverage but with the symptoms not occurring until years later.

Many property and casualty insurance companies insure a wide variety of risks, which in and of itself is a form of risk diversification. In addition, the expected payouts on claims can be estimated with a high degree of confidence if many policies are written on thousands of independent events. However, property damage claims from natural disasters and liability insurance claims are subject to fluctuating payouts and are very challenging to predict.

## Health Insurance

Health insurance companies provide coverage to policyholders for medical services that are not covered under a publicly funded health care system. Policyholders pay ongoing premiums and the insurance company will make payments for events such as necessary hospital treatment or prescription medication. Premiums may increase due to general increases in health care costs (similar to automobile insurance), but they typically will not increase due to the worsening of the policyholder's health (similar to life insurance). In some cases, insurance coverage may not be denied to individuals with pre-existing medical conditions. Some companies provide group health insurance plans through employers that cover both the employee and the employee's dependents.

## Risks Facing Insurance Companies

Major risks facing insurance companies include the following:

- *Insufficient funds to satisfy policyholders' claims.* The liability computations often provide a significant cushion, but it is always possible to have a sudden surge of payouts in a short period of time.
- *Poor return on investments.* Insurance companies often invest in fixed-income securities and if defaults suddenly increase, insurance companies will incur losses. Diversification of investments by industry sector and geography can help mitigate such losses.

- *Liquidity risk of investments.* Purchasing privately placed fixed-income securities, or publicly traded securities with a thinner market, may result in the inability to easily convert them to cash when most needed to satisfy a surge of claims.
- *Credit risk.* By transacting with banks and reinsurance companies, insurance companies face credit risk if the counterparty defaults on its obligations.
- *Operational risk.* Similar to banks, an insurance company faces losses due to failure of its systems and procedures or from external events outside the company's control (e.g., computer failure, human error).

## MORTALITY TABLES

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### LO 32.2: Describe the use of mortality tables and calculate the premium payment for a policy holder.

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An excerpt from mortality tables estimated by the U.S. Social Security Administration for 2013 is provided in Figure 1.

As an example, examine the row for a male aged 40. The second column indicates that the probability of a 40-year-old male dying within the next year is 0.002092 (or 0.2092%). The third column indicates that the probability of a male surviving to age 40 is 0.95908 (or 95.908%). The fourth column indicates that a 40-year-old male has a remaining life expectancy of 38.53 years so that, on average, he will live to age 78.53. The remaining three columns show the same estimates for a female and they appear slightly better than for a male.

Figure 1: Partial Mortality Table

Age (Years)	Male			Female		
	Probability of Death Within 1 Year	Survival Probability	Life Expectancy	Probability of Death Within 1 Year	Survival Probability	Life Expectancy
0	0.006519	1	76.28	0.005377	1	81.05
1	0.000462	0.99301	75.78	0.000379	0.99462	80.49
2	0.000291	0.99302	74.82	0.000221	0.99425	79.52
3	0.000209	0.99273	73.84	0.000162	0.99403	78.54
30	0.001467	0.97519	47.82	0.000664	0.98635	52.01
40	0.002092	0.95908	38.53	0.001287	0.97753	42.43
41	0.00224	0.95708	37.61	0.001393	0.97627	41.48
42	0.002418	0.95493	36.7	0.001517	0.97491	40.54
43	0.002629	0.95262	35.78	0.001662	0.97343	39.6
50	0.005038	0.9294	29.58	0.003182	0.95829	33.16
51	0.00552	0.92472	28.73	0.003473	0.95524	32.27
52	0.006036	0.91961	27.89	0.003767	0.95193	31.38
53	0.006587	0.91406	27.05	0.004058	0.94834	30.49
60	0.011197	0.86112	21.48	0.006545	0.91526	24.46
61	0.012009	0.85147	20.72	0.007034	0.90927	23.62
62	0.012867	0.84125	19.97	0.007607	0.90287	22.78
63	0.013772	0.83042	19.22	0.008281	0.896	21.95
70	0.023528	0.73461	14.24	0.015728	0.82864	16.43
71	0.025693	0.71732	13.57	0.017338	0.81561	15.68
72	0.028041	0.69889	12.92	0.019108	0.80147	14.95
73	0.030567	0.6793	12.27	0.021041	0.78616	14.23
80	0.059403	0.50629	8.2	0.043289	0.6388	9.64
90	0.167291	0.17735	4.03	0.132206	0.29104	4.8

Source: Social Security Administration, [www.ssa.gov/OACT/STATS/table4c6.html](http://www.ssa.gov/OACT/STATS/table4c6.html)

When examining the full table, the probability of death during the following year is a decreasing function of age until age 10 and then it increases. For an 80-year-old male, the probability of death within the next year is about 5.9% and increases to about 16.7% at age 90.

Some probabilities can be computed indirectly using other numbers in the table. For example, in the third column, the probability of a male surviving to age 70 is 0.73461 and the probability of the male surviving to age 71 is 0.71732. Therefore, the probability of death of a male between age 70 and 71 is  $0.73461 - 0.71732 = 0.01729$  (or about 1.73%). Given that a male reaches age 70, the probability of death within the following year is  $0.01729 / 0.73461 = 0.023536$  (or about 2.35%), which is consistent with the number in the second column.

Going further, the probability of the death of a 70-year-old male in the second year (between ages 71 and 72) is the probability that he does not die in the first year times the probability that he does die in the second year. Using the numbers in the second column, the probability is:  $(1 - 0.023528) \times 0.025693 = 0.025088$  (or about 2.51%).

With the information in the mortality tables, we can calculate the breakeven premium payment by equating the present value of the expected payout to the present value of the expected premium payments.

#### Example: Breakeven premium payments

The relevant interest rate for insurance contracts is 3% per annum (semiannual compounding applies), and all premiums are paid annually at the beginning of the year. A \$500,000 term insurance contract is being proposed for a 60-year-old male in average health. Assuming that payouts occur halfway throughout the year, **calculate** the insurance company's breakeven premium for a one-year term and a two-year term.

**Answer:**

*One-year term:*

The expected payout for a one-year term is  $0.011197 \times \$500,000 = \$5,598.50$ . Assuming the payout occurs in six months, the breakeven premium is:  $\$5,598.50 / 1.015 = \$5,515.76$ .

*Two-year term:*

The expected payout for a two-year term is the sum of the expected payouts in both the first year and the second year. The probability of death in the second year is  $(1 - 0.011197) \times 0.012009 = 0.011874$ , so the expected payout in the second year is  $0.011874 \times \$500,000 = \$5,937.27$ . If the payout occurs in 18 months, then the present value is  $\$5,937.27 / (1.015)^3 = \$5,677.91$ . The total present value of the payouts is then  $\$5,515.76 + \$5,677.91 = \$11,193.67$ .

The first premium payment occurs immediately (i.e., beginning of the first year) so it is certain to be received. However, the probability of the second premium payment being made at the beginning of the second year is the probability of not dying in the first year, which is  $1 - 0.011197 = 0.988803$ . The present value of the premium payments (using  $Y$  as the breakeven premium) is  $Y + (0.988803Y / 1.015^2) = 1.959793Y$ .

Computing the breakeven annual premium equates the present value of the payouts and the premium payments as follows:  $11,193.67 = 1.959793Y$ . Solving for  $Y$ , the breakeven annual premium is  $\$5,711.66$ .

## P&C INSURANCE RATIOS

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**LO 32.3: Calculate and interpret loss ratio, expense ratio, combined ratio, and operating ratio for a property-casualty insurance company.**

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Property and casualty insurance companies compute the following ratios:

- The **loss ratio** for a given year is the percentage of payouts versus premiums generated, usually between 60–80% and increasing over time.
- The **expense ratio** for a given year is the percentage of expenses versus premiums generated, usually between 25–30% and decreasing over time. The largest expenses are usually loss adjustments (e.g., claims investigation and assessing payout amounts) and selling (e.g., broker commissions).
- The **combined ratio** for a given year is equal to the sum of the loss ratio and the expense ratio.
- The **combined ratio after dividends** for a given year is equal to the combined ratio plus the payment of dividends to policyholders (if applicable).
- The **operating ratio** for a given year is the combined ratio (after dividends) less investment income. The mismatch of the cash inflows (generally earlier) and outflows (generally later) for many insurance companies allows them to earn interest income. For example, policyholders tend to pay their premiums upfront at the beginning of the year, but insurance companies tend to pay out claims throughout the year or after year-end.

## MORAL HAZARD AND ADVERSE SELECTION

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**LO 32.4: Describe moral hazard and adverse selection risks facing insurance companies, provide examples of each, and describe how to overcome the problems.**

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**Moral hazard** describes the risk to the insurance company that having insurance will lead the policyholder to act more recklessly than if the policyholder did not have insurance.

An example of moral hazard would be the existence of collision and liability coverage with automobile insurance. As a result of such coverage, some drivers would be willing to drive over the speed limits knowing that if an accident occurs, they would be covered for damage to the car and any resulting injury to a third party. Another example would be the existence of health insurance. As a result, some policyholders may request more health services than necessary.

Methods to mitigate against moral hazard include: deductibles (e.g., policyholder is responsible for a fixed amount of the loss), coinsurance provisions (e.g., insurance company will pay a fixed percentage of losses, less than 100%, over the deductible amount), and policy limits (e.g., fixed maximum payout).

**Adverse selection** describes the situation where an insurer is unable to differentiate between a good risk and a bad risk. By charging the same premiums to all policyholders, the insurer may end up insuring more bad risks (e.g., careless drivers, sick individuals).

Methods to mitigate against adverse selection include: (1) greater initial due diligence (e.g., mandatory physical examinations for life insurance, researching driving records for



automobile insurance) and (2) ongoing due diligence (e.g., updating driving records and adjusting premiums to reflect changing risk).

## MORTALITY RISK VS. LONGEVITY RISK

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**LO 32.5: Distinguish between mortality risk and longevity risk and describe how to hedge these risks.**

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**Mortality risk** refers to the risk of policyholders dying earlier than expected due to illness or disease, for example. From the perspective of the insurance company, the risk of losses increases due to the earlier-than-expected life insurance payout.

**Longevity risk** refers to the risk of policyholders living longer than expected due to better healthcare and healthier lifestyle choices, for example. From the perspective of the insurance company, the risk of losses increases due to the longer-than-expected annuity payout period.

## Hedging Mortality and Longevity Risks

There is a natural hedge (or offset) for insurance companies that deal with both life insurance products and annuity products. For example, longevity risk is bad for the annuity business but is good for the life insurance business due to the delayed payout (or no payout if the policyholder has term insurance and dies after the policy expires). Mortality risk is bad for the life insurance business but is good for the annuity business because of the earlier-than-expected termination of payouts.

To the extent that there is excessive net exposure to mortality risk, longevity risk, or both, an insurance company may consider **reinsurance contracts**. With this type of contract, the insurance company pays a fee to another insurance company to assume some or all of the risks that were originally insured.

Longevity derivatives are used to hedge longevity risk inherent in annuity contracts and defined benefit pensions. A good example would be a longevity bond (or a survivor bond) whereby the bond coupon is set to an amount that is linked to the number of people in a defined population group that are still alive.

## CAPITAL REQUIREMENTS FOR INSURANCE COMPANIES

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**LO 32.6: Evaluate the capital requirements for life insurance and property-casualty insurance companies.**

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A life insurance company might have the following summarized balance sheet composition:

- *Assets*: investments (80%), other assets (20%)
- *Liabilities and Equity*: policy reserves (85%), subordinated long-term debt (5%), equity capital (10%)

Under an asset-liability management approach, the life insurance company attempts to equate asset duration with liability duration. There is risk associated with both sides of the

balance sheet. On the asset side, corporate bonds comprise the bulk of the investments, so there is credit risk assumed. On the liability side, the policy reserves represent the present value of the future payouts as determined by actuaries. The risk is that the policy reserves are set too low if life insurance policyholders die too soon or annuity holders live too long. Equity capital represents contributed capital plus retained earnings and serves as a protection barrier if payouts are larger than loss reserves.

A P&C insurance company might have the following summarized balance sheet composition:

- *Assets*: investments (80%), other assets (20%)
- *Liabilities and Equity*: policy reserves (50%), unearned premiums (10%), subordinated long-term debt (5%), equity capital (35%)

On the asset side, the investments typically comprise of highly liquid bonds with shorter maturities than those used by life insurance companies. On the liability side, the unearned premiums represent prepaid insurance contracts whereby amounts are received but the coverage applies to future time periods; unearned premiums do not generally exist for life insurance companies. Finally, there is substantially more equity capital for a P&C insurance company than for a life insurance company. This is due to the highly unpredictable nature of claims (both timing and amount) for P&C insurance contracts.

## **GUARANTY SYSTEM FOR INSURANCE COMPANIES**

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### **LO 32.7: Compare the guaranty system and the regulatory requirements for insurance companies with those for banks.**

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In the United States, a **guaranty system** exists for both insurance companies and banks. Insurance companies are regulated at the state level while banks are regulated at the federal level.

For insurance companies, every insurer must be a member of the guaranty association in the state(s) in which it operates. If an insurance company becomes insolvent in a state, each of the other insurance companies must contribute an amount to the state guaranty fund based on the amount of premium income it earns in that state. The guaranty fund proceeds are distributed to the small policyholders of the insolvent company. In some cases, an annual limit may apply with regard to the contribution, which may contribute to a delay in accumulating sufficient funds to pay all of the policyholders. Most frequently, the policyholders of insolvent life insurance companies are transferred to other life insurance companies.

In contrast, the guaranty system for banks is a permanent fund to protect depositors and consists of amounts remitted by banks to the Federal Deposit Insurance Corporation (FDIC). No such permanent fund generally exists for insurance companies; therefore, insurance companies must make contributions whenever a default occurs.

## PENSION FUNDS

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**LO 32.8: Describe a defined benefit plan and a defined contribution plan for a pension fund and explain the differences between them.**

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Many companies establish pension plans on behalf of their employees with contributions being made by both parties. Upon retirement, the employee will receive periodic pension payments for the remainder of her life.

**Defined benefit plans** (i.e., employee benefit known, employer contribution unknown) explicitly state the amount of the pension that the employee will receive upon retirement. It is usually calculated as a fixed percentage times the number of years of employment times the annual salary for a specific period of time. There is significant risk borne by the employer because it is obligated to fund the benefit to the employee; therefore, when the present value of the pension obligation exceeds the market value of the pension assets, the employer must cover the deficiency. As a result, there is no risk borne by the employee (in theory). Additionally, some defined benefit plans may include one or more of the following features: (1) indexation of pension amounts to account for inflation, (2) continued pension payments (likely on a reduced basis) to the surviving spouse upon the death of a retired employee, or (3) a lump sum payment to an employee's dependents upon the death of a currently active employee.

**Defined contribution plans** (i.e., employer contribution known, employee benefit unknown) involve both employer and employee contributions being invested in one or more investment options selected by the employee. Upon retirement, the employee could opt to receive a lifetime pension (based on the ending value of the contributions) in the form of an annuity or, in some cases, simply to receive a lump sum. There is virtually no risk borne by the employer because it is obligated simply to make a set contribution and no more. The risk of underperformance of the plan's investments is borne solely by the employee.

A defined contribution plan involves one individual account associated with one employee. The individual pension is computed based only on the funds in that account. In contrast, a defined benefit plan involves one pooled account for all employees; all contributions go into and all payments come out of the one account.



## KEY CONCEPTS

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### LO 32.1

Three categories of insurance companies include life insurance, nonlife [property and casualty (P&C)] insurance, and health insurance. Life insurance companies usually provide long-term coverage and will make a specified payment to the policyholder's beneficiaries upon the death of the policyholder during the policy term. Term (temporary) life insurance provides a specified amount of insurance coverage for a fixed period of time. Whole (permanent) life insurance provides a specified amount of insurance coverage for the life of the policyholder.

Risks facing insurance companies include: (1) insufficient funds to satisfy policyholders' claims, (2) poor return on investments, (3) liquidity risk of investments, (4) credit risk, and (5) operational risk.

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### LO 32.2

Mortality tables can be used to compute life insurance premiums. Mortality tables include information related to the probability of an individual dying within the next year, the probability of an individual surviving to a specific age, and the remaining life expectancy of an individual of a specific age.

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### LO 32.3

P&C insurance companies compute the following ratios:

loss ratio + expense ratio = combined ratio

combined ratio + dividends = combined ratio after dividends

combined ratio after dividends – investment income = operating ratio

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### LO 32.4

Moral hazard describes the risk to the insurance company that having insurance will lead the policyholder to act more recklessly than if the policyholder did not have insurance. Methods to mitigate moral hazard include deductibles, coinsurance, and policy limits.

Adverse selection describes the situation where an insurer is unable to differentiate between a good risk and a bad risk. Methods to mitigate adverse selection include greater initial due diligence and ongoing due diligence.

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### LO 32.5

Mortality risk refers to the risk of policyholders dying earlier than expected. For the insurance company, the risk of losses increases due to the earlier-than-expected life

insurance payouts. Longevity risk refers to the risk of policyholders living longer than expected. For the insurance company, the risk of losses increases due to the longer-than-expected annuity payout period.

There is a natural hedge (or offset) for insurance companies that deal with both life insurance products and annuity products because longevity risk is bad for the annuity business but good for the life insurance business, and mortality risk is bad for the life insurance business but good for the annuity business. Other forms of hedging include reinsurance contracts with other insurance companies and longevity derivatives.

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### LO 32.6

Under an asset-liability management approach, the life insurance company attempts to equate asset duration with liability duration. There is risk associated with both sides of the balance sheet. Equity capital represents contributed capital plus retained earnings and serves as a protection barrier if payouts are larger than loss reserves.

For P&C insurance companies, assets typically comprise of highly liquid bonds with shorter maturities than those used by life insurance companies. On the liability side, there are unearned premiums (non-existent with life insurance companies) that represent prepaid insurance contracts whereby amounts are received but the coverage applies to future time periods. Finally, there is substantially more equity capital than for a life insurance company because of the highly unpredictable nature of claims for P&C insurance contracts.

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### LO 32.7

For insurance companies in the United States, every insurer must be a member of the guaranty association in the state(s) in which it operates. If an insurance company becomes insolvent in a state, then each of the other insurance companies must contribute an amount to the state guaranty fund based on the amount of premium income it earns in that state.

The guaranty system for banks in the United States is a permanent fund to protect depositors that consists of amounts remitted by banks to the Federal Deposit Insurance Corporation (FDIC). No such permanent fund exists for insurance companies.

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### LO 32.8

Defined benefit plans explicitly state the amount of the pension that the employee will receive upon retirement. It is usually calculated as a fixed percentage times the number of years of employment times the annual salary for a specific period of time. There is significant risk borne by the employer because it is obligated to fund the benefit to the employee.

Defined contribution plans involve both employer and employee contributions being invested in one or more investment options selected by the employee. There is virtually no risk borne by the employer because it is obligated simply to make a set contribution and no more. The risk of underperformance of the plan's investments is borne solely by the employee.

## CONCEPT CHECKERS

1. Which of the following forms of insurance is most likely subject to long-tail risk?
  - A. Health insurance.
  - B. Life insurance.
  - C. Liability insurance.
  - D. Property insurance.
  
2. The relevant interest rate for insurance contracts is 2% per annum (semiannual compounding applies) and all premiums are paid annually at the beginning of the year. A \$2,000,000 term insurance contract is being proposed for a 40-year-old male in average health. Assume that payouts occur halfway throughout the year. Using the mortality rates estimated by the U.S. Social Security Administration (in Figure 1 on page 13), which of the following amounts is closest to the insurance company's breakeven premium for a two-year term?
  - A. \$4,246.
  - B. \$4,287.
  - C. \$4,332.
  - D. \$8,482.
  
3. The following information pertains to a property and casualty (P&C) insurance company:

Investment income	5%
Dividends	2%
Loss ratio	74%
Expense ratio	23%

Based on the information provided, what is this company's operating ratio?

  - A. 90%.
  - B. 94%.
  - C. 97%.
  - D. 99%.
  
4. Which of the following problems would most likely be a concern for life insurance companies that are worried about differentiating between good risks and bad risks?
  - A. Adverse selection.
  - B. Catastrophic risk.
  - C. Longevity risk.
  - D. Moral hazard.
  
5. Which of the following statements regarding the capital requirements and regulation of insurance companies is correct?
  - A. Insurance companies are regulated at both the state and federal level.
  - B. The guaranty system for insurance companies consists of a permanent fund created from premiums paid by insurers.
  - C. Unearned premiums can be found on the balance sheets of both life insurance and property and casualty insurance companies.
  - D. The amount of equity on the balance sheet of a life insurance company is typically lower than that of a property and casualty insurance company.

## CONCEPT CHECKER ANSWERS

1. C Liability insurance is subject to long-tail risk, which is the risk of legitimate claims being submitted years after the insurance coverage has ended. An example could include exposure to cancer-causing substances during the period of coverage but with the symptoms not occurring until years later.

2. B One-year term:

The expected payout for a one-year term is  $0.002092 \times \$2,000,000 = \$4,184$ . Assuming the payout occurs in six months, the breakeven premium is  $\$4,184 / 1.01 = \$4,142.57$ .

Two-year term:

The expected payout for a two-year term is the sum of the expected payouts in both the first year and the second year. The probability of death in the second year is  $(1 - 0.002092) \times 0.00224 = 0.0022353$ , so the expected payout in the second year is  $0.0022353 \times \$2,000,000 = \$4,470.63$ . If the payout occurs in 18 months, then the present value is  $\$4,470.63 / (1.01)^3 = \$4,339.15$ . The total present value of the payouts is then  $\$4,142.57 + \$4,339.15 = \$8,481.72$ .

The first premium payment occurs immediately (i.e., beginning of the first year) so it is certain to be received. However, the probability of the second premium payment being made at the beginning of the second year is the probability of not dying in the first year, which is  $1 - 0.002092 = 0.997908$ . The present value of the premium payments is as follows (using Y as the breakeven premium):  $Y + (0.997908Y / 1.01^2) = 1.978245Y$ .

Computing the breakeven annual premium equates the present value of the payouts and the premium payments as follows:  $8,481.72 = 1.978245Y$ . Solving for Y, the breakeven annual premium is  $\$4,287.50$ .

Response A ( $\$4,246$ ) is not correct because it performs the computation on the assumption that all payouts occur at the end of the year instead of halfway throughout the year. Response C ( $\$4,332$ ) is not correct because it did not apply any discounting (at the 1% semiannual rate). Response D ( $\$8,482$ ) is not correct because it is simply the total present value of the payouts.

3. B The operating ratio is computed as follows:

$$\begin{aligned} & \text{loss ratio (74\%)} + \text{expense ratio (23\%)} + \text{dividends (2\%)} - \text{investment income (5\%)} \\ & = 94\% \end{aligned}$$

The combined ratio is computed as follows:

$$\text{loss ratio (74\%)} + \text{expense ratio (23\%)} = 97\%$$

The combined ratio after dividends is computed as follows:

$$\text{loss ratio (74\%)} + \text{expense ratio (23\%)} + \text{dividends (2\%)} = 99\%$$

4. A Adverse selection describes the situation where an insurer is unable to differentiate between a good risk and a bad risk. In the context of life insurance, by charging the same premiums to all policyholders (healthy and unhealthy individuals), the insurer may end up insuring more bad risks (e.g., unhealthy individuals). To mitigate adverse selection, a life insurance company might require physical examinations prior to providing coverage.
5. D Property and casualty insurance companies typically have a greater amount of equity than a life insurance company because of the highly unpredictable nature of P&C claims (both timing and amount).

Insurance companies are regulated at the state level only (and banks are regulated at the federal level only). The guaranty system for insurance companies is not a permanent fund; in contrast, banks have a permanent fund created from premiums paid by banks to the FDIC. On the liability side of a property and casualty insurance company's balance sheet, there are unearned premiums that represent prepaid insurance contracts whereby amounts are received but the coverage applies to future time periods. Unearned premiums do not exist with life insurance companies.

# MUTUAL FUNDS AND HEDGE FUNDS

Topic 33

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## EXAM FOCUS

Not every investor has the time or the skill to manage his own financial assets. For this reason, investors will sometimes hire a professional manager in the form of a mutual fund or perhaps a hedge fund. These pooled investment vehicles offer instant diversification and professional management to their investors. Mutual funds are often used by smaller investors while hedge funds are tools for wealthy individuals. Because hedge funds are limited only to those who can afford to lose their investment, they are subject to much less regulation. For the exam, be able to describe the various types of mutual funds and hedge funds along with their regulatory environments and typical fee structures.

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## TYPES OF MUTUAL FUNDS

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**LO 33.1: Differentiate among open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs).**

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**Mutual funds** are pooled investment vehicles that offer instant diversification for their investors. This diversification is very important because it spreads out risk to different sectors and asset classes. Most investors either do not have the time or the skill to properly diversify on their own. For this reason, investment vehicles like open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs) were created.

## Open-End Mutual Funds

Open-end mutual funds, which are often simply called *mutual funds*, are the most common pooled investment vehicle. Figure 1 shows the growth in open-end mutual fund assets since World War II. Essentially, investors are commingling their funds to be better diversified, to save on transaction fees, and to hire a professional management team. The professional management team will conduct research and ultimately invest commingled assets on behalf of their investors. These investors begin their investment by purchasing a set dollar amount of an open-end mutual fund and then they receive a proportional ownership interest (in the form of shares) in the mutual fund. This means that the number of shares goes up as new investors arrive and goes down as investors withdraw assets. When investors decide that they want to exit their investment in an open-end mutual fund, they can redeem their shares directly from the fund company, who will promptly send them either a check or a digital transfer of the value of their investment.



Figure 1: Growth of Mutual Fund Assets

<i>Year</i>	<i>Invested Assets (\$ billions)</i>
1940	0.5
1960	17.0
1980	134.8
2000	6,964.6
2015	15,652.0

Source: Investment Company Institute

At a high level, open-end mutual funds are broken down into four main categories: money market funds, equity funds, bond funds, and hybrid funds. Money market funds invest in short-term interest-bearing instruments, such as Treasury bills, commercial paper, and banker's acceptances. Money market investors are typically risk averse. This category is an alternative to interest-bearing bank accounts and is often the “cash” portion of an investor's asset allocation mix. Equity funds invest solely in stocks. Within this category you can find index funds that track a broad market index, such as the S&P 500 Index, funds that follow a certain style, such as medium company value funds, or sector funds, such as a health care sector fund. Bond funds invest only in fixed-income instruments, such as sovereign debt, corporate bonds, and asset-backed securities. Hybrid funds will blend stock and bond ownership into the same fund.

Open-end funds trade at the fund's **net asset value (NAV)**, which is essentially the sum of all assets owned minus any liabilities of the fund then divided by the shares outstanding. When investors decide they want to buy shares of an open-end mutual fund, they will transact at the next available NAV, which is not actually calculated until after the market closes at 4:00 pm in New York City. An investor who decides at 10:00 am that they want to buy shares will enter a buy order for a set dollar amount, but they will not know the price at which they will transact until after the market closes. For this reason, we say that open-end fund investors have poor price visibility. Since shares are transacted at an unknown price, investors cannot use stop orders or limit orders. They must place a market order to transact in shares of an open-end mutual fund.

Taxes are levied against open-end mutual fund investors as if they owned the diversified fund's holdings outright. If the underlying investment pays a dividend, then the investors must pay taxes on their proportional ownership interest in that dividend. The open-end fund may also buy and sell underlying investments and generate taxable short-term or long-term capital gains. These taxable events are also passed on to investors. Dividends and capital gains are distributed to investors typically toward the end of the calendar year, but they can be automatically reinvested in the fund to purchase more shares. Investors often choose reinvestment if they do not need the cash flow for current consumption.

The cost of investing is also a major consideration for any investment category. Open-end mutual funds have a management fee, an advertising surcharge (called a 12b-1 fee), and potentially a sales charge. The management fee covers the operational costs of the open-end mutual fund company, including the salaries of the management team. Management fees are typically around 1.0%, but they can be as high as 2.5–3.0% for international funds because they have increased complexity. The advertising surcharge is a stipend paid to the advisor who recommends the investment, and these fees can range from 0.0–1.0% with



the most common fee being 0.25%. Sales charges are commonly called *loads*. A **front-end load** is a set percentage that is charged to the investor when the asset is originally sold. Alternatively, some funds choose to charge a sales charge if an investor leaves a fund within a certain window of time. This is called a **back-end load**. Figure 2 shows the average cost of ownership for an open-end mutual fund per year over a five-year holding period.

**Figure 2: Average Total Cost of Ownership (% of Assets)**

<i>Country</i>	<i>Bond Funds</i>	<i>Equity Funds</i>
Australia	0.75	1.41
Austria	1.55	2.37
Belgium	1.60	2.27
Canada	1.84	3.00
Denmark	1.91	2.62
Finland	1.76	2.77
France	1.57	2.31
Germany	1.48	2.29
Italy	1.56	2.58
Luxembourg	1.62	2.43
Netherlands	1.73	2.46
Norway	1.77	2.67
Spain	1.58	2.70
Sweden	1.67	2.47
Switzerland	1.61	2.40
United Kingdom	1.73	2.48
United States	1.05	1.53
<i>Average</i>	1.39	2.09

Source: Khorana, Servaes, and Tufano, “Mutual Fund Fees Around the World,” *Review of Financial Studies* 22 (March 2009): 1279–1310.

### Closed-End Mutual Funds

Closed-end mutual funds are a similar concept to open-end funds with a few notable differences. The first difference is that closed-end funds tend to invest in niche areas like specific emerging markets, while open-end mutual funds tend to invest in broader areas like a diversified emerging markets fund. Consider the difference between the China Fund (CHN), which is a closed-end fund that only invests in China, and the Vanguard Emerging Markets Index Fund (VEMAX), which is an open-end mutual fund with only 28.7% invested in China as of October 1, 2016.

The second difference is that a purchase of shares in an open-end mutual fund will increase the number of shares outstanding because new shares are created, but a closed-end fund's number of shares remains static. Investors who desire to purchase shares of a closed-end fund do not transact directly with the fund company but rather with other investors. Recall that investors who want to close their investment position in an open-end fund can simply redeem their shares from the fund company. This is where the fund gets the name “open-end.”

The third difference is that closed-end fund investors cannot simply redeem their shares from the fund company. They must find another investor to buy their shares. This process is streamlined using a broker like Charles Schwab or Merrill Lynch.

The fourth difference is that, while open-end funds always transact at the next available NAV, a closed-end fund can transact at a price other than NAV. It is very common for a closed-end fund to trade at either a discount or a premium to its actual NAV. It then becomes important to know the historical norms for a closed-end fund's discount or premium before buying. For example, if a certain closed-end fund normally trades at a discount of 10% to its NAV but is now trading at a discount of only 3%, then it may still be overvalued and the investor would be wise to wait to make an investment.



*Professor's Note: In terms of trading, a closed-end fund behaves much like an individual stock. Investors can trade closed-end funds throughout the trading day, which means they have better price visibility and can utilize stop orders and limit orders if they so choose.*

## Exchange-Traded Funds

Exchange-traded funds (ETFs) represent an innovative twist on the open-end mutual fund. They enable instant diversification like an open-end fund, but they are exchange-traded, which means they trade throughout the day on the open market just like a closed-end fund does. Because they trade throughout the day, investors can utilize stop orders, limit orders, and even short selling in some cases.

A few ETFs also have call options and put options available. Unlike a closed-end fund, ETFs typically trade at their NAV. The vast majority of ETFs are passively managed index funds, although some new actively managed ETFs are beginning to come to market. One of the most widely known ETFs is the SPDR S&P 500 Index Fund (SPY).

Exchange-traded funds must disclose their holdings twice each day, which enables investors to have tremendous visibility into their underlying investments. Open-end mutual funds, on the other hand, disclose their holdings very infrequently, perhaps as delayed as once per quarter.

Another big difference is the management fees. Exchange-traded funds often have a considerably lower internal expense ratio, which means less of a hurdle for the investment to rise above. Lower fees equal higher potential after-fee returns for investors.

Because open-end funds, closed-end funds, and exchange-traded funds all solicit investment from small retail customers, they are subject to significant regulatory oversight. They are all regulated by the Securities and Exchange Commission (SEC) and must register with the SEC and provide a very detailed disclosure document, called a prospectus, to all investors prior to investing. The SEC also enforces the prevention of conflicts of interest, fraud, and excessive fees. Regulatory oversight theoretically helps protect investors and causes increased costs for the funds as they hire compliance specialists to ensure that all regulations are being followed.

## NET ASSET VALUE

### LO 33.2: Calculate the net asset value (NAV) of an open-end mutual fund.

In order to calculate the net asset value (NAV), the fund needs to know the current value of all investment holdings (including cash positions), any liabilities like management fees payable, and the total number of shares outstanding. Calculation of the NAV is shown as follows:

$$\text{NAV} = \frac{\text{fund assets} - \text{fund liabilities}}{\text{total shares outstanding}}$$

#### Example: Computing NAV

Consider an open-end mutual fund that owns \$1.1 billion in equities, \$350 million in bonds, and \$35 million in cash. They owe \$1.85 million in management fees payable at this point in the quarter and they have 39.635 million shares outstanding. Calculate this fund's NAV.

**Answer:**

$$\$37.42 = \frac{(\$1,100 + \$350 + \$35) - 1.85}{39.635}$$

Investors who wish to buy or sell this fund will transact at exactly \$37.42 per share, which is not calculated until after the market closes on the trading day in question. If they wanted to invest \$25,000, then they would buy exactly 668.092 (= \$25,000 / \$37.42) shares after the market closes on the relevant trading day.

Recall that the NAV for an open-end mutual fund is only calculated after the close of trading on any given day, while the NAV for closed-end funds and exchange-traded funds is calculated continuously throughout the day.

## HEDGE FUNDS VS. MUTUAL FUNDS

### LO 33.3: Explain the key differences between hedge funds and mutual funds.

Hedge funds and mutual funds share some common characteristics, but several nuances between them are very different. Both hedge funds and mutual funds offer professional management, instant diversification, and the ability to commingle funds with other investors. However, mutual funds are marketed to any and all investors, while hedge funds are restricted to only wealthy and sophisticated investors. Because of this, hedge funds

escape certain regulations that apply to mutual funds. Specifically, they do not need to provide the redemption of shares at any time the investor chooses, a daily calculated NAV, or the full disclosure of their investment policies and strategies. Hedge funds are also permitted to use leverage while mutual funds are not. Because hedge funds can use leverage and are also permitted to use both long and short investment strategies, they are considered to be an alternative investment class.



*Professor's Note: The term "hedge fund" implies that the fund is hedging some form of risk. This may be the case if the fund is using both long and short positions, but not all hedge funds focus on risk reduction. Some, like distressed debt funds, actually focus on risk enhancement.*

Hedge funds have become much more mainstream for institutional investors as of late. One attraction is that many hedge funds have registered in tax-favorable jurisdictions. For example, a little over 30% of all hedge funds are domiciled in the Cayman Islands. Institutional investors have been using hedge funds to invest in short-selling, convertible debt instruments, credit default swaps, distressed debt, non-investment grade bonds, and sometimes illiquid assets.

Since hedge funds are not required to redeem shares any time an investor requests, they have implemented advance notification requirements and lock-up periods for any withdrawal requests. The advance notification could mean that the investor must wait 90 days after requesting a withdrawal before they can expect to have access to their money. The **lock-up period** is a certain amount of time in which the investor is not able to withdraw his funds. This could be one year, two years, or some other customized time period. The advance notification time period and the lock-up period will be disclosed to investors before they invest. They exist for one key reason—many hedge fund investments are not very easy to unwind on short notice. Some hedge fund investments are illiquid, which means managers cannot sell them quickly and retain a proper value. In addition, some hedge fund investments are bets on certain asset mispricing, and those trades can take time to unwind.

## HEDGE FUND EXPECTED RETURNS AND FEE STRUCTURES

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**LO 33.4: Calculate the return on a hedge fund investment and explain the incentive fee structure of a hedge fund including the terms hurdle rate, high-water mark, and clawback.**

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While mutual funds charge fees as a set percentage of assets under management (AUM), hedge funds deploy a more complex compensation structure centered around incentive fees. These **incentive fees** are engineered to give hedge fund managers significant payouts based on their performance. The typical hedge fund fee structure is known as "**2 plus 20%**," which means that they charge a flat 2% of all assets that they manage plus an additional 20% of all profits above a specified benchmark. This compares to a typical American open-end equity mutual fund that charges roughly 1.5% and an exchange-traded fund that typically charges less than 0.5%. This fee differential can prove very lucrative for hedge fund managers. In 2015, the top 25 hedge funds earned in excess of \$12 billion dollars from incentive fees.

Hedge funds do soften the incentive fee structure with a few safeguards for investors. The first safeguard is the **hurdle rate**, which is the benchmark that must be beaten before incentive fees can be charged. The hurdle rate could be zero (used for absolute return strategies), Treasuries plus a premium, LIBOR plus a premium, or some other custom benchmark. It is usually not the S&P 500 Index.

The second safeguard is a **high-water mark clause**, which essentially states that previous losses must first be recouped and hurdle rates surpassed before incentive fees once again apply. Consider a hedge fund that just began with \$100 million in assets from investors. Their hurdle rate is the 10-year Treasury, which is currently yielding 1.5%. In the first year of operation, this hedge fund made some bad decisions and ended up losing \$10 million (ending balance of \$90 million). This means that the managers get to charge the 2% flat fee, but no incentive fees apply. Incentive fees would only have applied to any profits earned above a 1.5% return, meaning that only an ending balance higher than \$101.5 million would have triggered the 20% incentive fee. In year two, this hedge fund would need to get its fund up above \$103 million (two years of beating Treasuries) in order for incentive fees to apply. In this case, the high-water mark for year one is \$101.5 million, and for year two it is \$103 million.

The third safeguard for investors is a **clawback clause**, which enables investors to retain a portion of previously paid incentive fees in an escrow account that is used to offset investment losses should they occur.

The incentive fee structure of a hedge fund certainly encourages hedge fund managers to reach for profits, but this comes at the expense of also encouraging them to take risks. A hedge fund manager essentially owns a call option against the assets of the hedge fund and payoff for options are higher if volatility is higher. Consider an example where a hedge fund manager is presented with an opportunity that offers a 40% probability of returning 50% and a 60% probability of losing 50%. The expected return of the fund can be calculated as follows:

$$(0.4 \times 50\%) + (0.6 \times -50\%) = -10\%$$

In this example, the hedge fund manager might be willing to take a big risk (60% probability) of losing money, which would end in him only collecting his 2% flat fee. The alternative is that if he were to end up making a huge return with the lower probability event, then he would potentially earn a substantial incentive fee. If this hedge fund generates a 50% profit, then he could potentially earn fees of 11.6% [= 2% (flat fee) + 0.20 × 48% (incentive fee on return above the 2% flat fee)]. The expected payoff for fees then becomes 5.84%:

$$(0.4 \times 11.6\%) + (0.6 \times 2\%) = 5.84\%$$

From the investor's perspective, the expected payoff is -15.84%:

$$[0.4 \times (50\% - 11.6\%)] + [0.6 \times (-50\% - 2\%)] = 0.1536 - 0.312 = -15.84\%$$



The expected return for the hedge fund is 5.84% and the expected return for the hedge fund investor is -15.84%. When these two numbers are added together, we arrive back at the original return of -10%. This shows the disproportionate payoff for the hedge fund manager. Why would investors be willing to make this investment? Clearly they are hoping that the incentive fees will motivate the hedge fund manager to do everything within their power to produce significant returns for both the investor and the hedge fund manager.

## HEDGE FUND STRATEGIES

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**LO 33.5: Describe various hedge fund strategies, including long/short equity, dedicated short, distressed securities, merger arbitrage, convertible arbitrage, fixed income arbitrage, emerging markets, global macro, and managed futures, and identify the risks faced by hedge funds.**

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Hedge funds deploy numerous different strategies in their attempt to capture incentive fees. Not all hedge funds fall easily into a specific category, but the discussion in this section follows the classification system used by the Dow Jones, which provides indices to track various hedge fund strategies. Throughout this section, you will see the term **arbitrage**, which (in the hedge fund context) involves short selling an asset that is believed to be overvalued and buying an asset that is believed to be undervalued in an attempt to exploit a pricing differential.

### Long/Short Equity

Long/short equity hedge funds endeavor to find mispriced securities. Managers of a long/short equity fund spend a great deal of time conducting fundamental analysis on stocks, that are largely ignored by most analysts, in an attempt to find mispricings. They will buy (go long) a stock that they believe to be undervalued, and they will short sell (go short) a stock that they believe to be overvalued. Sometimes funds can have a net long bias or a net short bias depending on what opportunities they see in the markets. Funds can also be sector neutral, where they net long and short positions that cancel out sector exposure. *Market neutral funds* are where long and short positions make the fund ambivalent to market direction, and *factor neutral funds* are where positions are isolated from a specific factor like oil or interest rate policy.

### Dedicated Short

Dedicated short hedge funds are focused exclusively on finding a company that they think is overvalued and then short selling the stock. Traditionally, short sellers are looking for companies with weak financials, those that switch auditors frequently, those that delay SEC filings, those in industries with overcapacity, or those engaged in lawsuits that could go horribly wrong.

### Distressed Securities

Bonds with a credit rating of BB or less are considered to be “junk” bonds, while those with a CCC rating are considered to be “distressed.” Distressed bonds usually trade at deep discounts to par value and often offer yields upwards of 10% greater than a comparable

Treasury. Of course, an investment in a distressed bond could prove worthless if the wrong events happen. Distressed securities hedge funds are searching for distressed bonds with the potential to turn things around. Many of these distressed companies are in or close to being in bankruptcy proceedings. Some distressed bond investors passively wait for the investment to turn around, while others take an active approach to influencing the target company's reorganization. Distressed bond investors do their homework to figure out if they can gain an advantage by buying specific debt tranches. If they own more than one-third of any class of a bond, then they can block any reorganization plan that is not in their best interest. There is tremendous profit to be made in this area for investors who know what they are doing.

### Merger Arbitrage

Merger arbitrage hedge funds try to find arbitrage opportunities after mergers are announced. These are primarily positive deals where the managers are planning on the deal going through. There are two different types of mergers: *cash deals* and *stock deals*.

Consider an all-cash deal where company A announces that it will buy company B at \$50 per share. Pre-announcement, company B was trading at \$37.50 and post-announcement company B will typically be trading somewhere near \$48. Why not at \$50, which is where the deal was announced? It could be because some market participants are slow to adjust to the new information, but it could also be that there is some cushion left in company B's price on the chance that the deal falls through. Either way, a merger arbitrage fund would buy the shares of company B and wait for the full \$50 (or better) price to be achieved.

Now consider an all-stock deal where company A offers one share of its stock for every four shares of company B's stock. This could be a realistic ratio if company B had a considerably lower market capitalization than company A. In this case, a merger arbitrage fund would buy a certain amount of company B's shares and, at the same time, they would short sell one-quarter of this number of shares in company A's stock. This is because the acquirer usually pays too much and their stock usually goes down after a merger. The merger arbitrage strategy is a very lucrative strategy, but there is also great potential for insider trading issues. The SEC will actively pursue insider trading violations, so these managers must make certain that they are only factoring public information into their decision-making process.

### Convertible Arbitrage

Some hedge funds invest using convertible bonds, which are fixed-income instruments that can be converted into shares of stock if the stock price rises above a pre-specified value. If convertible bonds are not converted into shares of stock, then they simply retain their bond status and continue to offer interest payments and a certain principal repayment at maturity. This debt instrument conceptually merges a bond with a call option on the stock. Sometimes, if the convertible bond is also callable, the issuer will announce its intention to call the bond in order to force convertible bondholders to convert to stock. A conversion into stock will shift the investor from being a debtholder to an equity holder and will therefore reduce the debt burden of the issue without them actually repaying any debt. A convertible arbitrage hedge fund develops a sophisticated model to value convertible bonds that factors everything from default risk to interest rate risk. Sometimes they offset



investment risk by shorting the issuer's stock or by using more sophisticated assets like credit default swaps and interest rate swaps.

### Fixed Income Arbitrage

Fixed income arbitrage hedge funds attempt to exploit perceived mispricings in the realm of fixed-income securities. Some hedge funds try to find arbitrage opportunities in estimating shifts in the Treasury yield curve. Others look for overvalued and undervalued positions with U.S. Treasuries, with other country's sovereign debt obligations, or with credit default swap rates. However, this strategy is risky business. Recall that Long-Term Capital Management (LTCM) realized 40% plus returns using this strategy in the 1990s only to have their trades move the wrong way and cause such panic in the broader markets that several Wall Street banks had to bail them out. The LTCM scenario turned from a profitable fixed income arbitrage opportunity to a significant systemic risk overnight. Some have compared fixed income arbitrage to picking up nickels in front of a steamroller.

### Emerging Market

Emerging market hedge funds focus on investments in developing countries. These managers often expend great effort to research their investments by visiting potential investment targets, attending conferences, meeting with analysts, talking directly with management, and possibly hiring consultants with local knowledge. Some hedge funds choose to invest in developing country securities in their local market while others invest using *American depository receipts* (ADRs), which are certificates issued in America that provide ownership in foreign countries coupled with currency exposure. There are occasionally pricing discrepancies between the ADR and the underlying asset that an adept hedge fund manager can exploit as well. If managers decide to invest using emerging market debt, then they need to consider default risk because several countries have defaulted multiple times, including Russia, Argentina, Brazil, and Venezuela.

### Global Macro

Several of the most financially successful hedge fund managers have made their fortunes with a global macro hedge fund strategy. In this strategy, hedge fund managers attempt to profit from a global macroeconomic trend that they feel is not in equilibrium (priced correctly and rationally). They will place very large dollar bets on the equilibrium being reestablished. Typically, the investment focus of global macro funds is either on foreign exchange rates or on interest rates. The biggest challenge for these funds is that there is no way to know for certain when a perceived deviation from equilibrium will be corrected. There is a saying that the markets can stay irrational (out of equilibrium) longer than most investors can stay solvent. In other words, a deviation from equilibrium could take a long time to correct itself and some hedge funds will not be able to wait out the trend.

### Managed Futures

Managed futures hedge funds attempt to predict future movements in commodity prices based on either technical analysis or fundamental analysis. Technical analysis attempts to infer patterns from past price movements and use those patterns as a basis for predictions.

When technical analysis is used, fund managers will backtest their trading rules using historical data. Fundamental analysis studies economic, political, and other relevant measurable factors to determine a valuation for the given commodity and then buy or short sell based on the outcome of this fundamental research.

## HEDGE FUND PERFORMANCE AND MEASUREMENT BIAS

### LO 33.6: Describe hedge fund performance and explain the effect of measurement biases on performance measurement.

Hedge fund performance is not as easy to assess as mutual fund performance, which is readily available and accurately reported by numerous independent parties. Participation in hedge fund indices is voluntary. If the fund had good performance, then they will report their results to the index vendor. If they did not have good results, then they simply do not report their results to the index. In the Barclay's Hedge Fund Index, the data for August 2016 had 2,914 funds reporting information, while September 2016 only had 617. This is known as the **measurement bias** of hedge fund index reporting. When returns are reported by a hedge fund, the database is then backfilled with the fund's previous returns. This is known as **backfill bias** and it creates an issue with reliability for hedge fund benchmarks. It is very common for a hedge fund to have a string of several good years and then have a meltdown. LTCM, for example, reported returns (before fees) of 28%, 59%, 57%, and 17% in 1994, 1995, 1996, and 1997, respectively. However, in 1998, the fund lost virtually everything.

Prior to 2008, most hedge funds performed very well. However, after the financial crisis of 2007–2009, hedge funds have underperformed relative to the S&P 500. Figure 3 shows historical performance for the Barclay's Hedge Fund Index from 2006–2015 relative to the S&P 500 Index.

**Figure 3: Hedge Fund Historical Performance**

	<i>Barclay's Hedge Fund Index</i>	<i>S&amp;P 500 Index</i>
2006	12.39%	15.61%
2007	10.22%	5.48%
2008	–21.63%	–36.55%
2009	23.74%	25.94%
2010	10.88%	14.82%
2011	–5.48%	2.10%
2012	8.25%	15.89%
2013	11.12%	32.15%
2014	2.88%	13.52%
2015	0.04%	1.36%

Source: Barclay<sup>1</sup> and the Stern School of Business at NYU<sup>2</sup>

1. [http://www.barclayhedge.com/research/indices/ghs/Hedge\\_Fund\\_Index.html](http://www.barclayhedge.com/research/indices/ghs/Hedge_Fund_Index.html).

2. [http://pages.stern.nyu.edu/~adamodar/New\\_Home\\_Page/datafile/histretSP.html](http://pages.stern.nyu.edu/~adamodar/New_Home_Page/datafile/histretSP.html).

In Figure 3, we see that hedge funds outperformed the S&P 500 in two out of ten years from 2006–2015. One of the years of outperformance was during a significant negative year for the S&P 500 Index. This is one reason why hedge funds remain attractive diversifiers for pension funds and other interested investors. Protection during periods of stock market volatility is one hallmark of hedge funds as they actively pursue short selling when traditional mutual funds are not allowed to do so.

## KEY CONCEPTS

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### LO 33.1

There are three primary types of commingled pools of investments that are available to investors. They are open-end mutual funds, closed-end mutual funds, and exchange-traded funds (ETFs). Open-end funds transact at the next available net asset value (NAV), which occurs after the market has closed for the day. Shares may be redeemed directly from the fund company with an open-end fund. Closed-end funds transact throughout the trading day, but shares cannot be redeemed at the fund company and their price may differ substantially from their NAV—the shares must be bought or sold by other investors. Exchange-traded funds also trade throughout the day, but their shares do trade at the NAV. ETFs usually have the lowest internal fees, which is a big component of investment returns.

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### LO 33.2

The NAV is easily calculated as the total invested assets of the fund minus any liabilities (typically management fees payable) all divided by the total shares outstanding. The NAV for an open-end fund is set after the trading day is over, while the NAV for a closed-end fund and an exchange-traded fund is calculated continuously throughout the trading day. The NAV is used to determine the number of shares purchased or sold in a fund.

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### LO 33.3

Both mutual funds and hedge funds offer professional management, instant diversification, and the ability to commingle funds with other investors. However, there are some notable differences between mutual funds and hedge funds. Hedge funds are only marketed to wealthy and sophisticated investors. Because of this, hedge funds escape certain regulatory oversight, which enables them to avoid allowing investors to redeem shares at any time they want, calculating the NAV daily, and disclosing investment policies and strategies. They are also permitted to use leverage and short selling, which are commonly not permitted for mutual funds. In addition, hedge funds use lock-up periods to prevent investor withdrawals at the wrong time for the fund.

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### LO 33.4

Hedge funds commonly deploy a 2% and 20% incentive fee structure, where they earn management fees for investment results relative to a given hurdle rate. Investors are partially protected with the use of high-water marks and clawback clauses.

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### LO 33.5

There are many different types of hedge fund strategies. They all search for perceived mispricings in different corners of the markets and then try to exploit them for profit.

Long/short equity funds take both long and short positions in the equity markets, diversifying or hedging across sectors, regions, or market capitalizations, and have directional exposure to the overall market.

Dedicated short funds tend to take net short positions in equities, and their returns are negatively correlated with equities.

Distressed hedge funds invest across the capital structure of firms that are under financial or operational distress or are in the middle of bankruptcy. These hedge fund managers try to profit from an issuer's ability to improve its operation or come out of a bankruptcy successfully.

Merger arbitrage funds bet on spreads related to proposed merger and acquisition transactions.

Convertible arbitrage funds attempt to profit from the purchase of convertible securities and the shorting of corresponding stock.

Fixed income arbitrage funds try to obtain profits by exploiting inefficiencies and price anomalies between related fixed income securities.

Emerging market funds invest in currencies, debt, equities, and other instruments in countries with emerging or developing markets.

Global macro managers make large bets on directional movements in interest rates, exchange rates, commodities, and stock indices and do better during extreme moves in the currency markets.

Managed futures funds attempt to predict future movements in commodity prices based on either technical analysis or fundamental analysis.

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### LO 33.6

Hedge fund benchmarks are problematic due to measurement bias and backfill bias. Over the last ten years, reported hedge fund performance suggests that they have only beaten the S&P 500 Index in two of those years.

**CONCEPT CHECKERS**

1. Which of the following statements is not correct regarding investment funds available to all investors?
  - A. Open-end mutual funds always transact at the next available net asset value.
  - B. Stop orders can be used on closed-end funds.
  - C. Open-end mutual funds can be purchased with a limit order.
  - D. Short selling is available for some exchange-traded funds.
2. Which of the following characteristics is a key differentiator between mutual funds and hedge funds?
  - A. Professional asset management.
  - B. Immediate access to withdrawals from the fund.
  - C. Charging a fee for providing investment services.
  - D. Easy diversification for an investor.
3. What is the expected return to a hedge fund if the fund uses a standard 2 and 20 incentive fee structure with an investment that has a 35% probability of making 55% and a 65% probability of losing 45%?
  - A. 5.71%.
  - B. 6.12%.
  - C. 3.78%.
  - D. 5.28%.
4. Which type of hedge fund focuses on isolating mispricings in foreign exchange markets?
  - A. Fixed income arbitrage hedge funds.
  - B. Global macro hedge funds.
  - C. Managed futures hedge funds.
  - D. Convertible arbitrage hedge funds.
5. Which of the following statements is/are most accurate regarding hedge fund performance reporting?
  - I. When a hedge fund's performance is recorded in an index, all of its prior results are also included.
  - II. Hedge funds are permitted to self-select if their performance is reported in index averages.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.



**CONCEPT CHECKER ANSWERS**

1. C Open-end mutual funds have very low price transparency because they trade at the next available NAV, which is not calculated until after the market closes. As such, they can only be bought or sold using a market order. Closed-end funds can be bought or sold using stop orders and limit orders. In some cases, ETFs can be sold short.
2. B Mutual funds must offer immediate access to withdrawals from their fund. This is an SEC requirement. Hedge funds have advance notification and lock-up periods, which prevent immediate access to withdrawals from the fund.
3. A The hedge fund could potentially earn fees of 12.6% [2% (flat fee) +  $0.20 \times 53\%$  (incentive fee on return above the 2% flat fee)]. The expected payoff for fees then becomes 5.71% computed as follows:

$$(0.35 \times 12.6\%) + (0.65 \times 2\%) = 5.71\%$$

4. B Global macro funds focus on finding mispricings at the level of the global macro economy. They materialize in foreign exchange pricing and interest rates. Fixed income arbitrage funds focus on various mispricings with fixed-income securities. Managed futures funds focus on forecasting commodity prices. Convertible arbitrage funds focus on valuing convertible bonds.
5. C Statement I describes backfill bias and Statement II describes measurement bias. Backfill bias arises when the database is backfilled with the fund's previous returns. Measurement bias indicates that not all hedge funds report their performance to index providers.

# INTRODUCTION (OPTIONS, FUTURES, AND OTHER DERIVATIVES)

Topic 34

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## EXAM FOCUS

In this topic, we present the basic concepts of derivative securities and derivative markets. For the exam, know the basic derivative terms as well as the terms related to derivative markets. Also, be able to compute payoffs for the different derivative securities. Finally, be able to create a hedge and know how to take advantage of an arbitrage situation.

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## DERIVATIVE MARKETS

An **open outcry system** and **electronic trading system** are different forms of trading securities (matching buyers with sellers). The open outcry system (e.g., CBOT) is the more traditional system, which involves traders actually indicating their trades through hand signals and shouting. Electronic trading does not involve an actual “physical” exchange location, but rather involves matching buyers and sellers electronically via computers (e.g., NASDAQ).

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### LO 34.1: Describe the over-the-counter market, distinguish it from trading on an exchange, and evaluate its advantages and disadvantages.

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An **over-the-counter (OTC) market** differs from a traditional exchange. It is a customized trading market which utilizes telephone and computers to make trades. This market typically involves much larger trades than traditional exchanges. The most typical OTC trade is conducted over the phone. Since terms are not specified by an “exchange,” participants have more flexibility to negotiate the most mutually agreeable or attractive trade.

The OTC market is several times the size of the traditional exchange market. For example, in 2007, the OTC market was over \$500 trillion, while the exchange-traded market was under \$100 trillion.

Advantages of over-the-counter trading:

- Terms are not set by any exchange.
- Participants have flexibility to negotiate.
- In the event of a misunderstanding, calls are recorded.

Disadvantages of over-the-counter trading:

- OTC trading has more credit risk than exchange trading. Exchanges are organized in such a way that credit risk is eliminated.

## BASICS OF DERIVATIVE SECURITIES

### LO 34.2: Differentiate between options, forwards, and futures contracts.

An **option contract** is a contract that, in exchange for the option price, gives the option buyer the right, but not the obligation, to buy (sell) an asset at the exercise price from (to) the option seller within a specified time period, or depending on the type of option, a precise date (i.e., expiration date). A call option gives the option holder the right to purchase the underlying asset by a certain specified date for a specified (in advance) price. A put option gives the option holder the right to sell the underlying asset by a selected date for a pre-selected price.

A **forward contract** is a contract that specifies the price and quantity of an asset to be delivered sometime in the future. There is no standardization for forward contracts, and these contracts are traded in the over-the-counter market. One party takes the long position, agreeing to purchase the underlying asset at a future date for a specified price, while the other party is the short, agreeing to sell the asset on that same date for that same price. Forward contracts are often used in foreign exchange situations as these contracts can be used to hedge foreign currency risk.

A **futures contract** is a more formalized, legally binding agreement to buy/sell a commodity/financial instrument in a pre-designated month in the future, at a price agreed upon today by the buyer/seller. Futures contracts are highly standardized regarding quality, quantity, delivery time, and location for each specific commodity. These contracts are typically traded on an exchange.



*Professor's Note: Remember that a futures contract is an obligation/promise to actually complete a transaction, while an option is simply the right to buy/sell.*

### LO 34.3: Identify and calculate option and forward contract payoffs.

#### Call Option Payoff

The payoff on a **call option** to the option buyer is calculated as follows:

$$C_T = \max(0, S_T - X)$$

where:

$C_T$  = payoff on call option

$S_T$  = stock price at maturity

$X$  = strike price of option

The payoff to the option seller is  $-C_T$  [i.e.,  $-\max(0, S_T - X)$ ]. We should note that  $\max(0, S_t - X)$ , where time,  $t$ , is between 0 and  $T$ , is also the payoff if the owner decides to exercise the call option early (in the case of an American option as we will discuss later).

The price paid for the call option,  $C_0$ , is referred to as the **call premium**. Thus, the profit to the option buyer is calculated as follows:

$$\text{profit} = C_T - C_0$$

where:

$C_T$  = payoff on call option

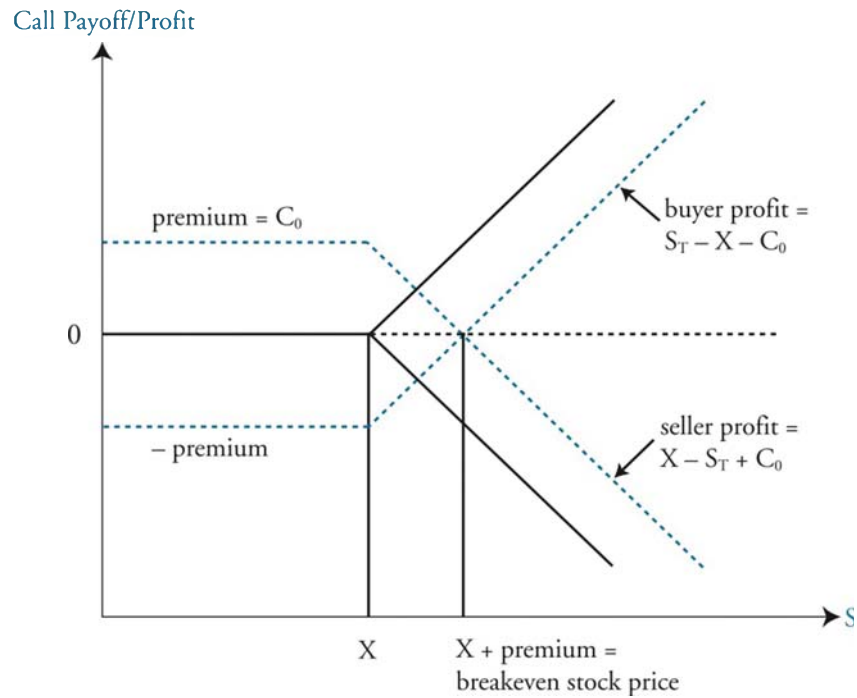
$C_0$  = call premium

Conversely, the profit to the option seller is:

$$\text{profit} = C_0 - C_T$$

Figure 1 depicts the payoff and profit for the buyer and seller of a call option.

**Figure 1: Profit Diagram for a Call at Expiration**



## Put Option Payoff

The payoff on a **put option** is calculated as follows:

$$P_T = \max(0, X - S_T)$$

where:

$P_T$  = payoff on put option

$S_T$  = stock price at maturity

$X$  = strike price of option

The payoff to the option seller is  $-P_T$  [i.e.,  $-\max(0, X - S_T)$ ]. We should note that  $\max(0, X - S_t)$ , where  $0 < t < T$ , is also the payoff if the owner decides to exercise the put option early.

The price paid for the put option,  $P_0$ , is referred to as the **put premium**. Thus, the profit to the option buyer is calculated as follows:

$$\text{profit} = P_T - P_0$$

where:

$P_T$  = payoff on put option

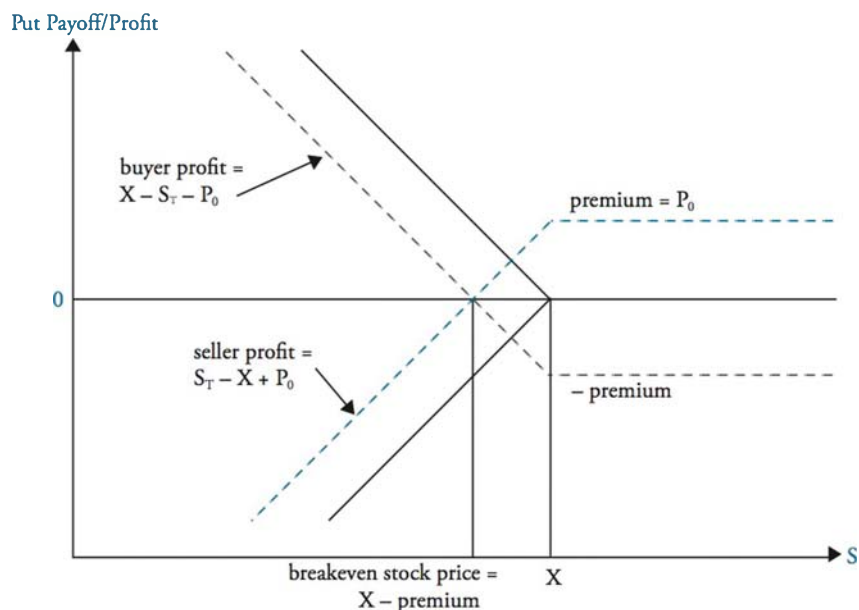
$P_0$  = put premium

The profit to the option seller is:

$$\text{profit} = P_0 - P_T$$

Figure 2 depicts the payoff and profit for the buyer and writer of a put option.

**Figure 2: Profit Diagram for a Put at Expiration**



**Example: Calculating profit and payoffs from options**

Compute the payoff and profit to a call buyer, a call writer, put buyer, and put writer if the strike price for both the put and the call is \$45, the stock price is \$50, the call premium is \$3.50, and the put premium is \$2.50.

**Answer:**

Call buyer:

$$\text{payoff} = C_T = \max(0, S_T - X) = \max(0, \$50 - \$45) = \$5$$

$$\text{profit} = C_T - C_0 = \$5 - \$3.50 = \$1.50$$

Call writer:

$$\text{payoff} = -C_T = -\max(0, S_T - X) = -\max(0, \$50 - \$45) = -\$5$$

$$\text{profit} = C_0 - C_T = \$3.50 - \$5 = -\$1.50$$

Put buyer:

$$\text{payoff} = P_T = \max(0, X - S_T) = \max(0, \$45 - \$50) = \$0$$

$$\text{profit} = P_T - P_0 = \$0 - \$2.50 = -\$2.50$$

Put writer:

$$\text{payoff} = -P_T = -\max(0, X - S_T) = -\max(0, \$45 - \$50) = \$0$$

$$\text{profit} = P_0 - P_T = \$2.50 - \$0 = \$2.50$$

**Forward Contract Payoff**

The payoff to a long position in a forward contract is calculated as follows:

$$\text{payoff} = S_T - K$$

where:

$S_T$  = spot price at maturity

$K$  = delivery price

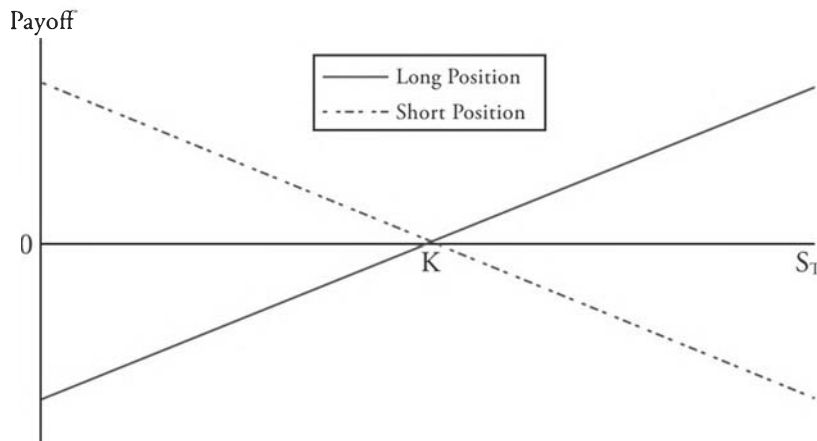
Conversely, the payoff to a short position in a forward contract is calculated as follows:

$$\text{payoff} = K - S_T$$



Figure 3 depicts the payoff for the long and short positions in a forward contract.

**Figure 3: Forward Contract Payoff**



**Example: Calculating forward contract payoffs**

**Compute** the payoff to the long and short positions in a forward contract given that the forward price is \$25 and the spot price at maturity is \$30.

**Answer:**

Payoff to long position:

$$\text{payoff} = S_T - K = \$30 - \$25 = \$5$$

Payoff to short position:

$$\text{payoff} = K - S_T = \$25 - \$30 = -\$5$$

## HEDGING STRATEGIES

**LO 34.4: Calculate and compare the payoffs from hedging strategies involving forward contracts and options.**

**Hedgers** use forward contracts and options to reduce or eliminate financial exposure. An investor or business with a long exposure to an asset can hedge exposure by either entering into a short futures contract or by buying a put option. An investor or business with a short exposure to an asset can hedge exposure by either entering into a long futures contract or by buying a call option.

Hedgers use forward contracts to lock in the price of the underlying security. Forward contracts do not require an initial investment, but hedgers give up any price movement that may have had positive results in the event that the position was left unhedged. Option contracts on the other hand function as insurance for the underlying by providing the downside protection that the hedger seeks and allowing for price movement in the direction that could yield positive results. This insurance does not come without a cost, as we described earlier, since hedgers are required to pay a premium to purchase options.

**Example: Hedging with a forward contract**

Suppose that a company based in the United States will receive a payment of €10M in three months. The company is worried that the euro will depreciate and is contemplating using a forward contract to hedge this risk. **Compute** the following:

1. The value of the €10M in U.S. dollars at maturity given that the company hedges the exchange rate risk with a forward contract at 1.25 \$/€.
2. The value of the €10M in U.S. dollars at maturity given that the company did not hedge the exchange rate risk and the spot rate at maturity is 1.2 \$/€.

**Answer:**

1. The value at maturity for the hedged position is:  
 $€10,000,000 \times 1.25 \text{ \$/€} = \$12,500,000$
2. The value at maturity for the unhedged position is:  
 $€10,000,000 \times 1.2 \text{ \$/€} = \$12,000,000$

### Example: Hedging with a put option

Suppose that an investor owns one share of ABC stock currently priced at \$30. The investor is worried about the possibility of a drop in share price over the next three months and is contemplating purchasing put options to hedge this risk. **Compute** the following:

1. The profit on the unhedged position if the stock price in three months is \$25.
2. The profit on the unhedged position if the stock price in three months is \$35.
3. The profit for a hedged stock position if the stock price in three months is \$25, the strike price on the put is \$30, and the put premium is \$1.50.
4. The profit for a hedged stock position if the stock price in three months is \$35, the strike price on the put is \$30, and the put premium is \$1.50.

**Answer:**

1. Profit =  $S_T - S_0 = \$25 - \$30 = -\$5$
2. Profit =  $S_T - S_0 = \$35 - \$30 = \$5$
3. Profit =  $S_T - S_0 + \max(0, X - S_T) - P_0$   
 $= \$25 - \$30 + \max(0, \$30 - \$25) - \$1.50 = -\$1.50$
4. Profit =  $S_T - S_0 + \max(0, X - S_T) - P_0$   
 $= \$35 - \$30 + \max(0, \$30 - \$35) - \$1.50 = \$3.50$



*Professor's Note: Notice that the max term is \$5 in Case #3 and \$0 in Case #4.*

## SPECULATIVE STRATEGIES

### LO 34.5: Calculate and compare the payoffs from speculative strategies involving futures and options.

**Speculators** have a different motivation for using derivatives than hedgers. They use derivatives to make bets on the market, while hedgers try to eliminate exposures.

The motivation for using futures in speculation is that the limited amount of initial investment creates significant **leverage**. The amount of investment required for futures is the amount of the initial margin required by the exchange. This is generally a small percentage of the notional value of the underlying, and Treasury securities can typically be posted as margin. Futures contracts can result in large gains or large losses, and contract payoffs are symmetrical.

Options also create significant leverage as investors only need to pay the option premium to purchase an option instead of the face value of the underlying. Options differ from futures in that options have asymmetrical payoffs. Gains can be quite large going long options, but losses from long option positions are limited to the option premium.

#### Example: Speculating with futures

An investor believes that the euro will strengthen against the dollar over the next three months and would like to take a position with a value of €250,000. He could purchase euros in the spot market at 0.80 \$/€ or purchase two futures contracts at 0.83 \$/€ with an initial margin of \$10,000. **Compute** the profit from the following:

1. Purchasing euros in the spot market if the spot rate in three months is 0.85 \$/€.
2. Purchasing euros in the spot market if the spot rate in three months is 0.75 \$/€.
3. Purchasing the futures contract if the spot rate in three months is 0.85 \$/€.
4. Purchasing the futures contract if the spot rate in three months is 0.75 \$/€.

**Answer:**

1. Profit = €250,000 × (0.85 \$/€ – 0.80 \$/€) = \$12,500
2. Profit = €250,000 × (0.75 \$/€ – 0.80 \$/€) = –\$12,500
3. Profit = €250,000 × (0.85 \$/€ – 0.83 \$/€) = \$5,000
4. Profit = €250,000 × (0.75 \$/€ – 0.83 \$/€) = –\$20,000

A summary of these four transactions is as follows:

	<i>Purchase Euros in Spot Market</i>	<i>Purchase Long Forward Position</i>
Investment	\$200,000	\$10,000
Profit if spot at maturity = 0.85 \$/€	\$12,500	\$5,000
Profit if spot at maturity = 0.75 \$/€	–\$12,500	–\$20,000

### Example: Speculating with options

An investor who has \$30,000 to invest believes that the price of stock XYZ will increase over the next three months. The current price of the stock is \$30. The investor could directly invest in the stock, or she could purchase 3-month call options with a strike price of \$35 for \$3. **Compute** the profit from the following:

1. Investing directly in the stock if the price of the stock is \$45 in three months.
2. Investing directly in the stock if the price of the stock is \$25 in three months.
3. Purchasing call options if the price of the stock is \$45 in three months.
4. Purchasing call options if the price of the stock is \$25 in three months.

#### Answer:

1. Number of stocks to purchase =  $\$30,000 / \$30 = 1,000$   
Profit =  $1,000 \times (\$45 - \$30) = \$15,000$
2. Profit =  $1,000 \times (\$25 - \$30) = -\$5,000$
3. Number of call options to purchase =  $\$30,000 / \$3 = 10,000$   
Profit =  $10,000 \times [\max(0, \$45 - \$35) - \$3] = \$70,000$
4. Profit =  $10,000 \times [\max(0, \$25 - \$35) - \$3] = -\$30,000$



*Professor's Note: Since option contracts are traded in amounts of 100 options, the transactions in #3 and #4 above would entail the purchase of 100 call option contracts (i.e.,  $10,000 / 100 = 100$ ).*

A summary of these four transactions is as follows:

	<i>Purchase Stock</i>	<i>Purchase Call Option</i>
# Shares/Call option	1,000	10,000
Profit if stock at maturity = \$45	\$15,000	\$70,000
Profit if spot at maturity = \$25	-\$5,000	-\$30,000

## ARBITRAGE OPPORTUNITIES

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**LO 34.6: Calculate an arbitrage payoff and describe how arbitrage opportunities are temporary.**

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**Arbitrageurs** are also frequent users of derivatives. Arbitrageurs seek to earn a risk-free profit in excess of the risk-free rate through the discovery and manipulation of mispriced securities. They earn a riskless profit by entering into equivalent offsetting positions in one or more markets. Arbitrage opportunities typically do not last long as supply and demand forces will adjust prices to quickly eliminate the arbitrage situation.

### Example: Arbitrage of stock trading on two exchanges

Assume stock DEF trades on the New York Stock Exchange (NYSE) and the Tokyo Stock Exchange (TSE). The stock currently trades on the NYSE for \$32 and on the TSE for ¥2,880. Given the current exchange rate is 0.0105 \$/¥, **determine** if an arbitrage profit is possible.

**Answer:**

Value in dollars of DEF on TSE = ¥2,880 × 0.0105 \$/¥ = \$30.24

Arbitrageur could purchase DEF on TSE for \$30.24 and sell on NYSE for \$32.

Profit per share = \$32 – \$30.24 = \$1.76

## RISK FROM DERIVATIVES

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**LO 34.7: Describe some of the risks that can arise from the use of derivatives.**

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Derivatives are versatile and can be used for hedging, arbitrage, and pure speculation. If, however, the “bet” one makes starts going in the wrong direction, the results can be catastrophic. Additionally, the risk exists that a trader with instructions to hedge a position may actually use derivatives to speculate. This risk is known as operational risk. Controls need to be carefully established and monitored within both financial and nonfinancial corporations to prevent misuse of derivatives. Risk limits should be set, and adherence to risk limits should be monitored.

## COMMON TERMS RELATED TO DERIVATIVES

The following section discusses common terms associated with derivatives. Many of these terms have been mentioned earlier. Understanding these concepts will be helpful going forward as you progress through the derivatives material.



A **derivative** security is a financial security (e.g., options) whose value is derived in part from another security's characteristics or value. This other security is referred to as the underlying asset. A derivative effectively "derives" its price from some other variable.

A **market maker** is the individual that "makes a market" in a security. The market maker maintains bid and offer prices in a given security and stands ready to buy or sell lots of said security, at publicly quoted prices.

A **spot contract** is an agreement to buy/sell an asset *today*. A **forward contract** specifies the price/quantity of an asset to be delivered on or before a future pre-specified date. A **futures contract** is a legally binding agreement to buy/sell a commodity or financial instrument in a designated future month at a previously agreed upon price by the buyer/seller.

A **call option** gives its holder the right to buy a specified number of shares of the underlying security at the given strike price, on or before the option contract's expiration date. A **put option** gives the investor the right to sell a fixed number of shares at a fixed price within a given pre-specified time period. An investor may wish to have the option to sell shares of a stock at a certain price and time in order to hedge an existing investment.

An American-styled option contract can be exercised any time between issue date and expiration date. In contrast, a European-styled option contract may be exercised only on the actual expiration date. **American options** will be worth more than **European options** when the right to early exercise is valuable, and they will have equal value when it is not.

A **long position** refers to actually owning the security, while a short position is when a person sells a security he does not own. An investor taking a short position anticipates a drop in price of the security.

The exercise, or **strike price**, is the price at which the security underlying an options contract may be bought/sold.

**Expiration date** is the last date on which an option may be exercised.

The **bid price** is the "quoted bid," or the highest price, which a dealer is willing to pay to purchase a security. This is essentially the available price at which an investor can sell shares of stock. The **offer price** is the price at which the security is offered for sale, also known as the "asking price." The **bid-ask spread** is the difference between the ask (a.k.a. offer) price and the bid price.

**Hedgers** reduce their risks typically through the use of forward contracts or options. By using forward contracts, the trader is attempting to neutralize risk by fixing the price the hedger will pay/receive for the underlying asset. Option contracts, in contrast, are more of an insurance policy.

**Speculators** want to take a position in the market and profit from this position. Speculators are effectively betting on future price movement. When a speculator uses futures, there is a large possible gain/loss. Speculating using options is less risky since the maximum loss is the cost of the option itself.

**Arbitrageurs** take offsetting positions in financial instruments in order to lock in a riskless profit.

## KEY CONCEPTS

### LO 34.1

The over-the-counter (OTC) market is used for large trades, and a typical OTC trade is conducted over the phone. Terms are not set by an “exchange,” giving traders more flexibility to negotiate mutually agreeable terms. The OTC market has more credit risk. Exchanges are organized to eliminate credit risk.

### LO 34.2

A call option gives its holder the right to buy a specified number of shares of the underlying security at the given strike price, on or before the option contract’s expiration date, while a put option is the right to sell a fixed number of shares at a fixed price within a given pre-specified time period.

A forward contract is an agreement to buy or sell an asset at a pre-selected future time for a certain price.

A futures contract is a more formalized, legally binding agreement to buy or sell a commodity or financial asset in a pre-designated month in the future, at a price agreed upon today by the buyer/seller.

### LO 34.3

The payoff on a call option to the option buyer is calculated as follows:

$$\text{Call}_T = \max(0, S_T - X)$$

where:

$S_T$  = stock price at maturity

$X$  = strike price of option

The payoff on a put option is calculated as follows:

$$\text{Put}_T = \max(0, X - S_T)$$

where:

$S_T$  = stock price at maturity

$X$  = strike price of option

The payoff to a long position in a forward contract is calculated as follows:

$$\text{payoff} = S_T - K$$

where:

$S_T$  = spot price at maturity

$K$  = delivery price

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**LO 34.4**

Hedgers use derivatives to control or eliminate a financial exposure. Futures lock in the price of the underlying security and do not allow for any upside potential. Options hedge negative price movements and allow for upside potential since they have asymmetric payouts.

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**LO 34.5**

Speculators use derivatives to make bets on the market. Futures require a small initial investment, which is the initial margin requirement. Futures contracts can result in large gains or large losses as futures have a symmetrical payout function.

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**LO 34.6**

Arbitrageurs seek to earn a riskless profit through the discovery and manipulation of mispriced securities. Riskless profit is earned by entering into equivalent offsetting positions in one or more markets. Arbitrage opportunities do not last long as the act of arbitrage brings prices back into equilibrium quickly.

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**LO 34.7**

Derivatives are versatile instruments and can be used for hedging, arbitrage, and pure speculation. Controls need to be carefully established to prevent misuse of derivatives. Risk limits must be carefully established and scrupulously enforced.

**CONCEPT CHECKERS**

1. Which of the following statements is an advantage of an exchange trading system?  
On an exchange system:
  - A. terms are not specified.
  - B. trades are made in such a way as to reduce credit risk.
  - C. participants have flexibility to negotiate.
  - D. in the event of a misunderstanding, calls are recorded between parties.
2. Which of the following statements regarding futures contracts is most likely correct?  
A business with a long exposure to an asset would hedge this exposure by either entering into a:
  - A. long futures contract or by buying a call option.
  - B. long futures contract or by buying a put option.
  - C. short futures contract or by buying a call option.
  - D. short futures contract or by buying a put option.
3. Which of the following statements is least likely correct regarding the use of derivatives?
  - A. Misuse of derivatives is not a very significant risk.
  - B. Risk limits for derivatives should be set, and adherence to these limits should be monitored.
  - C. Due to leverage inherent in derivatives, if a bet goes wrong, results can be catastrophic.
  - D. There is a risk that traders may use derivatives for unintended purposes.
4. An individual that maintains bid and offer prices in a given security and stands ready to buy or sell lots of said security is a(n):
  - A. hedger.
  - B. arbitrageur.
  - C. speculator.
  - D. market maker.
5. An agreement sold over an exchange to buy/sell a commodity or financial instrument at a designated future date is known as a(n):
  - A. spot contract.
  - B. option contract.
  - C. futures contract.
  - D. forward contract.

## CONCEPT CHECKER ANSWERS

1. B Exchanges are organized to reduce credit risk. The other answer choices are advantages of over-the-counter trading.
2. D A business with a long exposure to an asset would hedge the exposure by either entering into a short futures contract or by buying a put option.
3. A Misuse of derivatives can be a significant risk for firms that engage in derivatives trading.
4. D A market maker maintains bid and offer prices in a security and stands ready to buy or sell lots of the given security.
5. C A futures contract is an agreement sold on an exchange to buy/sell a commodity or financial instrument in a designated future month.

# MECHANICS OF FUTURES MARKETS

Topic 35

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## EXAM FOCUS

In this topic, candidates should focus on the terminology of futures markets, how futures differ from forwards, the mechanics of margin deposits, and the process of marking to market. Limit price moves, delivery options, and convergence of spot prices to futures prices are also likely exam concepts. Learn the ways a futures position can be terminated prior to contract expiration and understand how cash settlement is accomplished by the final mark to market at contract expiration.

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**LO 35.1: Define and describe the key features of a futures contract, including the asset, the contract price and size, delivery, and limits.**

**LO 35.9: Compare and contrast forward and futures contracts.**

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Futures contracts are exchange-traded obligations to buy or sell a certain amount of an underlying good at a specified price and date. The underlying asset varies from agricultural products to stock indices. Most futures positions are not held to take delivery of the underlying good. Instead, they are closed out or reversed prior to the settlement date.

The purchaser of a futures contract is said to have gone long or taken a **long position**, while the seller of a futures contract is said to have gone short or taken a **short position**. For each contract traded, there is a buyer and a seller. The long has contracted to buy the asset at the contract price at contract expiration, and the short has an obligation to sell at that price. Futures contracts are used by **speculators** to gain exposure to changes in the price of the asset underlying a futures contract. A **hedger**, in contrast, will use futures contracts to reduce exposure to price changes in the asset (i.e., hedge their asset price risk). An example is a wheat farmer who sells wheat futures to reduce the uncertainty about the price of wheat at harvest time.

**Open interest** is the total number of long positions in a given futures contract. It also equals the total number of short positions in a futures contract. An open interest of 200 would imply that there are 200 short positions in existence and 200 long positions in existence. It is possible, on any given day, for the trading volume on a contract to be higher than its open interest.

## TRADING FUTURES CONTRACTS

To illustrate how a futures contract is created, let's use a contract on gold as an example. Each contract represents 100 troy ounces and is quoted on a per-ounce basis. Suppose an investor instructs a broker to sell one futures contract on gold with an April delivery date. At about the same time another investor instructs a broker to buy an identical futures



contract. The seller of the futures contract has a short-futures position and is obligated to sell 100 ounces of gold at the futures price at contract expiration. The buyer of the futures contract has a long futures position and is obligated to buy 100 ounces of gold at the futures price at maturity. They agree on a price of \$993.60 per ounce. The two parties in this example have no idea of one another's existence because the clearinghouse (discussed in LO 35.4) takes the opposite side of every transaction. In the futures market there is always the same number of long and short positions. This means that if a long position wins, the corresponding short position loses.

## CHARACTERISTICS SPECIFIED IN A FUTURES CONTRACT

Futures contracts are similar to forward contracts in that both allow for a transaction to take place at a future date at a price agreed upon today. The difference between the two is that forward contracts are private, customized contracts, while futures trade on an organized exchange and have terms that are highly standardized. When a new futures contract is introduced to the marketplace, the futures exchange must specify the exact terms of the contract. Futures contract characteristics specified by the exchange include the following:

- *Quality of the underlying asset.* When the underlying asset for the contract is a financial asset, such as Japanese yen, the definition of the asset is straightforward. However, when the underlying asset is a commodity, there may be different levels of quality for that good available in the marketplace (e.g., different types of wheat). The futures exchange stipulates the quality of a good that will be acceptable for settling the contract.
- *Contract size.* The contract size specifies the quantity of the asset that must be delivered to settle a futures contract (e.g., one grain contract = 5,000 bushels).
- *Delivery location.* The exchange specifies the place where delivery will take place.
- *Delivery time.* Futures contracts are referred to by the month in which delivery is to take place (e.g., a December corn contract). Some contracts are not settled by delivery but by payment in cash, based on the difference between the futures price and the market price at settlement.
- *Price quotations and tick size.* The exchange determines how the price of a contract will be quoted as well as the minimum price fluctuation for the contract, which is referred to as the **tick size**. For example, grain is quoted in dollars per bushel, and the minimum tick size is  $\frac{1}{4}$  cent per bushel. Since a grain contract consists of 5,000 bushels, the minimum tick size is \$12.50 ( $= 5,000 \times \$0.0025$ ) per contract.
- *Daily price limits.* The exchange sets the maximum price movement for a contract during a day. For example, wheat cannot move more than \$0.20 from its close the preceding day, for a daily price limit of \$1,000. When a contract moves down by its daily price limit, it is said to be **limit down**. When the contract moves up by its price limit, it is said to be **limit up**.
- *Position limits.* The exchange sets a maximum number of contracts that a speculator may hold in order to prevent speculators from having an undue influence on the market. Such limits do not apply to hedgers.

## FUTURES/SPOT CONVERGENCE

### LO 35.2: Explain the convergence of futures and spot prices.

The spot (cash) price of a commodity or financial asset is the price for immediate delivery. The futures price is the price today for delivery at some future point in time (i.e., the maturity date). The **basis** is the difference between the spot price and the futures price.

$$\text{basis} = \text{spot price} - \text{futures price}$$

As the maturity date nears, the basis converges toward zero. At expiration, the spot price must equal the futures price because the futures price has become the price today for delivery today, which is the same as the spot. Arbitrage will force the prices to be the same at contract expiration.

#### Example: Why the futures price must equal the spot price at expiration

Suppose the current spot price of silver is \$4.65. **Demonstrate** by arbitrage that the futures price of a futures silver contract that expires in one minute must equal the spot price.

#### Answer:

Suppose the futures price was \$4.70. We could buy the silver at the spot price of \$4.65, sell the futures contract, and deliver the silver under the contract at \$4.70. Our profit would be  $\$4.70 - \$4.65 = \$0.05$ . Because the contract matures in one minute, there is virtually no risk to this arbitrage trade.

Suppose instead the futures price was \$4.61. Now we would buy the silver contract, take delivery of the silver by paying \$4.61, and then sell the silver at the spot price of \$4.65. Our profit is  $\$4.65 - \$4.61 = \$0.04$ . Once again, this is a riskless arbitrage trade.

Therefore, in order to prevent arbitrage, the futures price at the maturity of the contract must be equal to the spot price of \$4.65.

## OPERATION OF MARGINS

### LO 35.3: Describe the rationale for margin requirements and explain how they work.

**Margin** is cash or highly liquid collateral placed in an account to ensure that any trading losses will be met. Marking to market is the daily procedure of adjusting the margin account balance for daily movements in the futures price. The amount required to open a futures position is called the **initial margin**. The **maintenance margin** is the minimum margin

account balance required to retain the futures position. When the margin account balance falls below the maintenance margin, the investor gets a margin call, and he must bring the margin account back to the initial margin amount. The amount necessary to do this is called the **variation margin**.

#### Example: Margin trading

Let's return to our investor with the long gold contract. The investor entered the position at \$993.60. Each contract controls 100 troy ounces for a current market value of \$99,360. Assume that the initial margin is \$2,500, the maintenance margin is \$2,000, and the futures price drops to \$991.00 at the end of the first day and \$985.00 on the end of the second day. **Compute** the amount in the margin account at the end of each day for the long position and any variation margin needed.

#### Answer:

At the end of the first day, the loss is computed as  $(\$991 - \$993.6)100 = -\$260$ , so when the account is marked to market, \$260 is withdrawn from the buyer's margin account and \$260 deposited in the seller's margin account. The buyer's (long) margin account balance is now \$2,240 ( $= \$2,500 - \$260$ ). The margin account balance for the short position is now \$2,760 ( $= \$2,500 + \$260$ ).

At the end of the second day, the daily loss is  $(\$985 - \$991)100 = -\$600$ , and the buyer's margin account balance is reduced to \$1,640 ( $= \$2,240 - \$600$ ). At \$1,640 the investor will get a margin call since the margin account balance is less than the maintenance margin. The variation margin is the amount necessary to bring the margin account back up to the initial margin. In this case, it is \$860 ( $= \$2,500 - \$1,640$ ).

Depending on the client, brokers may require the posting of a balance in the margin account more than the maintenance margin requirements established by exchanges. For example, hedgers are usually required to post smaller margins than speculators. To ensure that the daily cash flows are withdrawn or contributed appropriately, the exchange has a clearinghouse.

## CLEARINGHOUSES IN FUTURES TRANSACTIONS

### LO 35.4: Describe the role of a clearinghouse in futures and over-the-counter market transactions.

Each exchange has a **clearinghouse**. The clearinghouse guarantees that traders in the futures market will honor their obligations. The clearinghouse does this by splitting each trade once it is made and acting as the opposite side of each position. The clearinghouse acts as the buyer to every seller and the seller to every buyer. By doing this, the clearinghouse allows either side of the trade to reverse positions at a future date without having to contact the other side of the initial trade. This allows traders to enter the market knowing that they will be able to reverse their position. Traders are also freed from having to worry about the

counterparty defaulting since the counterparty is now the clearinghouse. In the history of U.S. futures trading, the clearinghouse has never defaulted on a trade.

The clearinghouse has members that collateralize it, ensuring that no defaults take place. All trades eventually go through the clearinghouse members, who must have a **clearing margin** posted at the clearinghouse in the same way an investor has a margin account with a broker. This ensures that the clearinghouse is liquid enough at all times to honor all obligations under futures contracts.

## OVER-THE-COUNTER MARKETS

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### LO 35.5: Describe the role of collateralization in the over-the-counter market and compare it to the margining system.

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The over-the-counter (OTC) market includes the trading in all securities not listed on one of the registered exchanges. This market is subject to a good deal of credit risk since the party on the other side of an OTC contract could default on its payments. One way to reduce this credit risk is by means of **collateralization**. Collateralization is basically a marked to market feature for the OTC market where any loss is settled in cash at the end of the trading day. A cash payment is made to the party with a positive account balance. This is a similar system to trading on margin where the futures trader needs to restore funds if the value of the contract drops below the maintenance margin.

Recently passed legislation requires that some OTC transactions use clearinghouses. OTC market clearinghouses operate in a similar fashion to clearinghouses on futures exchanges. After two parties (X and Y) negotiate an OTC agreement, it is submitted to the clearinghouse for acceptance. Assuming the transaction is accepted, the clearinghouse will become the counterparty to both parties X and Y. Thus, the clearinghouse assumes the credit risk of both parties in an OTC transaction. This risk is managed by requiring the parties to post initial margin and any variation margins on a daily basis.

Historically, OTC markets have functioned as a series of bilateral agreements between parties. If a clearinghouse was instead used for every OTC transaction, each market participant would only deal with a central clearing party. However, because only some OTC transactions are currently required to use clearinghouses, in practice, the current OTC market is a mix of both bilateral agreements and transactions dealing with one or more clearinghouses.

Arguments for the use of clearinghouses in OTC markets include: (1) automatic posting of collateral, (2) reduction of financial system credit risk, and (3) increased transparency of OTC trades. Governments have pushed for the use of clearinghouses in OTC markets in an attempt to reduce systemic risk, which is the risk that a failure by a significant financial institution will impact other institutions and potentially lead to a collapse of the overall financial system. An example of systemic risk occurred during the 2007–2009 credit crisis when the OTC transactions for insurance corporation AIG led to large losses and an eventual bailout of the company by the U.S. government.



## NORMAL AND INVERTED FUTURES MARKET

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### LO 35.6: Identify the differences between a normal and inverted futures market.

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The **settlement price** is analogous to the closing price for a stock but is not simply the price of the last trade. It is an average of the prices of the trades during the last period of trading, called the closing period, which is set by the exchange. This feature of the settlement price prevents manipulation by traders. The settlement price is used to make margin calculations at the end of each trading day.

Depending on the direction of futures settlement prices, the market may be normal or inverted. Increasing settlement prices over time indicates a **normal market**. Conversely, decreasing settlement prices over time indicates an **inverted market**.

## THE DELIVERY PROCESS

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### LO 35.7: Describe the mechanics of the delivery process and contrast it with cash settlement.

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There are four ways to terminate a futures contract:

1. A short can terminate the contract by delivering the goods. When the long accepts this delivery, he pays the contract price to the short. This is called **delivery**. The location for delivery (for physical assets), terms of delivery, and details of exactly what is to be delivered are all specified in the **notice of intention to deliver** file. Each exchange has specific rules as to the conditions for making an intent to deliver. However, the price paid or received will be dictated by the settlement period on the exchange-determined last trading day of the contract.
2. In a **cash-settlement contract**, delivery is not an option. The futures account is marked to market based on the settlement price on the last day of trading.
3. You may make a **reverse**, or **offsetting**, trade in the futures market. With futures, the other side of your position is held by the clearinghouse—if you make an exact opposite trade (maturity, quantity, and good) to your current position, the clearinghouse will net your positions out, leaving you with a zero balance. This is how most futures positions are settled. The contract price can differ between the two contracts. If you initially are long one contract at \$970 per ounce of gold and subsequently sell (i.e., take the short position in) an identical gold contract when the price is \$950 per ounce, \$20 multiplied by the number of ounces of gold specified in the contract will be deducted from the margin deposit(s) in your account. The sale of the futures contract ends the exposure to future price fluctuations on the first contract. Your position has been *reversed*, or **closed out**, by a *closing* trade.
4. A position may also be settled through an **exchange for physicals**. Here you find a trader with an opposite position to your own and deliver the goods and settle up between yourselves, off the floor of the exchange (i.e., an ex-pit transaction). This is the sole exception to the federal law that requires that all trades take place on the floor of the exchange. You must then contact the clearinghouse and tell them what happened.

An exchange for physicals differs from a delivery in that the traders actually exchange the goods, the contract is not closed on the floor of the exchange, and the two traders privately negotiate the terms of the transaction. Regular delivery involves only one trader and the clearinghouse.

## TYPES OF ORDERS

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### LO 35.8: Evaluate the impact of different trading order types.

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There are several different types of orders in the marketplace:

**Market orders** are orders to buy or sell at the best price available. A **discretionary order** is a market order where the broker has the option to delay transaction in search of a better price.

**Limit orders** are orders to buy or sell away from the current market price. A *limit buy order* is placed below the current price. A *limit sell order* is placed above the current price. Limit orders have a time limit, such as instantaneous, one day, one week, one month, or good till canceled. Limit orders are turned over to the specialist by the commission broker.

**Stop-loss orders** are used to prevent losses or to protect profits. Suppose you own a stock currently selling for \$40. You are afraid that it may drop in price, and if it does, you want your broker to sell it, thereby limiting your losses. You would place a *stop loss sell* order at a specific price (e.g., \$35); if the stock price drops to this level, your broker will place a sell market order. A *stop loss buy* order is usually combined with a short sale to limit losses. If the stock price rises to the “stop” price, the broker enters a market order to buy the stock.

Variations on these order types also exist. **Stop-limit orders** are a combination of a stop and limit order. The stop price and limit price must be specified, so that once the stop level is reached, or bettered, the order would turn into a limit order and hopefully transact at the limit price. **Market-if-touched orders**, or MIT orders, are orders that would become market orders once a specified price is reached in the marketplace.

For those orders that remain outstanding until the designated price range is reached, the trader making the order needs to indicate the time period for the order (**time-of-day order**). **Good-till-canceled (GTC) orders** (a.k.a. **open orders**) are orders that remain open until they either transact or are canceled. A popular method of submitting a limit order is to have it automatically canceled at the end of the trading day in which it was submitted. **Fill-or-kill orders** must be executed immediately or the trade will not take place.

## REGULATORY, ACCOUNTING, AND TAX FRAMEWORKS

### Regulation

In the United States, the **Commodity Futures Trading Commission (CFTC)** is responsible for regulating futures markets. The CFTC licenses futures exchanges as well as traders who offer futures trading services to the public. It also approves new futures contracts and any revisions to existing futures contracts. When approving contracts, the agency ensures that each contract serves a useful economic purpose (e.g., for either hedging or speculating).



In addition, the CFTC is responsible for communicating prices to the public, addressing public complaints, and taking disciplinary actions against members who violate futures exchange rules.

Other regulatory bodies that influence the futures markets include the National Futures Association (NFA), the Securities and Exchange Commission (SEC), the Federal Reserve Board, and the U.S. Treasury Department. The SEC, Fed, and Treasury Department are mainly concerned with how futures trading impacts spot market trading in stocks and bonds. The NFA has a more prominent role by attempting to prevent fraud and ensuring that futures markets operate in the best interests of the public. Examples of futures trading fraud include cornering the market (i.e., taking excessive long positions while influencing the supply of the commodity underlying the long futures contracts) and front running (traders using privileged information to trade in their own accounts before customer accounts).

## Accounting

When accounting for changes in the market value of a futures contract, changes must be recognized when they occur. The exception to this accounting standard is when a futures contract is being used for hedging purposes. **Hedge accounting** specifies that gains/losses from a hedging instrument be recognized in the same period as gains/losses from the asset being hedged.

Under FAS 133 [Financial Accounting Standard Board (FASB) Statement No. 133], the fair market value of all derivative contracts must be included on the balance sheet. In addition to more position transparency, FAS 133 places stricter guidelines on the use of hedge accounting. To use this accounting method, it must be shown that the hedging instrument frequently and effectively offsets the intended risk exposure.

## Taxes

Regarding U.S. tax regulations, differences arise due to the nature of taxable gains/losses and the timing of realized gains/losses. For corporate taxpayers, capital gains are taxed at the same level as ordinary income and capital losses are restricted. For non-corporate taxpayers, capital gains are taxed at the same level as ordinary income, but long-term gains (investments held over one year) are subject to a maximum 15% tax rate. Another difference is that capital losses are deductible for non-corporate taxpayers.

For tax purposes, futures contracts are considered closed out at the end of each year. This gives rise to a 60/40 rule for non-corporate taxpayers where capital gains/losses are treated as 60% long term and 40% short term. This rule, however, does not apply to hedging activities. Using futures for hedging purposes must be declared on the same day the transaction is entered. Gains/losses on hedging transactions are taxed at the same rate as ordinary income.

## KEY CONCEPTS

### LO 35.1

A long (short) futures position obligates the owner to buy (sell) the underlying asset at a specified price and date. Most futures positions are reversed (or closed out) as opposed to satisfying the contract by making (or taking) delivery.

### LO 35.2

The spot price of a commodity or financial asset is the price for immediate delivery. The futures price is the price today for delivery at some future point in time (i.e., the maturity date). The basis is the difference between the spot price and the futures price. As the maturity date nears, the basis converges toward zero. Arbitrage will force the spot and futures prices to be the same at contract expiration.

### LO 35.3

Futures are traded on margin (leveraged):

- Initial margin is the necessary collateral to trade the futures.
- Maintenance margin is the minimum collateral amount required to retain trading privileges.
- Variation margin is the collateral amount that must be deposited to replenish the margin account back to the initial margin.

The futures market is a zero-sum game in that the short's losses are the long's gains and vice versa. Gains and losses due to changes in futures prices are computed at the end of each trading day in a process known as marking to market.

### LO 35.4

The clearinghouse maintains an orderly and liquid market by acting as the counterparty to each long or short futures position. In the over-the-counter (OTC) markets, the clearinghouse also becomes the counterparty to both parties in an OTC transaction.

### LO 35.5

Collateralization is a means of reducing credit risk in OTC contracts.

### LO 35.6

The futures settlement price is an average of the prices of the trades during the last period of trading, called the closing period. It is used to make margin calculations at the end of each trading day. Increasing settlement prices over time indicate a normal market, while decreasing settlement prices over time indicate an inverted market.

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**LO 35.7**

A short can terminate the futures contract by delivering the goods. When the long accepts this delivery, he pays the contract price to the short. This is known as the delivery process. In a cash-settlement contract, delivery is not an option.

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**LO 35.8**

Several different types of orders exist in the marketplace including: market, limit, stop-loss, stop-limit, and market-if-touched orders. Market orders are orders to buy or sell at the best price available. Limit orders are orders to buy or sell away from the current market price. Stop-loss orders are used to prevent losses or to protect profits. Stop-limit orders are a combination of a stop and limit order. Market-if-touched orders are orders that would become market orders once a specified price is reached.

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**LO 35.9**

Futures contracts are similar to forward contracts in that both allow for a transaction to take place at a future date at a price agreed upon today. The difference between the two is that forward contracts are private, customized contracts, while futures trade on an organized exchange and have terms that are highly standardized.

**CONCEPT CHECKERS**

1. When an investor is obligated to buy the underlying asset in a futures position, it is a:
  - A. basis trade.
  - B. long-futures position.
  - C. short-futures position.
  - D. hedged-futures position.
2. Which of the following are characteristics specified by a futures contract?
  - I. Asset quality and asset quantity.
  - II. Delivery arrangements and delivery time.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
3. An investor enters into a short position in a gold futures contract with the following characteristics:
  - The initial margin is \$3,000.
  - The maintenance margin is \$2,250.
  - The contract price is \$1,300.
  - Each contract controls 100 troy ounces.

If the price drops to \$1,295 at the end of the first day and \$1,290 at the end of the second day, which of the following is closest to the variation margin required at the end of the second day?

  - A. \$0.
  - B. \$250.
  - C. \$500.
  - D. \$1,000.
4. Which of the following items are functions of the clearinghouse?
  - I. Determine which contracts trade.
  - II. Receive margin deposits from brokers.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
5. It is possible that which of the following types of orders may never be executed?
  - A. Limit orders.
  - B. Market-if-touched (MIT) orders.
  - C. Stop-limit orders.
  - D. All of the above.

## CONCEPT CHECKER ANSWERS

1. B When an investor is obligated to buy the underlying asset in a futures position, it is a long futures position.
2. C Delivery time, asset quality, asset quantity, and delivery arrangements are all characteristics specified by the futures contract.
3. A Note that the investor in this question has a short position that profits from price declines. The short position margin account has increased by \$1,000 over the two days, so there is no variation margin required.
4. B The clearinghouse acts as buyer to every seller and seller to every buyer, thus virtually eliminating default risk. It also collects margin payments from clearing members (brokers). Determining which contracts will trade is a function of the exchange, not the clearinghouse.
5. D All of these orders require that the price reach a certain range before being activated. If the price never reaches that range, the order will never be activated.

# HEDGING STRATEGIES USING FUTURES

Topic 36

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## EXAM FOCUS

Futures contracts are used extensively for implementing hedging strategies. This topic presents the calculations for determining the optimal hedge ratio and shows how to use it to determine the number of futures contracts necessary to hedge a spot market exposure. This topic also addresses basis risk, the change in the relationship between spot prices and futures prices over a hedge horizon. Basis risk arises because an asset being hedged may not be exactly the same as the asset underlying the futures contract.

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## HEDGING WITH FUTURES

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**LO 36.1: Define and differentiate between short and long hedges and identify their appropriate uses.**

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A **short hedge** occurs when the hedger shorts (sells) a futures contract to hedge against a price decrease in the existing long position. When the price of the hedged asset decreases, the short futures position realizes a positive return, offsetting the decline in asset value. Therefore, a short hedge is appropriate when you have a long position and expect prices to decline.

A **long hedge** occurs when the hedger buys a futures contract to hedge against an increase in the value of the asset that underlies a short position. In this case, an increase in the value of the shorted asset will result in a loss to the short seller. The objective of the long hedge is to offset the loss in the short position with a gain from the long futures position. A long hedge is therefore appropriate when you have a short position and expect prices to rise.

## Advantages and Disadvantages of Hedging

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**LO 36.2: Describe the arguments for and against hedging and the potential impact of hedging on firm profitability.**

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The objective of hedging with futures contracts is to reduce or eliminate the price risk of an asset or a portfolio. For example, a farmer with a large corn crop that will be harvested in a few months could wait until the end of the growing season and sell his corn at the prevailing spot price, *or* he could sell corn futures and “lock in” the price of his corn at a predetermined rate. By taking a short position in a corn futures contract, the farmer eliminates—or at least reduces—exposure to fluctuating corn prices. This is an example of a *short hedge*, where the user locks in a future selling price.



Alternatively, a cereal company will need to purchase corn in the future. The company could wait to buy corn in the spot market and face the volatility of future corn spot prices or lock in its purchase price by buying corn futures in advance. This demonstrates an *anticipatory hedge*. The cereal company has an anticipated need for corn and buys corn futures to lock in the price of those future corn purchases. This is an example of a *long hedge*, where the user locks in a future purchasing price.

It is easy to see that the benefit from hedging leads to less uncertainty regarding future profitability. However, there are some arguments against hedging. The main issue is that hedging can lead to less profitability if the asset being hedged ends up increasing in value. The increase in value will be offset by a corresponding loss in the futures contract used for the hedge.

Another argument against hedging is the questionable benefit that accrues to shareholders. Clearly, hedging reduces risk for a company and its shareholders, but there is reason to believe that shareholders can more easily hedge risk on their own. A third argument deals with the nature of the hedging company's industry. For example, assume that prices in an industry frequently adjust for changes in input prices and exchange rates. If competitors do not hedge, then there is an incentive to keep the status quo. In this way, the company ensures that profitability will remain more stable than if it were to hedge frequent changes.

## BASIS RISK

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**LO 36.3: Define the basis and explain the various sources of basis risk, and explain how basis risks arise when hedging with futures.**

**LO 36.4: Define cross hedging, and compute and interpret the minimum variance hedge ratio and hedge effectiveness.**

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When all of the existing position characteristics match perfectly with those of the futures contract specifications, we have a perfect hedge. With a perfect hedge, the loss on a hedged position will be perfectly offset by the gain on the futures position. Perfect hedges are not very common. There are two major reasons why this is so: (1) the asset in the existing position is often not the same as that underlying the futures (e.g., we may be hedging a corporate bond portfolio with a futures contract on a U.S. Treasury bond), and (2) the hedging horizon may not match perfectly with the maturity of the futures contract. The existence of either one of these conditions leads to what is called **basis risk**.

The basis in a hedge is defined as the difference between the spot price on a hedged asset and the futures price of the hedging instrument (e.g., futures contract). Basis is calculated as:

$$\text{basis} = \text{spot price of asset being hedged} - \text{futures price of contract used in hedge}$$

When the hedged asset and the asset underlying the hedging instrument are the same, the basis will be zero at maturity.



*Professor's Note: This is the typical definition for basis (where basis equals spot price minus futures price). However, basis is also sometimes defined as: futures price minus spot price, mostly when dealing with financial asset futures.*

When the spot price increases faster than the futures price over the hedging horizon, basis increases and a strengthening of the basis is said to occur. When the futures price increases faster than the spot price and the basis decreases, a weakening of the basis occurs. When hedging, a change in basis is unavoidable. The change in basis over the hedge horizon is termed *basis risk*, and it can work either for or against a hedger.

To minimize basis risk, hedgers should select the contract on an asset that is most highly correlated with the spot position and a contract maturity that is closest to the hedging horizon. Contract liquidity must also be considered when selecting a futures contract for hedging.

Three sources of basis risk are: (1) interruption in the convergence of the futures and spot prices, (2) changes in the cost of carry, and (3) imperfect matching between the cash asset and the hedge asset. Let's discuss each of these sources in more detail.

1. *Interruption in the convergence of the futures and spot prices.* Normally, spot prices and futures prices will converge as the time to maturity decreases, and basis reduces to zero at maturity. However, if the position is unwound prior to maturity, the return to the futures position could be different from the return to the cash position. A more rapid convergence results in a more rapid transfer of margin payments, while a less rapid convergence would delay payments. An interruption in the convergence could result in payments from the seller to the buyer. All of these effects are types of basis risk.
2. *Changes in the cost of carry.* Significant basis risk can arise due to changes in the components of the cost of carry. The cost of carry includes storage and safekeeping, interest, insurance, and related costs. Perhaps the most volatile of these costs is interest costs. An increase in the interest rates increases the opportunity cost of holding the asset, so the cost of carry and, hence, the difference in the basis of the contract rises.
3. *Imperfect matching between the cash asset and the hedge asset.* Sometimes it may be more efficient to **cross hedge** or hedge a cash position with a hedge asset that is closely related but different from the cash asset. For example, Eurodollar deposits are closely related to T-bill rates and may be considered a good hedge. However, if there is a structural shock that changes the close relationship of these two assets, the position may not be hedged as effectively as originally believed. This is the most common form of basis risk. Other forms of mismatch include maturity or duration mismatches, liquidity mismatches, and credit risk mismatches:
  - *Maturity or duration mismatch.* Hedging a portfolio of mortgages with 10-year Treasury notes (T-notes) may seem reasonable if the effective duration of the mortgages matches the duration of the T-notes. However, if rates fall and the mortgages prepay faster (resulting in a shorter duration), the position will not be matched.
  - *Liquidity mismatch.* Hedging an illiquid asset with a more liquid one will result in greater basis risk. Although over the long term the prices may be comparable, the difference in liquidity may result in large gaps between the pricing of the two assets. Hence, basis risk is inversely proportional to the liquidity of the hedged asset.

- *Credit risk mismatch.* The widening or narrowing of credit spreads constitutes another form of basis risk when the credit risk of the hedged asset is different (or becomes different) from the credit risk of the hedge instrument.

All of these represent basis risk. The size and type of basis risk can vary during the term of the contract, even if the position is perfectly hedged at maturity.

## The Optimal Hedge Ratio

We can account for an imperfect relationship between the spot and futures positions by calculating an **optimal hedge ratio** that incorporates the degree of correlation between the rates.

A hedge ratio is the ratio of the size of the futures position relative to the spot position. The *optimal hedge ratio*, which minimizes the variance of the combined hedge position, is defined as follows:

$$HR = \rho_{S,F} \frac{\sigma_S}{\sigma_F}$$

This is also the beta of spot prices with respect to futures contract prices since:

$$\rho = \frac{\text{Cov}_{S,F}}{\sigma_S \sigma_F} \text{ and } \frac{\text{Cov}_{S,F}}{\sigma_S \sigma_F} \times \frac{\sigma_S}{\sigma_F} = \frac{\text{Cov}_{S,F}}{\sigma_F^2} = \beta_{S,F}$$

where:

- $\rho_{S,F}$  = the correlation between the spot prices and the futures prices
- $\sigma_S$  = the standard deviation of the spot price
- $\sigma_F$  = the standard deviation of the futures price

### Example: Minimum variance hedge ratio

Suppose a currency trader computed the correlation between the spot and futures to be 0.925, the annual standard deviation of the spot price to be \$0.10, and the annual standard deviation of the futures price to be \$0.125. Compute the hedge ratio.

**Answer:**

$$HR = 0.925 \times \frac{0.100}{0.125} = 0.74$$

The ratio of the size of the futures to the spot should be 0.74.

The **effectiveness of the hedge** measures the variance that is reduced by implementing the optimal hedge. This effectiveness can be evaluated with a coefficient of determination ( $R^2$ ) term where the independent variable is the change in futures prices and the dependent variable is the change in spot prices. Recall that  $R^2$  measures the goodness-of-fit of a regression. As shown previously, the beta of spot prices with respect to futures prices is equal

to the hedge ratio (HR), which is also the slope of this regression. The  $R^2$  measure for this simple linear regression is the square of the correlation coefficient ( $\rho^2$ ) between spot and futures prices.

## HEDGING WITH STOCK INDEX FUTURES

**LO 36.5: Compute the optimal number of futures contracts needed to hedge an exposure, and explain and calculate the “tailing the hedge” adjustment.**

A common hedging application is the hedging of equity portfolios using futures contracts on stock indices (index futures). In this application, it is important to remember that the hedged portfolio's beta serves as a hedge ratio when determining the correct number of contracts to purchase or sell. The number of futures contracts required to completely hedge an equity position is determined with the following formula:

$$\begin{aligned}\text{number of contracts} &= \beta_{\text{portfolio}} \times \left( \frac{\text{portfolio value}}{\text{value of futures contract}} \right) \\ &= \beta_{\text{portfolio}} \times \left( \frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}} \right)\end{aligned}$$

### Example: Hedging with stock index futures

You are a portfolio manager with a \$20 million growth portfolio that has a beta of 1.4, relative to the S&P 500. The S&P 500 futures are trading at 1,150, and the multiplier is 250. You would like to hedge your exposure to market risk over the next few months. **Identify** whether a long or short hedge is appropriate, and **determine** the number of S&P 500 contracts you need to implement the hedge.

**Answer:**

You are long the S&P 500, so you should construct a short hedge and sell the futures contract. The number of contracts to sell is equal to:

$$1.4 \times \frac{\$20,000,000}{1,150 \times 250} \approx 97 \text{ contracts}$$

### Tailing the Hedge

A hedger may actually over-hedge the underlying exposure if daily settlement is not properly accounted for. To correct for the possibility of over-hedging, a hedger can implement a **tailing the hedge** strategy. The extra step needed to carry out this strategy is to multiply the hedge ratio by the daily spot price to futures price ratio. In practice, it is not efficient to adjust the hedge for every daily change in the spot-to-futures ratio.

### Example: Tailing the hedge

Suppose that you would like to make a tailing the hedge adjustment to the number of contracts needed in the previous example. Assume that when evaluating the next daily settlement period you find that the S&P 500 spot price is 1,095 and the futures price is now 1,160. **Determine** the number of S&P 500 contracts needed after making a tailing the hedge adjustment.

**Answer:**

The number of contracts to sell is equal to:

$$1.4 \times [(\$20,000,000) / (1,150 \times 250)] \times (1,095 / 1,160) = 92 \text{ contracts}$$

## Adjusting the Portfolio Beta

### LO 36.6: Explain how to use stock index futures contracts to change a stock portfolio's beta.

Hedging an existing equity portfolio with index futures is an attempt to reduce the *systematic risk* of the portfolio. If the beta of the capital asset pricing model is used as the systematic risk measure, then hedging boils down to a reduction of the portfolio beta. Let  $\beta$  be our portfolio beta,  $\beta^*$  be our target beta after we implement the strategy with index futures,  $P$  be our portfolio value, and  $A$  be the value of the underlying asset (i.e., the stock index futures contract). To compute the appropriate number of futures, we use the following equation:

$$\text{number of contracts} = (\beta^* - \beta) \frac{P}{A}$$

This equation can result in either positive or negative values. Negative values indicate selling futures (decreasing systematic risk), and positive values indicate buying futures contracts (increasing systematic risk).

### Example: Adjusting portfolio beta

Suppose we have a well-diversified \$100 million equity portfolio. The portfolio beta relative to the S&P 500 is 1.2. The current value of the 3-month S&P 500 Index is 1,080. The portfolio manager wants to completely hedge the systematic risk of the portfolio over the next three months using S&P 500 Index futures. **Demonstrate** how to adjust the portfolio's beta.

**Answer:**

In this instance, our target beta,  $\beta^*$ , is 0, since a complete hedge is desired.

$$\text{number of contracts} = (0 - 1.2) \frac{100,000,000}{1,080 \times 250} = -444.44$$

The negative sign tells us we need to sell 444 contracts.

## ROLLING A HEDGE FORWARD

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**LO 36.7: Explain the term “rolling the hedge forward” and describe some of the risks that arise from this strategy.**

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When the hedging horizon is long relative to the maturity of the futures used in the hedging strategy, hedges have to be rolled forward as the futures contracts in the hedge come to maturity or expiration. Typically, as a maturity date approaches, the hedger must close out the existing position and replace it with another contract with a later maturity. This is called **rolling the hedge forward**.

When rolling a hedge forward, hedgers are not only exposed to the basis risk of the original hedge, they are also exposed to the basis risk of a new position each time the hedge is rolled forward. This is referred to as rollover basis risk, or simply **rollover risk**.



## KEY CONCEPTS

### LO 36.1

Hedging may be achieved by shorting futures to protect an underlying position against price deterioration or by buying futures to hedge against unanticipated price increases in an underlying asset.

### LO 36.2

Investors hedge with futures contracts to reduce or eliminate the price risk of an asset or a portfolio. The advantage of hedging is that it leads to less uncertainty regarding future profitability. The disadvantage of hedging is that it can lead to less profitability if the asset being hedged ends up increasing in value.

### LO 36.3

Basis risk is the risk that a difference may occur between the spot price of a hedged asset and the futures price of the contract used to implement the hedge. Basis risk is zero only when there is a perfect match between the hedged asset and the contract's underlying instrument in terms of maturity and asset type.

Three sources of basis risk are: (1) interruption in the convergence of the futures and spot prices, (2) changes in the cost of carry, and (3) imperfect matching between the cash asset and the hedge asset.

### LO 36.4

Sometimes it may be more efficient to cross hedge or hedge a cash position with a hedge asset that is closely related but different from the cash asset.

A hedge ratio is the ratio of the size of the futures position relative to the spot position necessary to provide a desired level of protection.

$$HR = \rho_{\text{spot,futures}} \times \frac{\sigma_{\text{spot}}}{\sigma_{\text{futures}}}$$

The effectiveness of the hedge measures the variance that is reduced by implementing the optimal hedge.

### LO 36.5

A common hedging application is the hedging of equity portfolios using futures contracts on stock indices (index futures). The number of futures contracts required to completely hedge an equity position is determined as follows:

$$\# \text{ of contracts} = \beta_{\text{portfolio}} \times \left( \frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}} \right)$$

**LO 36.6**

When hedging an equity portfolio with a short position in stock index futures, the beta of the portfolio is reduced. To change a stock portfolio's beta, use the following formula:

$$\text{number of contracts} = (\beta^* - \beta) \times \frac{\text{portfolio value}}{\text{value of futures contract}}$$

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**LO 36.7**

When the hedging horizon is longer than the maturity of the futures, the hedge must be rolled forward to retain the hedge. This exposes the hedger to rollover risk, the basis risk when the hedge is re-established.

## CONCEPT CHECKERS

Use the following data to answer Questions 1 and 2.

An equity portfolio is worth \$100 million with the benchmark of the Dow Jones Industrial Average. The Dow is currently at 10,000, and the corresponding portfolio beta is 1.2. The futures multiplier for the Dow is 10.

1. Which of the following is the closest to the number of contracts needed to double the portfolio beta?
  - A. 1,100.
  - B. 1,168.
  - C. 1,188.
  - D. 1,200.
2. To cut the beta in half, the correct trade is:
  - A. long 600 contracts.
  - B. short 600 contracts.
  - C. long 1,200 contracts.
  - D. short 1,200 contracts.
3. Which of the following situations describe a hedger with exposure to basis risk?
  - I. A portfolio manager for a large-cap growth fund knows he will be receiving a significant cash investment from a client within the next month and wants to pre-invest the cash using stock index futures.
  - II. A farmer has a large crop of corn he is looking to sell before June 30. The farmer uses a June futures contract to lock in his sales price.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
4. The standard deviation of price changes in a wheat futures contract is 0.6, while the standard deviation of changes in the price of wheat is 0.75. The covariance between the spot price changes and the futures price changes is 0.3825. Which of the following is closest to the optimal hedge ratio?
  - A. 0.478.
  - B. 0.850.
  - C. 1.063.
  - D. 1.250.

5. A large-cap value equity manager has a \$6,500,000 equity portfolio with a beta of 0.92. An S&P 500 futures contract is available with a current value of 1,175 and a multiplier of 250. What position should the manager take to completely hedge the portfolio's market risk?
- A. Short 20 contracts.
  - B. Short 22 contracts.
  - C. Short 24 contracts.
  - D. Long 22 contracts.

## CONCEPT CHECKER ANSWERS

1. **D**  $(2.4 - 1.2) \frac{100,000,000}{10,000 \times 10} = 1.2 \times 1,000 = 1,200$

where beta = 1.2, target beta = 2.4, A = 10 × 10,000, P = \$100 million

2. **B**  $(0.6 - 1.2) \frac{100,000,000}{10,000 \times 10} = -0.6 \times 1,000 = -600$

where beta = 1.2, target beta = 0.6, A = 10 × 10,000, P = \$100 million

3. **C** Both of these situations describe exposure to basis risk—the risk that the difference between the spot price and futures delivery price will change. The portfolio manager using futures to pre-invest the cash does not know the exact date he will receive the cash and may need to sell or hold the futures contract for a longer time period than intended. The farmer may need to sell his June futures contract early if he sells his corn earlier than the June futures expiration date.

4. **C** Notice in this problem, we were given the covariance but not the correlation. We can calculate the correlation using the formula learned back in the Quantitative Analysis material, as follows:

$$\rho = \frac{\text{COV}_{S,F}}{(\sigma_S)(\sigma_F)} = \frac{0.3825}{(0.75)(0.60)} = 0.85$$

Now that we have our correlation value, we can calculate the minimum hedge ratio as:

$$0.85 \left( \frac{0.75}{0.60} \right) = 1.0625, \text{ or, directly, } \frac{\text{Cov}_{S,F}}{\sigma_F^2} = \frac{0.3825}{0.6^2} = 1.0625$$

5. **A**  $0.92 \times \frac{6,500,000}{1,175 \times 250} \approx 20 \text{ contracts}$

Because the manager has a long position in the market, she will want to take a short position in the futures.

# INTEREST RATES

Topic 37

## EXAM FOCUS

Spot, or zero, rates are computed from coupon bonds using a method known as bootstrapping. Forward rates can then be computed from the spot or zero curve. For the exam, understand how to use the bootstrapping method and how to compute forward rates from spot rates. Also, be familiar with the discrete and continuous compounding methods. Note that the fixed income readings in Book 4 will provide more information on the calculation of spot and forward rates as well as constructing the spot and forward rate curves. Duration and convexity are also mentioned in this topic but will be discussed in much more detail in Book 4.

## TYPES OF RATES

**LO 37.1: Describe Treasury rates, LIBOR, and repo rates, and explain what is meant by the “risk-free” rate.**

Three interest rates play a key role in interest rate derivatives: Treasury rates, LIBOR, and repo rates. Keep in mind that interest rates increase as the credit risk of the underlying instrument increases.

- **Treasury rates.** Treasury rates are the rates that correspond to government borrowing in its own currency. They are considered risk-free rates.
- **LIBOR.** The London Interbank Offered Rate (LIBOR) is the rate at which large international banks fund their activities. Some credit risk exists with LIBOR.
- **Repo rates.** The “repo” or repurchase agreement rate is the implied rate on a repurchase agreement. In a repo agreement, one party agrees to sell a security to another with the understanding that the selling party will buy it back later at a specified higher price. The interest rate implied by the price differential is the repo rate. The most common repo is the overnight repurchase agreement. Longer-term agreements are called term repos. Depending on the parties and structure involved, there is some credit risk with repurchase agreements.



*Professor's Note: You may see reference to an inverse floater (a.k.a. reverse floater) on the exam. Just know that an inverse floater is a debt instrument whose coupon payments fluctuate inversely with the reference rate (e.g., LIBOR). For example, the inverse floater's coupon rate will increase when LIBOR decreases and vice versa.*

As mentioned, Treasury rates (such as T-bill and T-bond rates) are often considered the benchmark for nominal risk-free rates. However, derivative traders view these rates as being too low to be considered risk free (since part of the demand for these bonds comes from fulfilling regulatory requirements, which drives prices up and rates down). As a result, traders instead use LIBOR rates for short-term risk-free rates, because LIBOR better reflects a trader's opportunity cost of capital.



## COMPOUNDING

**LO 37.2: Calculate the value of an investment using different compounding frequencies.**

**LO 37.3: Convert interest rates based on different compounding frequencies.**

Derivative pricing often uses a framework called continuous time mathematics. In this framework, it is assumed that returns are continuously compounded. This is a theoretical construct only, as returns cannot literally be compounded continuously. Fortunately, converting discrete compounding to continuous compounding is straightforward.

If we have an initial investment of  $A$  that earns an annual rate  $R$ , compounded  $m$  times a year for  $n$  years, then it has a future value of:

$$FV_1 = A \left( 1 + \frac{R}{m} \right)^{m \times n}$$

If our same investment is continuously compounded over that period, it has a future value of:

$$FV_2 = Ae^{R \times n}$$

For any rate,  $R$ ,  $FV_2$  will always be greater than  $FV_1$ . The difference will decrease as  $m$  increases. In fact, as  $m$  becomes infinitely large, the difference goes to zero.

In most circumstances rates are discretely compounded, so we need to find the continuously compounded rate that gives the same future value. Using the previous two equations, the goal is to solve the following:

$$A \left( 1 + \frac{R}{m} \right)^{m \times n} = Ae^{R_c \times n}$$

where:

$R_c$  = the continuous rate

We can solve for  $R_c$  as:

$$R_c = m \times \ln \left( 1 + \frac{R}{m} \right)$$

We can also solve for  $R$  as:

$$R = m \left( e^{R_c / m} - 1 \right)$$



*Professor's Note: In order to algebraically solve for  $R$  or  $R_c$ , given one of the equations above, it is helpful to understand that  $e$  is the base of the natural log ( $\ln$ ). In other words, the natural log is the inverse function of the exponential function:  $e^{\ln(x)} = \ln(e^x) = x$ . So if you are given an equation such that  $R = e^x$ ;  $x$  will be equal to:  $\ln(R)$ .*

**Example: Computing continuous rates**

Suppose we have a 5% rate that is compounded semiannually. **Compute** the corresponding continuous rate. Repeat this for quarterly, monthly, weekly, and daily compounding.

**Answer:**

$$R_c = 2 \ln \left( 1 + \frac{0.05}{2} \right) = 0.049385$$

The results for other compounding frequencies are shown in Figure 1.

**Figure 1: Compounding Frequencies and Returns**

$m$	$R_c$
4	0.049690
12	0.049896
52	0.049976
250	0.049995

Notice that as  $m$  increases, the difference between the rates decreases.

**Example: Discrete compounding rate**

A loan is quoted at 12% annually with continuous compounding. Interest is paid monthly. **Calculate** the equivalent rate with monthly compounding.

**Answer:**

$$R = 12(e^{0.12/12} - 1) = 12.06\%$$

## SPOT (ZERO) RATES AND BOND PRICING

### LO 37.4: Calculate the theoretical price of a bond using spot rates.

**Spot rates** are the rates that correspond to zero-coupon bond yields. They are the appropriate discount rates for a single cash flow at a particular future time or maturity. Spot rates are also often called zero rates. Most interest rates that are observed in the market, such as coupon bond yields, are not spot rates.

## Bond Pricing

A coupon bond makes a series of cash flows. Each cash flow considered in isolation is equivalent to a zero-coupon bond. Using this interpretation, a coupon bond is a series of zero-coupon bonds, and its value, assuming continuous compounding and semiannual coupons, is:

$$B = \left( \frac{c}{2} \times \sum_{j=1}^N e^{-\frac{z_j}{2} \times j} \right) + \left( FV \times e^{-\frac{z_N}{2} \times N} \right)$$

where:

$c$  = the annual coupon

$N$  = the number of semiannual payment periods

$z_j$  = the bond equivalent spot rate that corresponds to  $j$  periods ( $j/2$  years) on a continuously compounded basis

$FV$  = the face value of the bond

Don't let this formula intimidate you. It simply says that the value of a bond is the present value of its cash flows, where each cash flow is discounted at the appropriate spot rate for its maturity. Notice that the negative sign on the rate just means that the coupon and principal payments are being discounted back to the present in a continuous fashion. The following example is a good illustration of the process.

### Example: Calculating bond price

**Compute** the price of a \$100 face value, 2-year, 4% semiannual coupon bond using the annualized spot rates in Figure 2.

**Figure 2: Spot Rates**

<i>Maturity (Years)</i>	<i>Spot Rate (%)</i>
0.5	2.5
1.0	2.6
1.5	2.7
2.0	2.9

**Answer:**

$$B = \left( \$2 \times e^{-\frac{0.025}{2} \times 1} \right) + \left( \$2 \times e^{-\frac{0.026}{2} \times 2} \right) + \left( \$2 \times e^{-\frac{0.027}{2} \times 3} \right) + \left( \$102 \times e^{-\frac{0.029}{2} \times 4} \right) = \$102.10$$

## Bond Yield

The yield of a bond is the single discount rate that equates the present value of a bond to its market price. You can use a financial calculator to compute bond yield, as in the following example.

**Example: Calculating bond yield**

Compute the yield for the bond in the previous example.

**Answer:**

$$\begin{aligned} \text{PMT} &= 2; N = 4; PV = -102.10; FV = 100; \text{CPT} \rightarrow I/Y = 1.456; \\ Y &= 1.456\% \times 2 \approx 2.91\% \end{aligned}$$

The bond's **par yield** is the rate which makes the price of a bond equal to its par value. When the bond is trading at par, the coupon will be equal to the bond's yield.

**BOOTSTRAPPING SPOT RATES**

The theoretical spot curve is derived by interpreting each Treasury bond (T-bond) as a package of zero-coupon bonds. Using the prices for each bond, the spot curve is computed using the bootstrapping methodology.

For example, suppose there is a T-bond maturing on a coupon date in exactly six months. Further assume that the bond is priced at 102.2969% of par and has a semiannual coupon of 6.125%. How is the corresponding spot rate computed? In this case, this is truly a zero-coupon bond, since there is only one cash flow, which occurs in six months. Simply solve for  $z_1$  in the bond valuation equation, given the price, as follows:

$$\$102.2969 = \left( \$100 + \frac{\$6.125}{2} \right) \times e^{-\frac{z_1}{2}}$$

Solving this for  $z_1$ :

$$z_1 = -2 \times \ln \left[ \frac{\$102.2969}{\left( \$100 + \frac{\$6.125}{2} \right)} \right] = 1.491\%$$

The 6-month spot rate on a bond equivalent basis is 1.491%. Also note that the yield to maturity did not need to be computed in this case because the yield to maturity (YTM) and the spot rate are the same.

How is the spot rate that corresponds to one year found? Suppose a T-bond that matures in one year is priced at 104.0469% of par and has a semiannual coupon of 6.25%. From the previous computation, the 6-month spot rate is known, so the bond valuation equation can be written as:

$$\$104.0469 = \left( \frac{\$6.25}{2} \times e^{-\frac{0.01491}{2}} \right) + \left( \$100 + \frac{\$6.25}{2} \right) \times e^{-\frac{z_2}{2} \times 2}$$

$$\Rightarrow z_2 = 0.02136 = 2.136\%$$

The 1-year spot rate with continuous compounding is 2.136%.

**Example: Bootstrapping spot rates**

Compute the corresponding spot rate curve using the information in Figure 3. Note that we've already computed the first two spot rates.

**Figure 3: Input Information to Bootstrap Spot Rates**

<i>Price as a Percentage of Par</i>	<i>Coupon</i>	<i>Semiannual Period</i>	<i>Maturity (Years)</i>
102.2969	6.125	1	0.5
104.0469	6.250	2	1.0
104.0000	5.250	3	1.5
103.5469	4.750	4	2.0

**Answer:**

The spot rates derived by bootstrapping are shown in Figure 4.

**Figure 4: Bootstrapped Spot Rate Curve**

<i>Price as a Percentage of Par</i>	<i>Coupon</i>	<i>Semiannual Period</i>	<i>Maturity (Years)</i>	<i>Spot Rates</i>
102.2969	6.125	1	0.5	1.491%
104.0469	6.250	2	1.0	2.136%
104.0000	5.250	3	1.5	2.515%
103.5469	4.750	4	2.0	2.915%

An alternative verification is to use the spot rates to check if they result in the same prices using the bond valuation equation. For example, using the spot rates will ensure computation of the same price for the 2-year bond:

$$B = \left( \frac{\$4.75}{2} \times e^{-\frac{0.01491}{2} \times 1} \right) + \left( \frac{\$4.75}{2} \times e^{-\frac{0.02136}{2} \times 2} \right) + \left( \frac{\$4.75}{2} \times e^{-\frac{0.02515}{2} \times 3} \right) + \left( \$100 + \frac{\$4.75}{2} \right) \times e^{-\frac{0.02915}{2} \times 4} = \$103.5469$$

This results in a bond price of \$103.5469. Notice that this is exactly the price of the 2-year bond.

## FORWARD RATES

### LO 37.5: Derive forward interest rates from a set of spot rates.

**Forward rates** are interest rates implied by the spot curve for a specified future period. The spot rates in Figure 4 are the appropriate rates that an investor should expect to realize for various maturities. Suppose an investor is faced with the following two investments, which are based on the spot curve in Figure 4.

1. Invest for two years at 2.915%.
2. Invest for a year at 2.136% and then roll over that investment for another year at the forward rate.

It does not matter which investment is chosen if they both offer the same return at the end of two years. This is the same as stating that both strategies give the same future value at the end of two years. Equating the two future values:

$$\frac{0.02915}{e^2} \times 4 = \frac{0.02136}{e^2} \times 2 \times e^{\frac{R_{\text{Forward}}}{2} \times 2}$$

where:

$R_{\text{Forward}}$  = the 1-year forward rate one year from now

As we will show, for the two strategies to be equal,  $R_{\text{Forward}}$  must be 3.693%.

We can simplify this calculation by using the following equation:

$$R_{\text{Forward}} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1} = R_2 + (R_2 - R_1) \times \left( \frac{T_1}{T_2 - T_1} \right)$$

where:

$R_i$  = the spot rate corresponding with  $T_i$  periods

$R_{\text{Forward}}$  = the forward rate between  $T_1$  and  $T_2$

For example, if the 1-year rate is 2.136% and the 2-year rate is 2.915%, the 1-year forward rate one year from now is:

$$R_{\text{Forward}} = 0.02915 + (0.02915 - 0.02136) \times \left( \frac{1}{2-1} \right) = 0.03694 = 3.694\%$$

This is the same forward rate (with slight rounding error) that was calculated before.

As a further example, consider the problem of finding the 1-year forward rate three years from now, given a 3-year spot rate of 7.424% and a 4-year spot rate of 8.216% (both continuously compounded annual rates). Based on the previous formula, the continuously compounded 1-year rate three years from now is:

$$0.08216 + (0.08216 - 0.07424) \times \frac{3}{4-3} = 0.10592$$



With this equation, generalizations can be made between the shape of the spot curve and the forward curve. The second term is always positive for an upward-sloping spot curve. Therefore, when there is an upward-sloping spot curve, the corresponding forward rate curve is upward-sloping and above the spot curve. Similarly, when there is a downward-sloping spot curve, the corresponding forward-rate curve is downward-sloping and below the spot curve.

## FORWARD RATE AGREEMENTS

### LO 37.6: Derive the value of the cash flows from a forward rate agreement (FRA).

A forward rate agreement (FRA) is a forward contract obligating two parties to agree that a certain interest rate will apply to a principal amount during a specified future time. Obviously, forward rates play a crucial role in the valuation of FRAs. The  $T_2$  cash flow of an FRA that promises the receipt or payment of  $R_K$  is:

$$\text{cash flow (if receiving } R_K) = L \times (R_K - R) \times (T_2 - T_1)$$

$$\text{cash flow (if paying } R_K) = L \times (R - R_K) \times (T_2 - T_1)$$

where:

$L$  = principal

$R_K$  = annualized rate on  $L$ , expressed with compounding period  $T_1 - T_2$

$R$  = annualized actual rate, expressed with compounding period  $T_1 - T_2$

$T_i$  = time  $i$ , expressed in years

The value of an FRA if we're receiving or paying is:

$$\text{value (if receiving } R_K) = L \times (R_K - R_{\text{Forward}}) \times (T_2 - T_1) \times e^{-R_2 \times T_2}$$

$$\text{value (if paying } R_K) = L \times (R_{\text{Forward}} - R_K) \times (T_2 - T_1) \times e^{-R_2 \times T_2}$$

where:

$R_{\text{Forward}}$  = forward rate between  $T_1$  and  $T_2$

Note that  $R_2$  is expressed as a continuously compounded rate.

#### Example: Computing the payoff from an FRA

Suppose an investor has entered into an FRA where he has contracted to pay a fixed rate of 3% on \$1 million based on the quarterly rate in three months. Assume that rates are compounded quarterly. Compute the payoff of the FRA if the quarterly rate is 1% in three months.

**Answer:**

For this FRA, the payoff will take place in six months. The net payoff will be the difference between the fixed-rate payment and the floating rate receipt. If the floating rate is 1% in three months, the payoff at the end of the sixth month will be:

$$\$1,000,000 (0.01 - 0.03)(0.25) = -\$5,000$$

**Example: Computing the value of an FRA**

Suppose the 3-month and 6-month LIBOR spot rates are 4% and 5%, respectively (continuously compounded rates). An investor enters into an FRA in which she will receive 8% (assuming quarterly compounding) on a principal of \$5,000,000 between months 3 and 6. Calculate the value of the FRA.

**Answer:**

$$R_{\text{Forward}} = 0.05 + (0.05 - 0.04) \times \left( \frac{1}{2 - 1} \right) = 0.06 = 6\%$$

$$R_{\text{Forward}} (\text{with quarterly compounding}) = 4 \times \left( e^{\frac{0.06}{4}} - 1 \right) = 0.060452 = 6.05\%$$

$$\text{value} = \$5,000,000 \times (0.0800 - 0.0605) \times (0.50 - 0.25) \times e^{-(0.05)(0.5)} = \$23,773$$

**DURATION****LO 37.7: Calculate the duration, modified duration and dollar duration of a bond.**

The duration of a bond is the average time until the cash flows on the bond are received. For a zero-coupon bond, this is simply the time to maturity. For a coupon bond, its duration will be necessarily shorter than its maturity. The weights on the time in years until each cash flow is to be received are the proportion of the bond's value represented by each of the coupon payments and the maturity payment. The formula for duration using continuously compounded discounting of the cash flows is:

$$\text{duration} = \sum_{i=1}^n t_i \left[ \frac{c_i e^{-yt_i}}{B} \right]$$

where:

$t_i$  = the time (in years) until cash flow  $c_i$  is to be received

$y$  = the continuously compounded yield (discount rate) based on a bond price of  $B$

The usefulness of the duration measure lies in the fact that the approximate change in a bond's price,  $B$ , for a parallel shift in the yield curve of  $\Delta y$  is:

$$\frac{\Delta B}{B} = -\text{duration} \times \Delta y$$

The change in yield is often expressed as a **basis point** change. One basis point is equivalent to 0.01%. So a 100 basis point change is a change of 1% in the yield.

**Modified duration** is used when the yield given is something other than a continuously compounded rate. When the yield is expressed as a semiannually compounded rate, for example, modified duration = duration/(1 + y/2). In general we can express this relation as: modified duration =  $\frac{\text{duration}}{1 + \frac{y}{m}}$ , where  $m$  is the number of compounding periods per year.

Note that as  $m$  goes to infinity (continuous compounding), the two measures are equal and there is no difference between the two.

On the exam, you may also see a reference to **dollar duration**. Dollar duration is simply modified duration multiplied by the price of the bond.

## CONVEXITY

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**LO 37.8: Evaluate the limitations of duration and explain how convexity addresses some of them.**

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Duration is a good approximation of price changes for an option-free bond, but it's only good for relatively small changes in interest rates. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more important, meaning that a linear estimate of price changes, such as duration, will contain errors.

In fact, the relationship between bond price and yield is not linear (as assumed by duration) but convex. This convexity shows that the difference between actual and estimated prices widens as the yield swings grow. That is, the widening error in the estimated price is due to the curvature of the actual price path. This is known as the **degree of convexity**.

Fortunately, the amount of convexity in a bond can be measured and used to supplement duration in order to achieve a more accurate estimate of the change in price. It's important to note that all convexity does is account for the amount of error in the estimated price change based on duration. In other words, it picks up where duration leaves off and converts the straight (estimated price) line into a curved line that more closely resembles the convex (actual price) line.

## Using Convexity to Improve Price Change Estimates

In order to obtain an estimate of the percentage change in price due to convexity, or the amount of price change that is not explained by duration, the following calculation will need to be made:

$$\text{convexity effect} = 1/2 \times \text{convexity} \times \Delta y^2$$

The convexity effect is typically quite small. However, remember that convexity is simply correcting for the error embedded in the duration, so you would expect convexity to have a much smaller effect than duration. Also note that for an option-free bond, the convexity effect is always positive, no matter which direction interest rates move. Thus, for option-free bonds, convexity is always added to duration to modify the price volatility errors embedded in duration. This decreases the drop in price (due to an increase in yields) and adds to the rise in price (due to a fall in yields).

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**LO 37.9: Calculate the change in a bond's price given its duration, its convexity, and a change in interest rates.**

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By combining duration and convexity, we can obtain a far more accurate estimate of the percentage change in the price of a bond, especially for large swings in yield. That is, you can account for the amount of convexity embedded in a bond by adding the convexity effect to the duration effect.

**Example: Estimating price changes with the duration/convexity approach**

**Estimate** the effect of a 100 basis point increase and decrease on a 10-year, 5%, option-free bond currently trading at par, using the duration/convexity approach. The bond has a duration of 7 and a convexity of 90.

**Answer:**

Using the duration/convexity approach:

percentage bond price change  $\approx$  duration effect + convexity effect

$$\Delta B_{+\Delta y} \approx [-7 \times 0.01] + [(1/2) \times 90 \times (0.01^2)]$$

$$\approx -0.07 + 0.0045 = -0.0655 = -6.55\%$$

$$\Delta B_{-\Delta y} \approx [-7 \times -0.01] + [(1/2) \times 90 \times (-0.01^2)]$$

$$\approx 0.07 + 0.0045 = 0.0745 = 7.45\%$$

## THEORIES OF THE TERM STRUCTURE

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**LO 37.10: Compare and contrast the major theories of the term structure of interest rates.**

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The **expectations theory** suggests that forward rates correspond to expected future spot rates. That is, forward rates are good predictors of expected future spot rates. In reality, the expectations theory fails to explain all future spot rate expectations. The **market segmentation theory** states that the bond market is segmented into different maturity sectors and that supply and demand for bonds in each maturity range dictate rates in that maturity range. The **liquidity preference theory** suggests that most depositors prefer short-term liquid deposits. In order to coax them to lend longer term, the intermediary will raise longer-term rates by adding a liquidity premium.

## KEY CONCEPTS

### LO 37.1

Three types of interest rates are particularly relevant in the interest rate derivative markets: Treasury rates, London Interbank Offered Rate (LIBOR), and repo rates. Treasury rates (such as T-bill and T-bond rates) are often considered the benchmark for nominal risk-free rates.

### LO 37.2

If we have an initial investment of  $A$  that earns an annual rate  $R$ , compounded  $m$  times a year for  $n$  years, then it has a future value of:

$$FV = A \left( 1 + \frac{R}{m} \right)^{m \times n}$$

### LO 37.3

In most circumstances, rates are discretely compounded so we need to find the continuously compounded rate that gives the same future value. The continuous rate can be solved as follows:

$$R_c = m \times \ln \left( 1 + \frac{R}{m} \right)$$

### LO 37.4

Zero (spot) rates correspond to the interest earned on a single cash flow at a single point in time. Bond prices are computed using the spot curve by discounting each cash flow at the appropriate spot rate.

The yield of a bond is the single discount rate that equates the present value of a bond to its market price.

Zero rates are computed using the bootstrapping methodology.

### LO 37.5

Forward rates are computed from spot rates. When the spot curve is upward-sloping, the corresponding forward rate curve is upward-sloping and above the spot curve. When the spot curve is downward-sloping, the corresponding forward rate curve is downward-sloping and below the spot curve.



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**LO 37.6**

A forward-rate agreement is a contract between two parties that an interest rate will apply to a specific principal during some future time period.

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**LO 37.7**

Duration and modified duration are the same when continuously compounded yields are used, and they both estimate the percentage price change of a bond from an absolute change in yield. Dollar duration is modified duration multiplied by the price of the bond.

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**LO 37.8**

Duration is only good for relatively small changes in interest rates. As rate changes grow larger, the curvature of the bond price/yield relationship becomes more important, meaning that a linear estimate of price changes, such as duration, will contain errors. The amount of convexity in a bond can be measured and used to supplement duration in order to achieve a more accurate estimate of the change in price.

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**LO 37.9**

The approximate change in a bond's price,  $B$ , for a parallel shift in the yield curve of  $\Delta y$  is:

$$\frac{\Delta B}{B} = -\text{duration} \times \Delta y$$

In order to obtain an estimate of the percentage change in price due to convexity, the following calculation will need to be made:

$$\text{convexity effect} = \frac{1}{2} \times \text{convexity} \times \Delta y^2$$

Combining duration and convexity creates a more accurate estimate of the percentage change in the price of a bond:

$$\text{percentage bond price change} \approx \text{duration effect} + \text{convexity effect}$$

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**LO 37.10**

The expectations theory suggests that forward rates correspond to expected future spot rates. The market segmentation theory states that bonds are segmented into different maturity sectors and that supply and demand dictate rates in the segmented maturity sectors. The liquidity preference theory suggests that longer-term rates incorporate a liquidity premium.

**CONCEPT CHECKERS**

1. What is the continuously compounded rate of return for an investment that has a value today of \$86.50 and will have a future value of \$100 in one year?  
A. 13.62%.  
B. 14.50%.  
C. 15.61%.  
D. 16.38%.
2. Assume that the continuously compounded 10-year spot rate is 5% and the 9-year spot rate is 4.9%. Which of the following is closest to the 1-year forward rate nine years from now?  
A. 4.1%.  
B. 5.1%.  
C. 5.9%.  
D. 6.0%.
3. An investor enters into a 1-year forward rate agreement (FRA) where she will receive the contracted rate on a principal of \$1 million. The contracted rate is a 1-year rate at 5%. Which of the following is closest to the cash flow if the actual rate is 6% at maturity of the underlying asset (loan)?  
A. -\$10,000.  
B. -\$1,000.  
C. +\$1,000.  
D. +\$10,000.
4. What is the bond price of a \$100 face value, 2.5-year, 3% semiannual coupon bond using the following annual continuously compounded spot rates:  $z_1 = 3\%$ ,  $z_2 = 3.1\%$ ,  $z_3 = 3.2\%$ ,  $z_4 = 3.3\%$ , and  $z_5 = 3.4\%$ ?  
A. \$97.27.  
B. \$97.83.  
C. \$98.15.  
D. \$98.99.
5. A \$100 face value, 1-year, 4% semiannual bond is priced at 99.806128. If the annualized 6-month spot rate ( $z_1$ ) is 4.1%, what is the 1-year spot rate ( $z_2$ )? (Both spots are continuously compounded rates.)  
A. 4.07%.  
B. 4.16%.  
C. 4.20%.  
D. 4.26%.

## CONCEPT CHECKER ANSWERS

1. B The formula to solve this problem is:

$$R_c = m \times \ln \left( 1 + \frac{R}{m} \right)$$

First, we need to compute  $R$  as the rate earned on the \$86.50 investment:

$$R = \frac{\$100 - \$86.50}{\$86.50} = 0.15607$$

This is essentially the effective rate earned over one year with annual compounding.

So,  $m = 1$ , and  $R_c = 1 \times \ln(1.15607) = 0.1450$ . Alternatively, since  $m = 1$ ,

$$\ln \left( \frac{100}{86.50} \right) = 0.1450 = 14.50\%$$

2. C  $R_{\text{Forward}} = R_2 + (R_2 - R_1) \times [T_1 / (T_2 - T_1)] = 0.05 + (0.05 - 0.049) \times [9 / (10 - 9)] = 5.9\%$
3. A  $\$1,000,000 (0.05 - 0.06)(1) = -\$10,000$
4. D  $B = 1.5 \times e^{[(-0.03/2) \times 1]} + 1.5 \times e^{[(-0.031/2) \times 2]} + 1.5 \times e^{[(-0.032/2) \times 3]} + 1.5 \times e^{[(-0.033/2) \times 4]} + 101.5 \times e^{[(-0.034/2) \times 5]} = 1.48 + 1.45 + 1.43 + 1.40 + 93.23 = \$98.99$
5. B  $B = 2 \times e^{[(-z_1/2) \times 1]} + 102 \times e^{[(-z_2/2) \times 2]}$ ;  $\$99.806128 = 2 \times e^{[(-0.041/2) \times 1]} + 102 \times e^{[(-z_2/2) \times 2]}$ ;  $\$97.846711 = 102 \times e^{[(-z_2/2) \times 2]}$ ;  $z_2 = 0.0415707 = 4.16\%$

# DETERMINATION OF FORWARD AND FUTURES PRICES

Topic 38

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## EXAM FOCUS

Both forward and futures contracts are obligations regarding a future transaction. Because the difference in pricing between these contract types is small, forward contract pricing and futures contract pricing are often presented interchangeably. The basic model for forward prices is the cost-of-carry model, which essentially connects the forward price to the cost incurred from purchasing and storing the underlying asset until the contract maturity date. Cash flows over the life of the contract are easily incorporated into the pricing model. Futures contracts contain delivery options that benefit the short seller of the contract. These delivery options must be incorporated into the futures pricing model.

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## INVESTMENT AND CONSUMPTION ASSETS

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### LO 38.1: Differentiate between investment and consumption assets.

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An **investment asset** is an asset that is held for the purpose of investing. This type of asset is held by many different investors for the sake of investment. Examples of investment assets include stocks and bonds. A **consumption asset** is an asset that is held for the purpose of consumption. Examples of consumption assets include commodities such as oil and natural gas.

## SHORT-SELLING AND SHORT SQUEEZE

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### LO 38.2: Define short-selling and calculate the net profit of a short sale of a dividend-paying stock.

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**Short sales** are orders to sell securities that the seller does not own. Short selling is also known as “shorting” and is possible with some investment assets. For a short sale, the short seller (1) simultaneously borrows and sells securities through a broker, (2) must return the securities at the request of the lender or when the short sale is closed out, and (3) must keep a portion of the proceeds of the short sale on deposit with the broker.

The short seller may be forced to close his position if the broker runs out of securities to borrow. This is known as a **short squeeze**, and the seller will need to close his short position immediately.

Why would anyone ever want to sell securities short? The seller thinks the current price is too high and that it will fall in the future, so the short seller hopes to sell high and then buy low. If a short sale is made at \$30 per share and the price falls to \$20 per share, the short seller can buy shares at \$20 to replace the shares borrowed and keep \$10 per share as profit.

Two rules currently apply to short selling:

1. The short seller must pay all dividends due to the lender of the security.
2. The short seller must deposit collateral to guarantee the eventual repurchase of the security.

**Example: Net profit of a short sale of a dividend-paying stock**

Assume that trader Alex Rodgers sold short XYZ stock in March by borrowing 200 shares and selling them for \$50/share. In April, XYZ stock paid a dividend of \$2/share. Calculate the net profit from the short sale assuming Rodgers bought back the shares in June for \$40/share in order to replace the borrowed shares and close out his short position.

**Answer:**

The cash flows from the short sale on XYZ stock are as follows:

March: borrow 200 shares and sell them for \$50/share	+\$10,000
April: short seller dividend payment to lender of \$2/share	–\$400
June: buyback shares for \$40/share to close short position	–\$8,000
Total net profit =	+\$1,600

## FORWARD AND FUTURES CONTRACTS

**LO 38.3:** Describe the differences between forward and futures contracts and explain the relationship between forward and spot prices.

**LO 38.4:** Calculate the forward price given the underlying asset's spot price, and describe an arbitrage argument between spot and forward prices.

**LO 38.9:** Calculate, using the cost-of-carry model, forward prices where the underlying asset either does or does not have interim cash flows.

Futures contracts and forward contracts are *similar* in that both:

- Can be either deliverable or cash settlement contracts.
- Are priced to have zero value at the time an investor enters into the contract.

Futures contracts *differ* from forward contracts in the following ways:

- Futures contracts trade on organized exchanges. Forwards are private contracts and do not trade on an exchange.
- Futures contracts are highly standardized. Forwards are customized contracts satisfying the needs of the parties involved.

- A single clearinghouse is the counterparty to all futures contracts. Forwards are contracts with the originating counterparty.
- The government regulates futures markets. Forward contracts are usually not regulated.

## FORWARD PRICES

The pricing model used to compute forward prices makes the following assumptions:

- No transaction costs or short-sale restrictions.
- Same tax rates on all net profits.
- Borrowing and lending at the risk-free rate.
- Arbitrage opportunities are exploited as they arise.

For the development of a forward pricing model, we will use the following notation:

- $T$  = time to maturity (in years) of the forward contract.
- $S_0$  = underlying asset price today ( $t = 0$ ).
- $F_0$  = forward price today.
- $r$  = continuously compounded risk-free annual rate.

The forward price may be written as:

### Equation 1

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$$F_0 = S_0 e^{rT}$$

The right-hand side of Equation 1 is the cost of borrowing funds to buy the underlying asset and carrying it forward to time  $T$ . Equation 1 states that this cost must equal the forward price. If  $F_0 > S_0 e^{rT}$ , then arbitrageurs will profit by selling the forward and buying the asset with borrowed funds. If  $F_0 < S_0 e^{rT}$ , arbitrageurs will profit by selling the asset, lending out the proceeds, and buying the forward. Hence, the equality in Equation 1 must hold. Note that this model assumes perfect markets.

As it turns out, actual short sales are not necessary for Equation 1 to hold. All that is necessary is a sufficient number of investors who are not only holding the investment asset but also are willing to sell the asset if the forward price becomes too low. In the event that the forward price is too low, the investor will sell the asset and take a long position in the forward contract. This is important since the arbitrage relationship in Equation 1 must hold for all investment assets even though short selling is not available for every asset.

#### Example: Computing a forward price with no interim cash flows

Suppose we have an asset currently worth \$1,000. The current continuously compounded rate is 4% for all maturities. **Compute** the price of a 6-month forward contract on this asset.

**Answer:**

$$F_0 = \$1,000e^{0.04(0.5)} = \$1,020.20$$



## Forward Price With Carrying Costs

If the underlying pays a known amount of cash over the life of the forward contract, a simple adjustment is made to Equation 1. Since the owner of the forward contract does not receive any of the cash flows from the underlying asset between contract origination and delivery, the present value of these cash flows must be deducted from the spot price when calculating the forward price. This is most easily seen when the underlying asset makes a periodic payment. With this in mind, we let  $I$  represent the *present value* of the cash flows over  $T$  years. Equation 1 then becomes:

### Equation 2

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$$F_0 = (S_0 - I) e^{rT}$$

The same arbitrage arguments used for Equation 1 are used here. The only modification is that the arbitrageur must account for the known cash flows.

#### Example: Forward price when underlying asset has a cash flow

**Compute** the price of a 6-month forward on a coupon bond worth \$1,000 that pays a 5% coupon semiannually. A coupon is to be paid in three months. Assume the risk-free rate is 4%.

**Answer:**

The cost of carry (income) in this case is computed as:

$$I = 25e^{-0.04(0.25)} = \$24.75125$$

Using Equation 2:

$$F_0 = (\$1,000 - \$24.75125)e^{0.04(0.5)} = \$994.95$$

## The Effect of a Known Dividend

When the underlying asset for a forward contract pays a dividend, we assume that the dividend is paid continuously. Letting  $q$  represent the continuously compounded dividend yield paid by the underlying asset expressed on a per annum basis, Equation 1 becomes:

### Equation 3

---

$$F_0 = S_0 e^{(r-q)T}$$

Once again, the same arbitrage arguments are used to prove that Equation 3 must be true.

**Example: Forward price when the underlying asset pays a dividend**

Compute the price of a 6-month forward contract for which the underlying asset is a stock index with a value of 1,000 and a continuous dividend yield of 1%. Assume the risk-free rate is 4%.

**Answer:**

Using Equation 3:

$$F_0 = 1,000e^{(0.04-0.01)0.5} = 1,015.11$$

**VALUE OF A FORWARD CONTRACT**

The initial value of a forward contract is zero. After its inception, the contract can have a positive value to one counterparty (and a negative value to the other). Since the forward price at every moment in time is computed to prevent arbitrage, the value at inception of the contract must be zero. The forward contract can take on a non-zero value only after the contract is entered into and the obligation to buy or sell has been made. If we denote the obligated delivery price after inception as  $K$ , then the value of the long contract on an asset with no cash flows is computed as  $S_0 - Ke^{-rT}$ ; with cash flows (with present value  $I$ ) it is  $S_0 - I - Ke^{-rT}$ ; and with a continuous dividend yield of  $q$ , it is  $S_0e^{-qT} - Ke^{-rT}$ .

**Example: Value of a stock index forward contract**

Using the stock index forward in the previous example, **compute** the value of a long position if the index increases to 1,050 immediately after the contract is purchased.

**Answer:**

In this case,  $K = 1,015.11$  and  $S_0 = 1,050$ , so the value is:

$$1,050e^{-0.01(0.5)} - 1,015.11e^{-0.04(0.5)} = 49.75$$

**CURRENCY FUTURES**


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**LO 38.6: Calculate a forward foreign exchange rate using the interest rate parity relationship.**


---

**Interest rate parity (IRP)** states that the forward exchange rate,  $F$  (measured in domestic per unit of foreign currency), must be related to the spot exchange rate,  $S$ , and to the interest rate differential between the domestic and the foreign country,  $r - r_F$

The general form of the interest rate parity condition is expressed as:

$$F = S e^{(r - r_f)T}$$

This equation is a no-arbitrage relationship. Using our notation from earlier, we can state the interest rate parity relationship as:

#### Equation 4

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$$F_0 = S_0 e^{(r - r_f)T}$$

Note that this is equivalent to Equation 3 with  $r_f$  replacing  $q$ . Just as the continuous dividend yield  $q$  was used to adjust the cost of carry, we use the continuous yield on a foreign currency deposit here.

#### Example: Currency futures pricing

Suppose we wish to **compute** the futures price of a 10-month futures contract on the Mexican peso. Each contract controls 500,000 pesos and is quoted in terms of dollar/peso. Assume that the continuously compounded risk-free rate in Mexico ( $r_f$ ) is 14%, the continuously compounded risk-free rate in the United States is 2%, and the current exchange rate is 0.12.

**Answer:**

Applying Equation 4:

$$F_0 = \$0.12 e^{(0.02 - 0.14) \frac{10}{12}} = \$0.10858 / \text{peso}$$



*Professor's Note: The concept of interest rate parity will show up again in the foreign exchange risk topic (Topic 49).*

## FORWARD PRICES VS. FUTURES PRICES

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### LO 38.5: Explain the relationship between forward and futures prices.

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The most significant difference between forward contracts and futures contracts is the daily marking to market requirement on futures contracts. When interest rates are known over the life of a contract,  $T$ , forward and futures prices can be shown to be the same. Various relationships can be derived, depending on the assumptions made between the value of the underlying and the level of change in interest rates. In general, when  $T$  is small, the price differences are usually very small and can be ignored. Empirical research comparisons of forwards and futures prices are mixed. Some studies conclude a significant difference and others do not. The important concept to understand here is that assuming the two are the same is an approximation, and under certain circumstances the approximation can be inaccurate.

## COMMODITY FUTURES

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**LO 38.7: Define income, storage costs, and convenience yield.**

**LO 38.8: Calculate the futures price on commodities incorporating income/storage costs and/or convenience yields.**

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*Professor's Note: Topic 45 later in this book is devoted to commodity forwards and futures. In that topic, you will learn more about storage costs and convenience yield as well as the arbitrage relationships that must hold with commodity futures.*

### Income and Storage Costs

When the underlying is considered a consumption asset, the pricing relationships developed above do not adequately capture all the necessary characteristics of the asset. *Consumption assets have actual storage costs associated with them.* These costs increase the carrying costs. The costs can be expressed either as a known cash flow or as a yield. Let  $U$  denote the present value of known storage cost over the life of the forward contract. Equation 1 then becomes:

#### Equation 5

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$$F_0 = (S_0 + U)e^{rT}$$

If we express the storage costs in terms of a continuous yield,  $u$ :

#### Equation 6

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$$F_0 = S_0 e^{(r+u)T}$$

The arbitrage relationships are the same except we need to account for the additional carrying costs over  $T$  years. However, when the owner of these assets is reluctant to sell the asset, Equations 5 and 6 are replaced by:

#### Equation 7

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$$F_0 \leq (S_0 + U)e^{rT}$$

And:

#### Equation 8

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$$F_0 \leq S_0 e^{(r+u)T}$$

## CONVENIENCE YIELD

Equations 7 and 8 suggest there is a *benefit to owning the underlying consumable asset compared to owning the futures contract*. If we introduce a **convenience yield**,  $y$ , to balance Equations 7 and 8, we have:

$$F_0 e^{yT} = (S_0 + U) e^{rT} = S_0 e^{(r+u)T}$$

This formula can be reduced to:

### Equation 9

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$$F_0 = S_0 e^{(r+u-y)T}$$

In other words, the convenience yield is simply the yield required to produce an equality and is thus a measure of the benefit of owning spot, or physical, consumption commodities.

## DELIVERY OPTIONS IN THE FUTURES MARKET

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**LO 38.10: Describe the various delivery options available in the futures markets and how they can influence futures prices.**

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Some futures contracts grant **delivery options** to the short—options on what, where, and when to deliver. Some Treasury bond contracts give the short a choice of several bonds that are acceptable to deliver and options as to when to deliver during the expiration month. Physical assets, such as gold or corn, may offer a choice of delivery locations to the short. These options can be of significant value to the holder of the short position in a futures contract.

As shown in the previous discussion on commodity futures, if the cost of carrying the asset is greater than the convenience yield (benefit from holding the physical asset), it is ideal for the short position to deliver the contract early. This scenario suggests that the futures price will increase over time; hence, the short has an incentive to deliver early. The opposite relationship holds true when the cost of carry is less than the convenience yield. In this case, the short position will delay delivery since the futures price is expected to fall over time.

## FUTURES AND EXPECTED FUTURE SPOT PRICES

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**LO 38.11: Explain the relationship between current futures prices and expected future spot prices, including the impact of systematic and nonsystematic risk.**

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The cost of carry model is a widely used method for estimating the appropriate price of a futures contract, but other theories exist for explaining the futures price. One intuitively appealing model expresses the futures price as a function of the expected spot price ( $S_T$ ).

$$F_0 = E(S_T)$$

For obvious reasons, this is called the **expectations model** and states that the current futures price for delivery at time  $T$  is equal to the expected spot price at time  $T$ . Similar to the no-arbitrage rule, this model acts to keep the current futures price in line with the expected spot rate at that time. If the futures price is less than the expected price, aggressive buying of the futures would push up the futures price. If the futures price is greater than the expected spot rate, aggressive selling of the futures would lead to lower the futures price. Although intuitively appealing, other factors probably play a role in the pricing mechanism. Indeed, if the expectations model limited traders to a risk-free rate of return, there would be no incentive to buy or sell contracts.

### Cost of Carry vs. Expectations

Economist John Maynard Keynes found the expectations model to be flawed precisely because it provided no justification for speculators to enter the market. Futures contracts provide a mechanism to transfer risk from those who need to hedge their positions (e.g., farmers who are long the commodities) to speculators. In order to entice speculators to bear the risk of these contracts, there has to exist an expectation of profit greater than the risk-free rate. For this to occur, the futures contract price must be less than the expected spot rate at maturity [ $F_0 < E(S_T)$ ] and must continually increase during the term of the contract. Keynes referred to this as **normal backwardation**. This relationship suggests that the asset underlying the futures contract exhibits positive systematic risk, since this is the risk that remains after diversifying away all nonsystematic risk.

On the other side of the contracts are those who are users of the commodity who want to shift some of the risk of rising market prices to speculators. They wish to purchase futures contracts from speculators. The speculators have to be enticed into assuming this risk by the expectation of profits that would exceed the risk-free rate. From this perspective, the futures price must be higher than the expected spot price at maturity [ $F_0 > E(S_T)$ ] and must continually decrease during the term of the contract. Keynes referred to this expectation as **contango** (a.k.a. normal contango). This relationship suggests that the asset underlying the futures contract exhibits negative systematic risk.

### CONTANGO AND BACKWARDATION

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**LO 38.12: Define and interpret contango and backwardation, and explain how they relate to the cost-of-carry model.**

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**Backwardation** refers to a situation where the futures price is below the spot price. For this to occur, there must be a significant benefit to holding the asset. Backwardation might occur if there are benefits to holding the asset that offset the opportunity cost of holding the asset (the risk-free rate) and additional net holding costs.

**Contango** refers to a situation where the futures price is above the spot price. If there are no benefits to holding the asset (e.g., dividends, coupons, or convenience yield), contango will occur because the futures price will be greater than the spot price.



*Professor's Note: In this case, the reference to backwardation and contango refers to the relationship between the futures price and the current spot price, not the expected spot price.*



## KEY CONCEPTS

### LO 38.1

An investment asset is an asset that is held for the purpose of investing. A consumption asset is an asset that is held for the purpose of consumption.

### LO 38.2

Short sales are orders to sell securities that the seller does not own. A short squeeze results if the broker runs out of securities to borrow.

### LO 38.3

Forward and futures contracts are similar because they are both future obligations to transact an asset on some future date. Forward contracts do not trade on an exchange, are not standardized, and do not normally close out prior to expiration.

The relationship between forward and spot prices is as follows:

$$F = S_0 e^{rT}$$

### LO 38.4

The cost-of-carry model is used to price forward and futures contracts. It states that the total cost of carrying the underlying asset to expiration must be the futures price. Any other price results in arbitrage.

### LO 38.5

When interest rates are known over the life of a contract, forward and futures prices can be shown to be the same. Various relationships can be derived, depending on the assumptions made between the value of the underlying and the level of change in interest rates.

### LO 38.6

Interest rate parity states that the forward exchange rate,  $F$  (measured in domestic per unit of foreign currency), must be related to the spot exchange rate,  $S$ , and to the interest rate differential between the domestic and the foreign country:

$$F = S_0 e^{(r_{DC} - r_{FC})T}$$

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**LO 38.7**

Consumption assets have actual storage costs (known as carrying costs) associated with them.

If there is a benefit to owning the underlying consumable asset compared to owning the futures, the futures price will incorporate a convenience yield.

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**LO 38.8**

Futures price with storage costs,  $u$ :  $F = S_0 e^{(r+u)T}$

Futures price with convenience yield,  $y$ :  $F = S_0 e^{(r+u-y)T}$

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**LO 38.9**

The futures price or cost-of-carry model is easily accommodated for interim cash flows from the underlying asset. If the underlying asset pays a known amount of cash,  $I$ , over the life of the forward contract, a simple adjustment is made to the cost-of-carry model:

$$F = (S_0 - I)e^{rT}$$

When the underlying asset pays a dividend,  $q$ , we assume that the dividend is paid continuously:

$$F = S_0 e^{(r-q)T}$$

---

**LO 38.10**

Physical assets, such as gold or corn, may offer a choice of delivery locations to the short. These options can be of significant value to the holder of the short position in a futures contract. Futures contracts are typically “offset” by buying or selling a contract before the delivery date. Only a small percentage of contracts result in physical delivery.

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**LO 38.11**

The expectations model states that the current futures price for delivery at time  $T$  is equal to the expected spot price at time  $T$ . This model acts to keep the current futures price in line with the expected spot rate at that time.

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**LO 38.12**

Contango is the situation in which the futures price is above the current spot price. Backwardation is the opposite relationship.

## CONCEPT CHECKERS

Use the following data to answer Questions 1 and 2.

An investor has an asset that is currently worth \$500, and the continuously compounded risk-free rate at all maturities is 3%.

1. Which of the following is the closest to the no-arbitrage price of a 3-month forward contract?
  - A. \$496.26.
  - B. \$500.00.
  - C. \$502.00.
  - D. \$503.76.
2. If the asset pays a continuous dividend of 2%, which of the following is the closest to the no-arbitrage price of a 3-month forward contract?
  - A. \$494.24.
  - B. \$498.75.
  - C. \$501.25.
  - D. \$506.29.
3. A bond pays a semiannual coupon of \$40 and has a current value of \$1,109. The next payment on the bond is in four months and the interest rate is 6.50%. Using the continuous time model, the price of a 6-month forward contract on this bond is closest to:
  - A. \$995.62.
  - B. \$1,011.14.
  - C. \$1,035.65.
  - D. \$1,105.20.
4. The owner of 300,000 bushels of corn wishes to hedge his position for a sale in 150 days. The current price of corn is \$1.50/bushel and the contract size is 5,000 bushels. The interest rate is 7%, compounded daily. The storage cost for the corn is \$18/day. Assume the cost of storage as a percentage of the contract per year is 1.46%. The price for the appropriate futures contract used to hedge the position is closest to:
  - A. \$6,635.
  - B. \$7,248.
  - C. \$7,656.
  - D. \$7,765.
5. Backwardation refers to a situation where:
  - A. spot prices are above futures prices.
  - B. spot prices are below futures prices.
  - C. expected future spot prices are above futures prices.
  - D. expected future spot prices are below futures prices.

## CONCEPT CHECKER ANSWERS

1. D Using Equation 1:

$$500e^{(0.03)(0.25)} = \$503.76$$

where  $S = 500$ ,  $T = 0.25$ , and  $r = 0.03$

2. C Using Equation 3:

$$500e^{(0.03-0.02)0.25} = \$501.25$$

3. D Use the formula  $F_0 = (S_0 - I)e^{rT}$ , where  $I$  is the present value of \$40 to be received in 4 months, or 0.333 years. At a discount rate of 6.50%:

$$I = \$40 \times e^{-0.065 \times 0.333} = \$39.14$$

$$F_0 = (\$1,109 - 39.14) \times e^{(0.065 \times 0.5)} = \$1,105.20$$

4. D Since both the interest and the storage costs compound on a daily basis, a continuous time model is appropriate to approximate the price of the contract.

The cost of storage as a percentage of the contract per year is:

$$u = 365 \times \frac{18}{1.50 \times 300,000} = 0.0146$$

Using Equation 6, the futures price per bushel is:

$$F = \$1.50 \times e^{(0.07 + 0.0146)(150/365)} = \$1.553 \times 5,000 \text{ bushels per contract} = \$7,765.34$$

5. A Backwardation refers to a situation where spot prices are higher than futures prices. Significant monetary benefits of the asset or a relatively high convenience yield can lead to this result.

# INTEREST RATE FUTURES

Topic 39

## EXAM FOCUS

In this topic, we examine Treasury bonds (T-bonds) and Eurodollar futures contracts. These instruments are two of the most popular interest rate futures contracts that trade in the United States. Be able to define the cheapest-to-deliver bond for T-bonds and know how to use the convexity adjustment for Eurodollar futures. Duration-based hedging using interest rate futures is also discussed. Be familiar with the equation to calculate the number of contracts needed to conduct a duration-based hedge.

## DAY COUNT CONVENTIONS

**LO 39.1: Identify the most commonly used day count conventions, describe the markets that each one is typically used in, and apply each to an interest calculation.**

Day count conventions play a role when computing the interest that accrues on a fixed income security. When a bond is purchased, the buyer must pay any **accrued interest** earned through the settlement date.

$$\text{accrued interest} = \text{coupon} \times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$$

In the United States, there are three commonly used day count conventions.

1. U.S. Treasury bonds use **actual/actual**.
2. U.S. corporate and municipal bonds use **30/360**.
3. U.S. money market instruments (Treasury bills) use **actual/360**.

The following examples demonstrate the use of day count conventions when computing accrued interest.

### Example: Day count conventions

Suppose there is a semiannual-pay bond with a \$100 par value. Further assume that coupons are paid on March 1 and September 1 of each year. The annual coupon is 6%, and it is currently July 13. **Compute** the accrued interest of this bond as a T-bond and a U.S. corporate bond.

**Answer:**

The T-bond uses actual/actual (in period), and the reference (March 1 to September 1) period has 184 days. There are 134 actual days from March 1 to July 13, so the accrued interest is:

$$\frac{134}{184} \times \$3 = \$2.1848$$

The corporate bond uses 30/360, so the reference period now has 180 days. Using this convention, there are 132 (= 30 × 4 + 12) days from March 1 to July 13, so the accrued interest is:

$$\frac{132}{180} \times \$3 = \$2.20$$

## QUOTATIONS FOR T-BONDS

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**LO 39.3: Differentiate between the clean and dirty price for a US Treasury bond; calculate the accrued interest and dirty price on a US Treasury bond.**

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T-bond prices are quoted relative to a \$100 par amount in dollars and 32nds. So a 95–05 is 95 5/32, or 95.15625. The quoted price of a T-bond is not the same as the cash price that is actually paid to the owner of the bond. In general:

$$\text{cash price} = \text{quoted price} + \text{accrued interest}$$

### Clean and Dirty Prices

The cash price (a.k.a. **invoice price** or **dirty price**) is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond (a.k.a. **quoted price** or **clean price**) plus the accrued interest. This relationship is shown in the equation above. Conversely, the clean price is the cash price less accrued interest:

$$\text{quoted price} = \text{cash price} - \text{accrued interest}$$

This relationship can also be expressed as:

$$\text{clean price} = \text{dirty price} - \text{accrued interest}$$



**Example: Calculate the cash price of a bond**

Assume the bond in the previous example is a T-bond currently quoted at 102–11.  
**Compute the cash price.**

**Answer:**

$$\text{cash price} = \$102.34375 + \$2.1848 = \$104.52855$$

For a \$100,000 par amount, this is \$104,528.55.

## QUOTATIONS FOR T-BILLS

### LO 39.2: Calculate the conversion of a discount rate to a price for a US Treasury bill.

T-bills and other money-market instruments use a discount rate basis and an actual/360 day count. A T-bill with a \$100 face value with  $n$  days to maturity and a cash price of  $Y$  is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

This is referred to as the discount rate in annual terms. However, this discount rate is not the actual rate earned on the T-bill. The following example shows the calculation of the annualized yield on a T-bill, given its price.

**Example: Calculating the cash price on a T-bill**

Suppose you have a 180-day T-bill with a discount rate, or quoted price, of five (i.e., the annualized rate of interest earned is 5% of face value). If face value is \$100, what is the true rate of interest and the cash price?

**Answer:**

Interest is equal to \$2.5 ( $= \$100 \times 0.05 \times 180 / 360$ ) for a 180-day period. The true rate of interest for the period is therefore 2.564% [ $= 2.5 / (100 - 2.5)$ ].

Cash price:  $5 = (360 / 180) \times (100 - Y)$ ;  $Y = \$97.5$ .

## TREASURY BOND FUTURES

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**LO 39.4: Explain and calculate a US Treasury bond futures contract conversion factor.**

**LO 39.5: Calculate the cost of delivering a bond into a Treasury bond futures contract.**

**LO 39.6: Describe the impact of the level and shape of the yield curve on the cheapest-to-deliver Treasury bond decision.**

---

In a T-bond futures contract, any government bond with more than 15 years to maturity on the first of the delivery month (and not callable within 15 years) is deliverable on the contract. This produces a large supply of potential bonds that are deliverable on the contract and reduces the likelihood of market manipulation. Since the deliverable bonds have very different market values, the Chicago Board of Trade (CBOT) has created **conversion factors**. The conversion factor defines the price received by the short position of the contract (i.e., the short position is delivering the contract to the long). Specifically, the cash received by the short position is computed as follows:

$$\text{cash received} = (\text{QFP} \times \text{CF}) + \text{AI}$$

where:

QFP = quoted futures price (most recent settlement price)

CF = conversion factor for the bond delivered

AI = accrued interest since the last coupon date on the bond delivered

Conversion factors are supplied by the CBOT on a daily basis. Conversion factors are calculated as: (discounted price of a bond – accrued interest) / face value. For example, if the present value of a bond is \$142, accrued interest is \$2, and face value is \$100, the conversion factor would be:  $(142 - 2) / 100 = 1.4$ .

### Cheapest-to-Deliver Bond

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver. The procedure to determine which bond is the cheapest-to-deliver (CTD) is as follows:

$$\text{cash received by the short} = (\text{QFP} \times \text{CF}) + \text{AI}$$

$$\text{cost to purchase bond} = (\text{quoted bond price} + \text{AI})$$

The CTD bond minimizes the following: quoted bond price – (QFP × CF). This expression calculates the cost of delivering the bond.

### Example: The cheapest-to-deliver bond

Assume an investor with a short position is about to deliver a bond and has four bonds to choose from which are listed in the following table. The last settlement price is \$95.75 (this is the quoted futures price). **Determine** which bond is the cheapest-to-deliver.

<i>Bond</i>	<i>Quoted Bond Price</i>	<i>Conversion Factor</i>
1	99	1.01
2	125	1.24
3	103	1.06
4	115	1.14

**Answer:**

Cost of delivery:

Bond 1:  $99 - (95.75 \times 1.01) = \$2.29$

Bond 2:  $125 - (95.75 \times 1.24) = \$6.27$

Bond 3:  $103 - (95.75 \times 1.06) = \$1.51$

Bond 4:  $115 - (95.75 \times 1.14) = \$5.85$

Bond 3 is the cheapest-to-deliver with a cost of delivery of \$1.51.

Finding the cheapest-to-deliver bond does not require any arcane procedures but could involve searching among a large number of bonds. The following guidelines give an indication of what type of bonds tend to be the cheapest-to-deliver under different circumstances:

- When yields > 6%, CTD bonds tend to be low-coupon, long-maturity bonds.
- When yields < 6%, CTD bonds tend to be high-coupon, short-maturity bonds.
- When the yield curve is upward sloping, CTD bonds tend to have longer maturities.
- When the yield curve is downward sloping, CTD bonds tend to have shorter maturities.

## TREASURY BOND FUTURES PRICE

**LO 39.7: Calculate the theoretical futures price for a Treasury bond futures contract.**

Recall the cost-of-carry relationship, where the underlying asset pays a known cash flow, as was presented in the previous topic. The futures price is calculated in the following fashion:

$$F_0 = (S_0 - I)e^{rT}$$

where:

$I$  = present value of cash flow

We can use this equation to calculate the theoretical futures price when accounting for the CTD bond's accrued interest and its conversion factor.

**Example: Theoretical futures price**

Suppose that the CTD bond for a Treasury bond futures contract pays 10% semiannual coupons. This CTD bond has a conversion factor of 1.1 and a quoted bond price of 100. Assume that there are 180 days between coupons and the last coupon was paid 90 days ago. Also assume that Treasury bond futures contract is to be delivered 180 days from today, and the risk-free rate of interest is 3%. Calculate the theoretical price for this T-bond futures contract.

**Answer:**

The cash price of the CTD bond is equal to the quoted bond price plus accrued interest. Accrued interest is computed as follows:

$$AI = \text{coupon} \times \left( \frac{\text{number of days from last coupon to settlement date}}{\text{number of days in coupon period}} \right)$$

$$AI = 5 \times \frac{90}{180} = 2.5$$

$$\text{cash price} = 100 + 2.5 = 102.5$$

Since the next coupon will be received 90 days from today, that cash flow should be discounted back to the present using the familiar present value equation which discounts the cash flow using the risk-free rate:

$$5e^{-0.03 \times (90/365)} = \$4.96$$

Using the cost-of-carry model, the cash futures price (which expires 180 days from today) is then calculated as follows:

$$F_0 = (102.5 - 4.96)e^{(0.03)(180/365)} = 98.99$$

We are not done, however, since the futures contract expires 90 days after the last coupon payment. The quoted futures price at delivery is calculated after subtracting the amount of accrued interest (recall: QFP = cash futures price – AI).

$$98.99 - \left( 5 \times \frac{90}{180} \right) = \$96.49$$

Finally, the conversion factor is utilized, producing a theoretical price for this T-bond futures contract of:

$$QFP = \frac{96.49}{1.1} = \$87.72$$

## EURODOLLAR FUTURES

**LO 39.8: Calculate the final contract price on a Eurodollar futures contract.**

**LO 39.9: Describe and compute the Eurodollar futures contract convexity adjustment.**

The 3-month eurodollar futures contract trades on the Chicago Mercantile Exchange (CME) and is the most popular interest rate futures in the United States. This contract settles in cash and the minimum price change is one “tick,” which is a price change of one basis point, or \$25 per \$1 million contract. Eurodollar futures are based on a eurodollar deposit (a eurodollar is a U.S. dollar deposited outside the United States) with a face amount of \$1 million. The interest rate underlying this contract is essentially the 3-month (90-day) forward LIBOR. If  $Z$  is the quoted price for a eurodollar futures contract, the contract price is:

$$\text{eurodollar futures price} = \$10,000[100 - (0.25)(100 - Z)]$$

For example, if the quoted price,  $Z$ , is 97.8:

$$\text{contract price} = \$10,000[100 - (0.25)(100.0 - 97.8)] = \$994,500$$

### Convexity Adjustment

The corresponding 90-day forward LIBOR (on an annual basis) for each contract is  $100 - Z$ . For example, assume that the previous eurodollar contract was for a futures contract that matured in six months. Then the 90-day forward LIBOR six months from now is approximately 2.2% ( $100 - 97.8$ ). However, the daily marking to market aspect of the futures contract can result in differences between actual forward rates and those implied by futures contracts. This difference is reduced by using the convexity adjustment. In general, long-dated eurodollar futures contracts result in implied forward rates larger than actual forward rates. The two are related as follows:

$$\text{actual forward rate} = \text{forward rate implied by futures} - \left(\frac{1}{2} \times \sigma^2 \times T_1 \times T_2\right)$$

where:

$T_1$  = the maturity on the futures contract

$T_2$  = the time to the maturity of the rate underlying the contract (90 days)

$\sigma$  = the annual standard deviation of the change in the rate underlying the futures contract, or 90-day LIBOR

Notice that as  $T_1$  increases, the convexity adjustment will need to increase. So as the maturity of the futures contract increases, the necessary convexity adjustment increases. Also, note that the  $\sigma$  and the  $T_2$  are largely dictated by the specifications of the futures contract.

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**LO 39.10: Explain how Eurodollar futures can be used to extend the LIBOR zero curve.**


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Forward rates implied by convexity-adjusted eurodollar futures can be used to produce a LIBOR spot curve (also called a LIBOR zero curve since spot rates are sometimes referred to as zero rates). Recall the equation presented previously in Topic 37, which was used to generate the shape of the *futures* rate curve:

$$R_{\text{Forward}} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

where:

$R_i$  = spot rate corresponding with  $T_i$  periods

$R_{\text{Forward}}$  = the forward rate between  $T_1$  and  $T_2$

This forward rate equation can be rearranged to solve for the *spot* rate for the next time period ( $T_2$ ):

$$R_2 = \frac{R_{\text{Forward}}(T_2 - T_1) + R_1 T_1}{T_2}$$

Given the first LIBOR spot rate ( $R_1$ ) and the length of each forward contract period, we can calculate the next spot rate ( $R_2$ ). The rate at  $T_2$  can then be used to find the rate at  $T_3$  and so on. The end result is a generated LIBOR spot (zero) curve.

### DURATION-BASED HEDGING

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**LO 39.11: Calculate the duration-based hedge ratio and create a duration-based hedging strategy using interest rate futures.**


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The objective of a **duration-based hedge** is to create a combined position that does not change in value when yields change by a small amount. In other words, a position that has a duration of zero needs to be produced. The combined position consists of our portfolio with a hedge horizon value of  $P$  and a futures position with a contract value of  $F$ . Denote the duration of the portfolio at the hedging horizon as  $D_P$  and the corresponding duration of the futures contract as  $D_F$ . Using this notation, the duration-based hedge ratio can be expressed as follows:

$$N = -\frac{P \times D_P}{F \times D_F}$$

where:

$N$  = number of contracts to hedge

The minus sign suggests that the futures position is the opposite of the original position. In other words, if the investor is long the portfolio, he must short  $N$  contracts to produce a position with a zero duration.

### Example: Duration-based hedge

Assume there is a 6-month hedging horizon and a portfolio value of \$100 million. Further assume that the 6-month T-bond contract is quoted at 105–09, with a contract size of \$100,000. The duration of the portfolio is 10, and the duration of the futures contract is 12. Outline the appropriate hedge for small changes in yield.

**Answer:**

$$N = -\frac{100,000,000 \times 10}{105,281.25 \times 12} = -791.53$$

Rounding up to the nearest whole number means the manager should short 792 contracts.

## LIMITATIONS OF DURATION

### LO 39.12: Explain the limitations of using a duration-based hedging strategy.

The price/yield relationship of a bond is convex, meaning it is nonlinear in shape. Duration measures are linear approximations of this relationship. Therefore, as the change in yield increases, the duration measures become progressively less accurate. Moreover, duration implies that all yields are perfectly correlated. Both of these assumptions place limitations on the use of duration as a single risk measurement tool. When changes in interest rates are both large and nonparallel (i.e., not perfectly correlated), duration-based hedge strategies will perform poorly.



## KEY CONCEPTS

### LO 39.1

Day count conventions play a role when computing the interest that accrues on a fixed income security. When a bond is purchased, the buyer must pay any accrued interest earned through the settlement date. The most common day count conventions are Actual/Actual, 30/360, and Actual/360.

### LO 39.2

T-bills are quoted on a discount rate basis. A T-bill with a \$100 face value with  $n$  days to maturity and a cash price of  $Y$  is quoted as:

$$\text{T-bill discount rate} = \frac{360}{n}(100 - Y)$$

### LO 39.3

For a U.S. Treasury bond, the dirty price is the price that the seller of the bond must be paid to give up ownership. It includes the present value of the bond plus the accrued interest. Conversely, the clean price is the dirty price less accrued interest.

### LO 39.4

Since deliverable bonds have very different market values, the Chicago Board of Trade (CBOT) has created conversion factors. Conversion factors are supplied by the CBOT on a daily basis. They are calculated as:

$$(\text{bond discounted price} - \text{accrued interest}) / \text{face value}$$

### LO 39.5

The conversion factor system is not perfect and often results in one bond that is the cheapest (or most profitable) to deliver. The cheapest-to-deliver (CTD) bond is the bond that minimizes the following:

$$\text{quoted bond price} - (\text{quoted futures price} \times \text{conversion factor})$$

### LO 39.6

When the yield curve is not flat, there is a single bond that is the cheapest-to-deliver (CTD). When the yield curve is upward sloping, CTD bonds tend to have longer maturities. When the yield curve is downward sloping, CTD bonds tend to have shorter maturities.

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### LO 39.7

The theoretical price for a T-bond futures contract is calculated as:

$$(\text{cash futures price} - \text{accrued interest}) / \text{conversion factor}$$

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### LO 39.8

Eurodollar contracts are based on LIBOR and are quoted on a discount rate basis. If  $Z$  is the quoted price for a eurodollar futures contract, the contract price is:

$$\text{eurodollar futures price} = \$10,000 \times [100 - (0.25) \times (100 - Z)]$$

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### LO 39.9

Long-dated eurodollar contracts must be adjusted for convexity before being used to estimate the corresponding forward rates. As the maturity of the futures contract increases, the necessary convexity adjustment increases.

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### LO 39.10

Forward rates implied by convexity-adjusted eurodollar futures can be used to produce a LIBOR spot curve. The following equation is used to generate the shape of the futures rate curve:

$$R_{\text{Forward}} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

where :

$R_i$  = spot rate corresponding with  $T_i$  periods

$R_{\text{Forward}}$  = the forward rate between  $T_1$  and  $T_2$

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### LO 39.11

Duration can be used to compute the number of futures contracts needed to implement a duration-based hedging strategy. The duration-based hedge ratio can be expressed as follows:

$$\text{number of contracts} = - \frac{\text{portfolio value} \times \text{duration}_{\text{portfolio}}}{\text{futures value} \times \text{duration}_{\text{futures}}}$$

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### LO 39.12

The effectiveness of duration-based hedging strategies is limited when there are large changes in yield or nonparallel shifts in the yield curve.

## CONCEPT CHECKERS

1. Assume a 6-month hedging horizon and a portfolio value of \$30 million. Further assume that the 6-month Treasury bond (T-bond) contract is quoted at 100–13, with a contract size of \$100,000. The duration of the portfolio is 8, and the duration of the futures contract is 12. Which of the following is closest to the appropriate hedge for small changes in yield?
  - A. Long 298 contracts.
  - B. Short 298 contracts.
  - C. Long 199 contracts.
  - D. Short 199 contracts.
  
2. Which of the following items limits the use of duration as a risk metric?
  - I. It assumes the price/yield relationship is linear.
  - II. It assumes interest rate volatility is constant.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
  
3. Consider day count convention and, specifically, the following example: A semiannual bond with \$100 face value has a 4% coupon. Today is August 3. Assume coupon dates of March 1 and September 1. Which of the following statements is true?
  - A. Corporate bonds accrue more interest in July than T-bonds.
  - B. Corporate bonds accrue more interest from March 1 to September 1 than September 1 to March 1.
  - C. Corporate bonds accrue more interest than T-bonds for this period (March 1 to August 3).
  - D. The T-bond accrued interest is \$1.76 for this period (March 1 to August 3).
  
4. Assume an investor is about to deliver a short bond position and has four options to choose from which are listed in the following table. The settlement price is \$92.50 (i.e., the quoted futures price). Determine which bond is the cheapest-to-deliver.

<i>Bond</i>	<i>Quoted Bond Price</i>	<i>Conversion Factor</i>
1	98	1.02
2	122	1.27
3	105	1.08
4	112	1.15

- A. Bond 1.
- B. Bond 2.
- C. Bond 3.
- D. Bond 4.

5. Assume the cash price on a 90-T-bill is quoted as 98.75. The discount rate is closest to:
- A. 4%.
  - B. 7%.
  - C. 6%.
  - D. 5%.

## CONCEPT CHECKER ANSWERS

1. D 
$$N = -\frac{(\$30,000,000 \times 8)}{(\$100,406.25 \times 12)} = -199$$

The appropriate hedge is to short 199 contracts.

2. A The limitations of duration include: (1) that it is valid for only *small changes in yield*, (2) that it assumes the price/yield relationship is linear, and (3) it assumes that changes in yield are the same across all maturities and risk levels (i.e., they're perfectly correlated).
3. C July accrued T-bond interest is  $31/184 = 0.1685$ ; July accrued corporate bond interest is  $30/180 = 0.1667$ . T-bonds accrue  $155/184 = 0.8424 \times \$2 = \$1.6848$ ; C-bonds accrue  $152/180 = 0.8444 \times \$2 = \$1.6889$ .

4. A Cost of delivery:

Bond 1:  $98 - (92.50 \times 1.02) = \$3.65$

Bond 2:  $122 - (92.50 \times 1.27) = \$4.53$

Bond 3:  $105 - (92.50 \times 1.08) = \$5.10$

Bond 4:  $112 - (92.50 \times 1.15) = \$5.63$

Bond 1 is the cheapest-to-deliver with a cost of delivery of \$3.65.

5. D The discount rate on a U.S. T-bill is calculated using the following equation:

$$\text{discount rate} = \frac{360}{n} \times (100 - \text{cash price})$$

$$\text{discount rate} = \frac{360}{90} \times (100 - 98.75) = 5\%$$

# SWAPS

Topic 40

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## EXAM FOCUS

An interest rate swap is an agreement between two parties to exchange interest payments based on a specified principal over a period of time. In a plain vanilla interest rate swap, one of the interest rates is floating, and the other is fixed. Swaps can be used to efficiently alter the interest rate risk of existing assets and liabilities. A currency swap exchanges interest rate payments in two different currencies. For valuation purposes, swaps can be thought of as a long and short position in two different bonds or as a package of forward rate agreements. Credit risk in swaps cannot be ignored.

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## MECHANICS OF INTEREST RATE SWAPS

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### LO 40.1: Explain the mechanics of a plain vanilla interest rate swap and compute its cash flows.

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The most common interest rate swap is the **plain vanilla interest rate swap**. In this swap arrangement, Company X agrees to pay Company Y a periodic fixed rate on a notional principal over the tenor of the swap. In return, Company Y agrees to pay Company X a periodic floating rate on the same notional principal. Both payments are in the same currency. Therefore, only the net payment is exchanged. Most interest rate swaps use the London Interbank Offered Rate (LIBOR) as the reference rate for the floating leg of the swap. Finally, since the payments are based in the same currency, there is no need for the exchange of principal at the inception of the swap. This is why it is called notional principal.

For example, companies X and Y enter into a 2-year plain vanilla interest rate swap. The swap cash flows are exchanged semiannually, and the reference rate is 6-month LIBOR. The LIBOR rates are shown in Figure 1. The fixed rate of the swap is 3.784%, and the notional principal is \$100 million. We will compute the cash flows for Company X, the fixed payer of this swap.

Figure 1: 6-Month LIBOR

<i>Beginning of Period</i>	<i>LIBOR</i>
1	3.00%
2	3.50%
3	4.00%
4	4.50%
5	5.00%

The first cash flow takes place at the end of period one and uses the LIBOR at the beginning of that same period. In other words, at the beginning of each period, both payments for the end of the period are known. The gross cash flows for the end of the first period for both parties are calculated in the following manner:

$$\text{floating} = \$100 \text{ million} \times 0.03 \times 0.5 = \$1.5 \text{ million}$$

$$\text{fixed} = \$100 \text{ million} \times 0.03784 \times 0.5 = \$1.892 \text{ million}$$

Note that 0.5 is the semiannual day count. The net payment for Company X is an outflow of \$0.392 million. Note that we are ignoring the many day-count and business-day conventions associated with swaps. Figure 2 shows the other cash flows.

**Figure 2: Swap Cash Flows**

<i>End of Period</i>	<i>LIBOR at Beginning of Period</i>	<i>Floating Cash Flow</i>	<i>Fixed Cash Flow</i>	<i>Net X Cash Flow</i>
1	3.00%	\$1,500,000	\$1,892,000	–\$392,000
2	3.50%	\$1,750,000	\$1,892,000	–\$142,000
3	4.00%	\$2,000,000	\$1,892,000	\$108,000
4	4.50%	\$2,250,000	\$1,892,000	\$358,000

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**LO 40.2: Explain how a plain vanilla interest rate swap can be used to transform an asset or a liability and calculate the resulting cash flows.**

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Let's continue with companies X and Y. Suppose that X has a 2-year floating-rate liability, and Y has a 2-year fixed-rate liability. After they enter into the swap, interest rate risk exposure from their liabilities has completely changed for each party. X has transformed the floating-rate liability into a fixed-rate liability, and Y has transformed the fixed-rate liability to a floating-rate liability. Note that X pays fixed and receives floating, so X's liability becomes fixed.

Similarly, assume that X has a fixed-rate asset and Y has a floating-rate asset tied to LIBOR. After entering into the swap, X has transformed the fixed-rate asset into a floating-rate asset, and Y has transformed the floating-rate asset into a fixed-rate asset.

## FINANCIAL INTERMEDIARIES

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**LO 40.3: Explain the role of financial intermediaries in the swaps market.**

**LO 40.4: Describe the role of the confirmation in a swap transaction.**

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In many respects, swaps are similar to forwards:

- Swaps typically require no payment by either party at initiation.
- Swaps are custom instruments.



- Swaps are not traded in any organized secondary market.
- Swaps are largely unregulated.
- Default risk is an important aspect of the contracts.
- Most participants in the swaps market are large institutions.
- Individuals are rarely swap market participants.

There are swap intermediaries who bring together parties with needs for the opposite side of a swap. Dealers, large banks, and brokerage firms, act as principals in trades just as they do in forward contracts. In many cases, a swap party will not be aware of the other party on the offsetting side of the swap since both parties will likely only transact with the intermediary. **Financial intermediaries**, such as banks, will typically earn a spread of about 3 to 4 basis points for bringing two nonfinancial companies together in a swap agreement. This fee is charged to compensate the intermediary for the risk involved. If one of the parties defaults on its swap payments, the intermediary is responsible for making the other party whole.

**Confirmations**, as drafted by the International Swaps and Derivatives Association (ISDA), outline the details of each swap agreement. A representative of each party signs the confirmation, ensuring that they agree with all swap details (such as tenor and fixed/floating rates) and the steps taken in the event of default.

## COMPARATIVE ADVANTAGE

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**LO 40.5: Describe the comparative advantage argument for the existence of interest rate swaps and evaluate some of the criticisms of this argument.**

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Let's return to companies X and Y and assume that they have access to borrowing for two years as specified in Figure 3.

**Figure 3: Borrowing Rates for X and Y**

<i>Company</i>	<i>Fixed Borrowing</i>	<i>Floating Borrowing</i>
Y	5.0%	LIBOR + 10 bps
X	6.5%	LIBOR + 100 bps

Company Y has an **absolute advantage** in both markets but a comparative advantage in the fixed market. Notice that the differential between X and Y in the fixed market is 1.5%, or 150 basis points (bps), and the corresponding differential in the floating market is only 90 basis points. When this is the case, Y has a comparative advantage in the fixed market, and X has a comparative advantage in the floating market. When a **comparative advantage** exists, a swap arrangement will reduce the costs of both parties. In this example, the net potential borrowing savings by entering into a swap is the difference between the differences, or 60 bps. In other words, by entering into a swap, the total savings shared between X and Y is 60 bps.

To better understand where the 60 bps comes from, suppose Y borrows fixed at 5% for two years, X borrows floating for two years at LIBOR + 1%, and then X and Y enter into a swap to transform their liabilities. Specifically, X pays Y fixed and Y pays X floating based on LIBOR. If we assume the net savings is split evenly, the net borrowing costs for X are then 6.2% and LIBOR – 20 bps for Y. Each has saved 30 bps for a total of 60 bps. If an intermediary were used, part of the 60 bps would be used to pay the bid-ask spread.

## PROBLEMS WITH COMPARATIVE ADVANTAGE

A problem with the comparative advantage argument is that it assumes X can borrow at LIBOR + 1% over the life of the swap. It also ignores the credit risk taken on by Y by entering into the swap. If X were to raise funds by borrowing directly in the capital markets, no credit risk is taken, so perhaps the savings is compensation for that risk. The same criticisms exist when an intermediary is involved.

## VALUING INTEREST RATE SWAPS

### The Discount Rate

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**LO 40.6: Explain how the discount rates in a plain vanilla interest rate swap are computed.**

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Since a swap is nothing more than a sequence of cash flows, its value is determined by discounting each cash flow back to the valuation date. The question is, what is the appropriate *discount rate* to use? It turns out that the forward rates implied by either forward rate agreements (FRAs) or the convexity-adjusted Eurodollar futures are used to produce a LIBOR spot curve. The swap cash flows are then discounted using the corresponding spot rate from this curve. The following connection between forward rates and spot rates exists when continuous compounding is used:

$$R_{\text{forward}} = R_2 + (R_2 - R_1) \frac{T_1}{T_2 - T_1}$$

where:

$R_i$  = spot rate corresponding with  $T_i$  years

$R_{\text{forward}}$  = forward rate between  $T_1$  and  $T_2$

We will utilize this equation later when we value an interest rate swap using a sequence of forward rate agreements.

### Valuing an Interest Rate Swap With Bonds

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**LO 40.7: Calculate the value of a plain vanilla interest rate swap based on two simultaneous bond positions.**

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Let's return to our two companies, X and Y, in our 2-year swap arrangement. From X's perspective, there are two series of cash flows—one fixed going out and one floating coming in. Essentially, X has a long position in a floating-rate note (since it is an inflow) and a short position in a fixed-rate note (since it is an outflow). From Y's perspective, it is exactly the opposite—Y has a short position in a floating-rate note (since it is an outflow) and a long position in a fixed-rate note (since it is an inflow).

If we denote the present value of the fixed-leg payments as  $B_{\text{fix}}$  and the present value of the floating-leg payments as  $B_{\text{flt}}$ , the value of the swap can be written for both X and Y as:

$$V_{\text{swap}}(X) = B_{\text{flt}} - B_{\text{fix}}$$

$$V_{\text{swap}}(Y) = B_{\text{fix}} - B_{\text{flt}}$$

Note that  $V_{\text{swap}}(X) + V_{\text{swap}}(Y) = 0$ . This is by design since the two positions are mirror images of one another. At inception of the swap, it is convention to select the fixed payment so that  $V_{\text{swap}}(X) = V_{\text{swap}}(Y) = 0$ . As expected floating rates in the future change, the swap value for each party is no longer zero.

Valuing an interest rate swap in terms of bond positions involves understanding that the value of a floating rate bond will be equal to the notional amount at any of its periodic settlement dates when the next payment is set to the market (floating) rate. Since  $V_{\text{swap}} = \text{Bond}_{\text{fixed}} - \text{Bond}_{\text{floating}}$ , we can value the fixed-rate bond using the spot rate curve and then discount the next (known) floating-rate payment plus the notional amount at the current discount rate. The following example illustrates this method.

#### Example: Valuing an interest rate swap

Consider a \$1 million notional swap that pays a floating rate based on 6-month LIBOR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot LIBOR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The LIBOR at the last payment date was 5.0%. Calculate the value of the swap to the fixed-rate receiver using the bond methodology.

**Answer:**

$$B_{\text{fixed}} = \left( \text{PMT}_{\text{fixed}, 3 \text{ months}} \times e^{-(r \times t)} \right) + \left( \text{PMT}_{\text{fixed}, 9 \text{ months}} \times e^{-(r \times t)} \right) + \left[ (\text{notional} + \text{PMT}_{\text{fixed}, 15 \text{ months}}) \times e^{-(r \times t)} \right]$$

$$\begin{aligned} B_{\text{fixed}} &= \left( \$30,000 \times e^{-(0.054 \times 0.25)} \right) + \left( \$30,000 \times e^{-(0.056 \times 0.75)} \right) + \left[ (\$1,000,000 + \$30,000) \times e^{-(0.058 \times 1.25)} \right] \\ &= \$29,598 + \$28,766 + \$957,968 = \$1,016,332 \end{aligned}$$

$$\begin{aligned} B_{\text{floating}} &= \left[ \text{notional} + \left( \text{notional} \times \frac{r_{\text{floating}}}{2} \right) \right] \times e^{-(r \times t)} \\ &= \left[ \$1,000,000 + \left( \$1,000,000 \times \frac{0.05}{2} \right) \right] \times e^{-(0.054 \times 0.25)} = \$1,011,255 \end{aligned}$$

$$V_{\text{swap}} = (B_{\text{fixed}} - B_{\text{floating}}) = \$1,016,332 - \$1,011,255 = \$5,077$$

Figure 4 sums up the payments and present value factors.

**Figure 4: Valuing an Interest Rate Swap With Two Bond Positions**

<i>Time</i>	<i>Fixed Cash Flow</i>	<i>Floating Cash Flow</i>	<i>Present Value Factor</i>	<i>PV Fixed CF</i>	<i>PV Floating CF</i>
0.25 (3 months)	30,000	1,025,000	0.9866*	29,598	1,011,255
0.75 (9 months)	30,000		0.9589*	28,766	
1.25 (15 months)	1,030,000		0.9301*	957,968	
Total				1,016,332	1,011,255

\* Note that some rounding has occurred.

Again we see that the value of the swap =  $1,016,332 - 1,011,255 = \$5,077$ .

### Valuing an Interest Rate Swap With FRAs

**LO 40.8: Calculate the value of a plain vanilla interest rate swap from a sequence of forward rate agreements (FRAs).**

At settlement, the payment made on a forward rate agreement is the notional amount multiplied by the difference between a market (floating) rate such as LIBOR and the contract (fixed) rate specified in the FRA. This is identical to a periodic payment on an interest rate swap when the reference floating rates and notional principal amounts are the same and the swap fixed rate is equal to the contract rate specified in the FRA. Viewed this way, we can see that an interest rate swap is equivalent to a series of FRAs. One way to value a swap would be to use expected forward rates to forecast the expected net cash flows and then discount these expected cash flows at the corresponding spot rates, consistent with forward rate expectations.

#### Example: Valuing an interest rate swap with FRAs

Consider the previous example on valuing an interest rate swap with two bond positions. An investor has a \$1 million notional swap that pays a floating rate based on 6-month LIBOR and receives a 6% fixed rate semiannually. The swap has a remaining life of 15 months with pay dates at 3, 9, and 15 months. Spot LIBOR rates are as follows: 3 months at 5.4%; 9 months at 5.6%; and 15 months at 5.8%. The LIBOR at the last payment date was 5.0%. Calculate the value of the swap to the fixed-rate receiver using the FRA methodology.

**Answer:**

To calculate the value of the swap, we'll need to find the floating rate cash flows by calculating the expected forward rates via the LIBOR based spot curve.

The first floating rate cash flow is calculated in a similar fashion to the previous example.

LIBOR rate (last payment date): 5%.

Floating rate cash flow in 3 months:  $1,000,000 \times 0.05 / 2 = \$25,000$ .

The second floating rate cash flow is calculated by finding the forward rate that corresponds to the period between 3 months and 9 months. To calculate forward rate for the period between 3 and 9 months, use the previously mentioned forward rate formula:

$$R_{\text{forward}} = R_2 + (R_2 - R_1) \frac{T_1}{T_2 - T_1}$$

$$R_{\text{forward}} = 0.056 + (0.056 - 0.054) \frac{0.25}{0.75 - 0.25} = 0.057 = 5.7\%$$

This rate is a continuously compounded rate, so we need to find the equivalent forward rate with semiannual compounding:

$$R_{\text{forward (SC)}} = 2 \times [e^{(0.057/2)} - 1] = 0.05782 = 5.782\%$$

Floating rate cash flow in 9 months:  $1,000,000 \times 0.05782 / 2 = \$28,910$

The third floating rate cash flow is calculated by finding the forward rate that corresponds to the period between 9 months and 15 months.

$$R_{\text{forward}} = 0.058 + (0.058 - 0.056) \frac{0.75}{1.25 - 0.75} = 0.061 = 6.1\%$$

$$R_{\text{forward (SC)}} = 2 \times [e^{(0.061/2)} - 1] = 0.06194 = 6.1939\%$$

Floating rate cash flow in 15 months:  $1,000,000 \times 0.061939 / 2 = \$30,969$

**Figure 5: Valuing an Interest Rate Swap Based on a Sequence of FRAs**

<i>Time</i>	<i>Fixed Cash Flow</i>	<i>Floating Cash Flow</i>	<i>Present Value Factor</i>	<i>PV Fixed CF</i>	<i>PV Floating CF</i>
0.25 (3 months)	30,000	25,000	0.9866*	29,598	24,665
0.75 (9 months)	30,000	28,910	0.9589*	28,766	27,721
1.25 (15 months)	30,000	30,969	0.9301*	27,902	28,803
Total				86,266	81,189

\* Note that some rounding has occurred.

The value of the swap based on a sequence of FRAs =  $86,266 - 81,189 = \$5,077$ .

As you can see from the previous two examples, valuing a swap based on a sequence of forward rate agreements produces the same result as valuing a swap based on two simultaneous bond positions.

## CURRENCY SWAPS

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**LO 40.9: Explain the mechanics of a currency swap and compute its cash flows.**

**LO 40.11: Calculate the value of a currency swap based on two simultaneous bond positions.**

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A **currency swap** exchanges both principal and interest rate payments with payments in different currencies. The exchange rate used in currency swaps is the spot exchange rate. The valuation and application of currency swaps is similar to the interest rate swap. However, since the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in the interest rate swap.

Suppose we have two companies, A and B, that enter into a fixed-for-fixed currency swap with periodic payments annually. Company A pays 6% in Great Britain pounds (GBP) to Company B and receives 5% in U.S. dollars (USD) from Company B. Company A pays a principal amount to B of USD175 million, and B pays GBP100 million to A at the outset of the swap. Notice that A has effectively borrowed GBP from B and so it must pay interest on that loan. Similarly, B has borrowed USD from A. The cash flows in this swap are actually more easily computed than in an interest rate swap since both legs of the swap are fixed. Every period (12 months), A will pay GBP6 million to B, and B will pay USD8.75 million to A. At the end of the swap, the principal amounts are re-exchanged.

From Company A's perspective, there are two series of cash flows: one fixed GBP cash flow stream going out and one fixed USD cash flow stream coming in. Essentially, A has a long position in a USD-denominated note (since it's an inflow) and a short position in a GBP-denominated note (since it's an outflow).

If we denote the present value of the GBP-denominated payments as  $B_{\text{GBP}}$  and the present value of the USD payments as  $B_{\text{USD}}$ , the value of the swap in USD to Company A is:

$$V_{\text{swap}}(\text{USD}) = B_{\text{USD}} - (S_0 \times B_{\text{GBP}})$$

where:

$S_0$  = spot rate in USD per GBP



**Example: Calculate the value of a currency swap**

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD1.50 = GBP1. Value the currency swap just discussed assuming the swap will last for three more years.

**Answer:**

$$B_{\text{USD}} = 8.75e^{-0.02 \times 1} + 8.75e^{-0.02 \times 2} + 183.75e^{-0.02 \times 3} = \text{USD}190.03 \text{ million}$$

$$B_{\text{GBP}} = 6e^{-0.04 \times 1} + 6e^{-0.04 \times 2} + 106e^{-0.04 \times 3} = \text{GBP}105.32 \text{ million}$$

$$V_{\text{swap}} (\text{to A in USD}) = 190.03 - (1.5 \times 105.32) = \text{USD}32.05 \text{ million}$$

**LO 40.12: Calculate the value of a currency swap based on a sequence of FRAs.**

The value of a currency swap can also be calculated based on a sequence of FRAs.

**Example: Value of a currency swap with FRAs**

Suppose the yield curves in the United States and Great Britain are flat at 2% and 4%, respectively, and the current spot exchange rate is USD1.50 = GBP1.

**Compute** the value of the currency swap discussed previously using a sequence of FRAs to Company A. Assume the swap will last for three more years.

The corresponding forward rates are as follows:

**Figure 6: Forward Rates**

Year 1	\$1.47/£
Year 2	\$1.44/£
Year 3	\$1.41/£



*Professor's Note: The year 1 forward rate is calculated as follows:*

*$F_1 = 1.5e^{(0.02-0.04) \times 1} = \$1.47/\text{£}$ . Interest rate parity suggests that the dollar will appreciate relative to the pound, so the \$/£ forward rate will decline (i.e., it will take fewer USD to buy 1 GBP). We will discuss interest rate parity in the foreign exchange risk topic (Topic 49).*



**Answer:**

Figure 7 denotes the cash flows and forward rates for this currency swap.

**Figure 7: Valuing a Currency Swap Based on a Sequence of FRAs**

<i>Time</i>	<i>USD Cash Flow</i>	<i>GBP Cash Flow</i>	<i>Forward Rate</i>	<i>\$ Value of £</i>	<i>Net Cash Flows</i>	<i>PV of Net CF</i>
1	8.75	6	1.47	8.82	−0.07	−0.069
2	8.75	6	1.44	8.64	0.11	0.106
3	8.75	6	1.41	8.46	0.29	0.273
3	175	100	1.41	141	34	32.02
Total						32.33*

\* Note some rounding has occurred.

Ignoring the rounding differences, we see that the value of the currency swap to Company A is 32 million using both the two simultaneous bond positions and the forward rate agreements.

### Using a Currency Swap to Transform Existing Positions

**LO 40.10: Explain how a currency swap can be used to transform an asset or liability and calculate the resulting cash flows.**

Currency swaps can be combined with existing positions to completely alter the risk of a liability or an asset. For example, suppose that Company A has a dollar-based liability. By entering into a currency swap, the liability has become a pound-based liability at the GBP fixed (or floating) rate.

### Comparative Advantage

Comparative advantage is also used to explain the success of currency swaps. Typically, a domestic borrower will have an easier time borrowing in his own currency. This often results in comparative advantages that can be exploited by using a currency swap. The argument is directly analogous to that used for interest rate swaps. Suppose A and B have the 5-year borrowing rates in the United States and Germany (EUR) shown in Figure 8.

**Figure 8: Borrowing Rates**

<i>Borrowing Rates for A and B</i>		
<i>Company</i>	<i>USD Borrowing</i>	<i>EUR Borrowing</i>
A	5.0%	7.0%
B	6.0%	7.5%

Company A needs EUR, and Company B needs USD. Company A has an absolute advantage in both markets but a comparative advantage in the USD market. Notice that the differential between A and B in the USD market is 1%, or 100 basis points (bps), and the corresponding differential in the EUR market is only 50 basis points. When this is the case, A has a comparative advantage in the USD market, and B has a comparative advantage in the EUR market. The net potential borrowing savings by entering into a swap is the difference between the differentials, or 50 bps. In other words, by entering into a currency swap, the savings for both A and B totals 50 bps.

## SWAP CREDIT RISK

### LO 40.13: Describe the credit risk exposure in a swap position.

Because  $V_{\text{swap}}(A) + V_{\text{swap}}(B) = 0$ , whenever one side of a swap has a positive value, the other side must be negative. For example, if  $V_{\text{swap}}(A) > 0$ ,  $V_{\text{swap}}(B) < 0$ . As  $V_{\text{swap}}(A)$  increases in value,  $V_{\text{swap}}(B)$  must become more negative. This results in increased credit risk to A since the likelihood of default increases as B has larger and larger payments to make to A. However, the potential losses in swaps are generally much smaller than the potential losses from defaults on debt with the same principal. This is because the value of swaps is generally much smaller than the value of the debt.

## OTHER TYPES OF SWAPS

### LO 40.14: Identify and describe other types of swaps, including commodity, volatility and exotic swaps.

In an **equity swap**, the return on a stock, a portfolio, or a stock index is paid each period by one party in return for a fixed-rate or floating-rate payment. The return can be the capital appreciation or the total return including dividends on the stock, portfolio, or index.

In order to reduce equity risk, a portfolio manager might enter into a 1-year quarterly pay S&P 500 index swap and agree to receive a fixed rate. The percentage increase in the index each quarter is netted against the fixed rate to determine the payment to be made. If the index return is negative, the fixed-rate payer must also pay the percentage decline in the index to the portfolio manager. Uniquely among swaps, equity swap payments can be floating on both sides and the payments are not known until the end of the quarter. With interest rate swaps, both the fixed and floating payments are known at the beginning of the period for which they will be paid.

A swap on a single stock can be motivated by a desire to protect the value of a position over the period of the swap. To protect a large capital gain in a single stock, and to avoid a sale for tax or control reasons, an investor could enter into an equity swap as the equity-returns payer and receive a fixed rate in return. Any decline in the stock price would be paid to the investor at the settlement dates, plus the fixed-rate payment. If the stock appreciates, the investor must pay the appreciation less the fixed payment.

A **swaption** is an option which gives the holder the right to enter into an interest rate swap. Swaptions can be American- or European-style options. Like any option, a swaption is purchased for a premium that depends on the strike rate (the fixed rate) specified in the swaption.

Firms may enter into **commodity swap** agreements where they agree to pay a fixed rate for the multi-period delivery of a commodity and receive a corresponding floating rate based on the average commodity spot rates at the time of delivery. Although many commodity swaps exist, the most common use is to manage the costs of purchasing energy resources such as oil and electricity.

A **volatility swap** involves the exchanging of volatility based on a notional principal. One side of the swap pays based on a pre-specified volatility while the other side pays based on historical volatility.

As you can see, many different types of swaps exist. Some additional examples include: accrual swaps, cancelable swaps, index amortizing rate swaps, and constant maturity swaps. Swaps are also sometimes created for exotic structures. An example of an **exotic swap** was between Procter and Gamble and Banker's Trust where P&G's payments were based on the commercial paper rate.

## KEY CONCEPTS

### LO 40.1

A plain vanilla interest rate swap exchanges floating-rate payments (LIBOR) for fixed-rate payments over the life of the swap. The floating rate payments at time  $t$  in a plain vanilla interest rate swap are computed using the floating rate at time  $t - 1$ .

### LO 40.2

Interest rate swaps can be combined with existing asset and liability positions to drastically change the interest rate risk.

### LO 40.3

A swap dealer or financial intermediary facilitates the ability to enter into swaps.

### LO 40.4

Confirmations outline the details of each swap agreement. A representative of each party signs the confirmation, ensuring that they agree with all swap details and the steps taken in the event of default.

### LO 40.5

The comparative advantage argument suggests that when one of two borrowers has a comparative advantage in either the fixed- or floating-rate market, both borrowers will be better off by entering into a swap to exploit the advantage. The comparative advantage argument is flawed in that it assumes rates can be borrowed for the life of the swap. It also ignores the credit risk associated with the swap that does not exist if funds were raised directly in the capital markets.

### LO 40.6

Since a swap is nothing more than a sequence of cash flows, its value is determined by discounting each cash flow back to the valuation date. The cash flows are discounted using the corresponding spot rate from the LIBOR spot curve.

### LO 40.7

The value of a swap to the fixed-rate receiver at a point in time is the difference between the present value of the remaining fixed-rate payments and the present value of the remaining floating-rate payments.

### LO 40.8

Valuing a swap based on a sequence of forward rate agreements (FRAs) produces the same result as valuing a swap based on two simultaneous bond positions.

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**LO 40.9**

A currency swap exchanges interest rate payments in two different currencies. The exchange rate used in currency swaps is the spot exchange rate.

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**LO 40.10**

Currency swaps can be combined with existing positions to completely alter the risk of a liability or an asset.

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**LO 40.11**

Since the principals in a currency swap are not the same currency, they are exchanged at the inception of the currency swap so that they have equal value using the spot exchange rate. Also, the periodic cash flows throughout the swap are not netted as they are in an interest rate swap.

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**LO 40.12**

In addition to valuing a currency swap based on two simultaneous bond positions, the value of a currency swap can also be calculated based on a sequence of FRAs.

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**LO 40.13**

Credit risk is an important factor in existing swap positions, although potential losses are usually smaller than that with debt agreements.

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**LO 40.14**

Many different types of swaps exist. Examples of swaps, in addition to interest rate swaps and currency swaps, include: equity swaps, commodity swaps, and volatility swaps.

## CONCEPT CHECKERS

Use the following data to answer Question 1.

Two companies, C and D, have the borrowing rates shown in the following table.

Borrowing Rates for C and D		
<i>Company</i>	<i>Fixed Borrowing</i>	<i>Floating Borrowing</i>
C	10%	LIBOR + 50 bps
D	12%	LIBOR + 100 bps

1. According to the comparative advantage argument, what is the total potential savings for C and D if they enter into an interest rate swap?
  - A. 0.5%.
  - B. 1.0%.
  - C. 1.5%.
  - D. 2.0%.
  
2. Which of the following is most accurate regarding the credit risk of a currency swap? As the value of the:
  - I. domestic currency leg increases, so does the credit risk of the domestic currency payer.
  - II. foreign currency leg increases, so does the credit risk of the foreign currency payer.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
  
3. Which of the following would properly transform a floating-rate liability to a fixed-rate liability? Enter into a pay:
  - A. foreign currency swap.
  - B. fixed interest rate swap.
  - C. domestic currency swap.
  - D. floating interest rate swap.
  
4. Use the following information to determine the value of the swap to the floating rate payer using the bond methodology. Assume we are at the floating rate reset date.
  - \$1 million notional value, semiannual, 18-month maturity.
  - Spot LIBOR rates: 6 months, 2.6%; 12 months, 2.65%; 18 months, 2.75%.
  - The fixed rate is 2.8%, with semiannual payments.
  - A. -\$66.
  - B. \$476.
  - C. \$3,425.
  - D. \$5,077.

5. Suppose Company X pays 5% annually (in euros) to Company Y and receives 4% annually (in dollars). Company X pays a principal amount of \$150 million to Y, and Y pays a €100 million to X at the inception of the swap. Assume the yield curve is flat in the United States and in Germany (Europe). The U.S. rate is 3%, and the German rate is 5%. The current spot exchange rate is \$1.45/€. What is the value of the currency swap to Company X using the bond methodology if it is expected to last for two more years?
- A. \$3.53 million.
  - B. \$52.98 million.
  - C. \$8.09 million.
  - D. \$12.74 million.



## CONCEPT CHECKER ANSWERS

1. C The difference of the differences is  $(12\% - 10\%) - [\text{LIBOR} + 1\% - (\text{LIBOR} + 0.5\%)] = 1.5\%$ .
2. D As one currency (A) appreciates relative to another currency (B), the value of a currency swap increases on behalf of the currency A payer. As a result, the credit risk of the currency B payer increases.
3. B The fixed interest rate swap will allow for the conversion of a floating-rate liability to a fixed-rate liability.
4. B  $B_{\text{fix}} = [\$14,000 \times e^{-(0.026 \times 0.5)}] + [\$14,000 \times e^{-(0.0265 \times 1.0)}] + [(\$1,000,000 + \$14,000) \times e^{-(0.0275 \times 1.5)}] = \$13,819 + \$13,634 + \$973,023 = \$1,000,476$

Note that we are at a (semiannual) reset date, so the floating rate portion has a value equal to the notional amount.

$$V_{\text{swap}} = (B_{\text{fix}} - B_{\text{floating}}) = \$1,000,476 - \$1,000,000 = \$476$$

5. C  $B_{\$} = 6e^{-0.03 \times 1} + 156e^{-0.03 \times 2} = \$5.82 + \$146.92 = \$152.74$

$$B_{\text{€}} = 5e^{-0.05 \times 1} + 105e^{-0.05 \times 2} = \text{€}4.76 + \text{€}95.00 = \text{€}99.76$$

$$V_{\text{swap}} (\text{to X}) = 152.74 - (1.45 \times 99.76) = \$8.09 \text{ million}$$

# MECHANICS OF OPTIONS MARKETS

Topic 41

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## EXAM FOCUS

Stock options give the owner the right, but not the obligation, to buy or sell a stock at a specific price on or before a specific date. Call options give the owner the right to buy the stock, and put options give the owner the right to sell the stock. An option is exercised when the owner executes the right to buy or sell the stock. This topic covers the basic mechanics of option trading. You should understand the different kinds of options and the system by which exchange-traded options are bought and sold.

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## OPTION TYPES

### LO 41.1: Describe the types, position variations, and typical underlying assets of options.

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Option contracts have asymmetric payoffs. The buyer of an option has the right to exercise the option but is not obligated to exercise. Therefore, the maximum loss for the buyer of an option contract is the loss of the price (premium) paid to acquire the position, while the potential gains in some cases are theoretically infinite. Because option contracts are a zero-sum game, the seller of the option contract could incur substantial losses, but the maximum potential gain is the amount of the premium received for writing the option. *American options* may be exercised at any time up to and including the contract's expiration date, while *European options* can be exercised only on the contract's expiration date.

To understand the potential returns, we need to introduce the standard symbols used to represent the relevant factors:

- $X$  = strike price or exercise price specified in the option contract (a fixed value)
- $S_t$  = price of the underlying asset at time  $t$
- $C_t$  = the market value of a call option at time  $t$
- $P_t$  = the market value of a put option at time  $t$
- $t$  = the time subscript, which can take any value between 0 and  $T$ , where  $T$  is the maturity or expiration date of the option

## Call Options

A *call option* gives the *owner* the right, but not the obligation, to buy the stock from the seller of the option. The owner is also called the *buyer* or the holder of the *long position*. The buyer benefits, at the expense of the option *seller*, if the underlying stock price is greater than the exercise price. The option *seller* is also called the *writer* or holder of the *short position*.

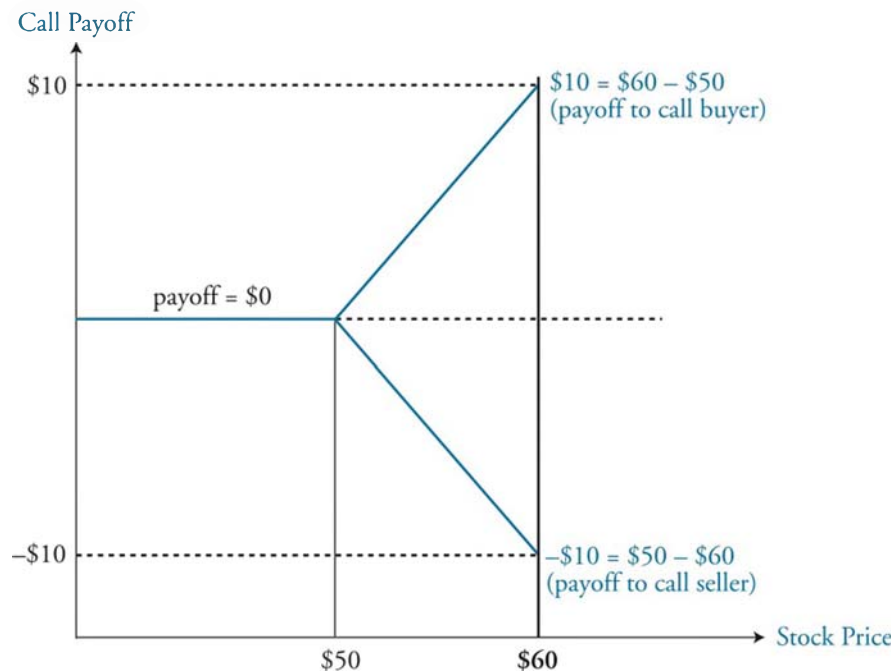
At maturity time  $T$ , if the price of the underlying stock is less than or equal to the strike price of a call option (i.e.,  $S_T \leq X$ ), the payoff is zero, so the option owner would not exercise the option. On the other hand, if the stock price is higher than the exercise price (i.e.,  $S_T > X$ ) at maturity, then the payoff of the call option is equal to the difference between the market price and the strike price ( $S_T - X$ ). The “payoff” (at the option’s maturity) to the call option seller, is the mirror image (opposite sign) of the payoff to the buyer.

Because of the linear relationships between the value of the option and the price of the underlying asset, simple graphs can clearly illustrate the possible value of option contracts at the expiration date. Figure 1 illustrates the payoff of a call with an exercise price equal to 50.



*Professor's Note: An option payoff graph ignores the initial cost of the option.*

**Figure 1: Payoff of Call With Exercise Price Equal to \$50**



**Example: Payoff of a call option**

An investor writes an at-the-money call option on a stock with an exercise price of 50 ( $X = 50$ ). If the stock price rises to \$60, what will be the *payoff* to the owner and seller of the call option?

**Answer:**

The call option may be exercised with the holder of the long position buying the stock from the writer at 50 for a \$10 gain. The payoff to the option buyer is \$10, and the payoff to the option writer is *negative* \$10. This is illustrated in Figure 1 and, as mentioned, does not include the premium paid for the option.

This example shows just how easy it is to determine option payoffs. At expiration time  $T$  (the option's maturity), the payoff to the option owner, represented by  $C_T$ , is:

$$C_T = S_T - X \quad \text{if} \quad S_T > X$$

$$C_T = 0 \quad \text{if} \quad S_T \leq X$$

Another popular way of writing this is with the “max (0, variable)” notation. If the variable in this expression is greater than zero, then  $\max(0, \text{variable}) = \text{variable}$ ; if the variable's value is less than zero, then  $\max(0, \text{variable}) = 0$ . Thus, letting the variable be the quantity  $S_0 - X$ , we can write:

$$C_T = \max(0, S_T - X)$$

The payoff to the option seller is the negative value of these numbers. In what follows, we will always talk about payoff in terms of the option owner unless otherwise stated. We should note that  $\max(0, S_t - X)$ , where  $0 < t < T$ , is also the payoff if the owner decides to exercise the call option early. In this topic, we will only consider time  $T$  in our analysis.

Although our focus here is not to calculate  $C_t$ , we should clearly define it as the initial cost of the call when the investor purchases at time 0, which is  $T$  units of time before  $T$ .  $C_0$  is also called the premium. Thus, we can write that the profit to the owner at  $t = T$  is:

$$\text{profit} = C_T - C_0$$

This says that at time  $T$ , the owner's profit is the option payoff minus the premium paid at time 0. Incorporating  $C_0$  into Figure 1 gives us the profit diagram for a call at expiration, and this is Figure 2.

Figure 2 illustrates an important point, which is that the profit to the owner is negative when the stock price is less than the exercise price plus the premium. At expiration, we can say that:

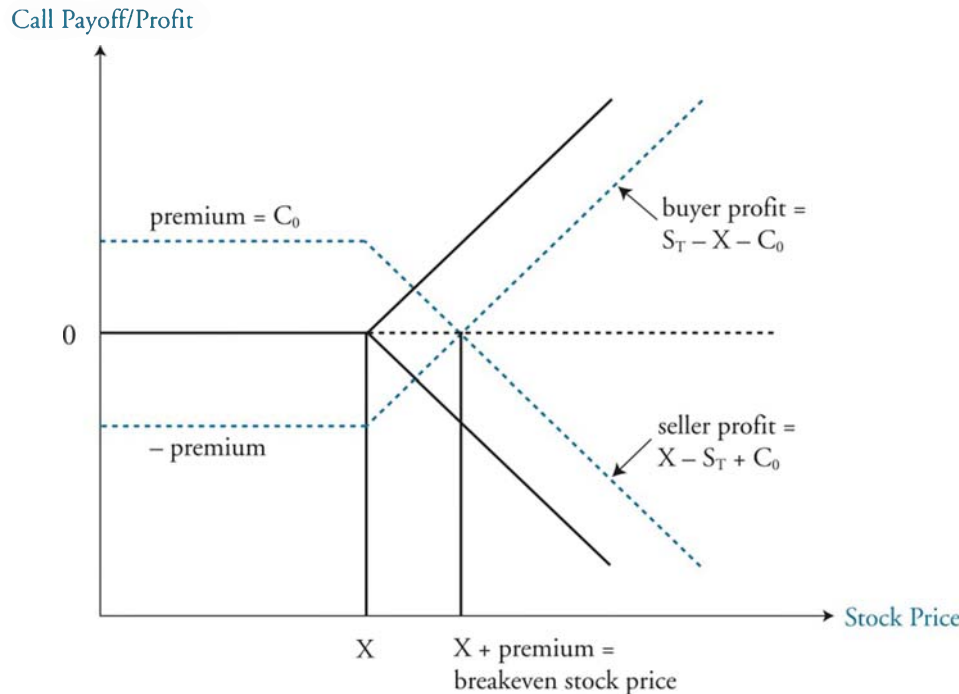
$$\text{if } S_T < X + C_0 \quad \text{then: call buyer profit} < 0 < \text{call seller profit}$$

if  $S_T = X + C_0$  then: call buyer profit = 0 = call seller profit

if  $S_T > X + C_0$  then: call buyer profit > 0 > call seller profit

The **breakeven price** is a very descriptive term that we use for  $X + C_0$ , or  $X + \text{premium}$ .

Figure 2: Profit Diagram for a Call at Expiration



## Put Options

If you understand the properties of a call, the properties of a put should come to you fairly easily. A put option gives the owner the right to sell a stock to the seller of the put at a specific price. At expiration, the buyer benefits if the price of the underlying is less than the exercise price  $X$ :

$$\begin{aligned} P_T &= X - S_T & \text{if } S_T < X \\ P_T &= 0 & \text{if } X \leq S_T \end{aligned}$$

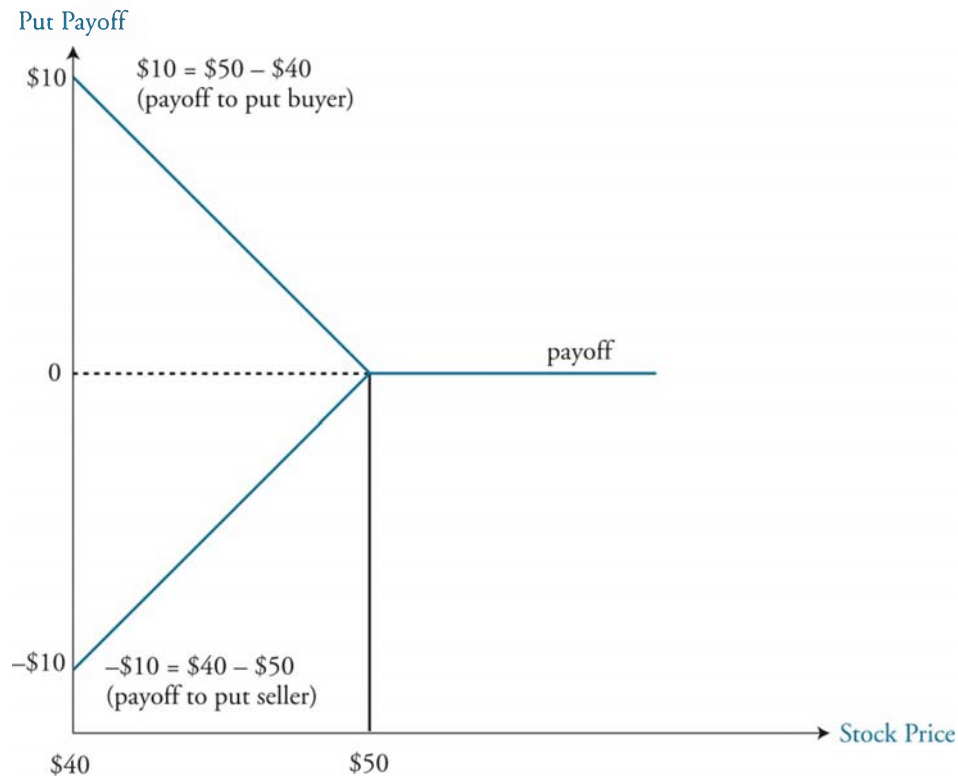
or:

$$P_T = \max(0, X - S_T)$$

For example, an investor writes a put option on a stock with a strike price of  $X = 50$ . If the stock stays at \$50 or above, the payoff of the put option is zero (because the holder may receive the same or better price by selling the underlying asset on the market rather than exercising the option). But if the stock price falls below \$50, say to \$40, the put option may be exercised with the option holder buying the stock from the market at \$40 and selling it to the put writer at \$50 for a \$10 gain. The writer of the put option must pay the put price of \$50 when it can be sold in the market at only \$40, resulting in a \$10 loss. The gain to the option holder is the same magnitude as the loss to the option writer. Figure 3 illustrates this example, excluding the initial cost of the put and

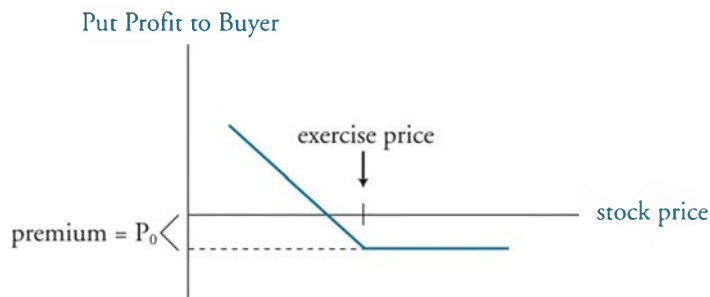
transaction costs. Figure 4 includes the cost of the put (but not transaction costs) and illustrates the profit to the put owner.

Figure 3: Put Payoff to Buyer and Seller



Given the “mirror image quality” that results from the “zero-sum game” nature of options, we often just draw the profit to the buyer as shown in Figure 4. Then, we can simply remember that each positive (negative) value is a negative (positive) value for the seller.

Figure 4: Put Profit to Buyer



The breakeven price for a put position upon expiration is the exercise price minus the premium paid,  $X - P_0$ .

## UNDERLYING ASSETS

Exchange-traded options trade on four primary assets: individual stocks, foreign currency, stock indices, and futures.

**Stock options.** Stock options are typically exchange-traded, American-style options. Each option contract is normally for 100 shares of stock. For example, if the last trade on a call option occurred at \$3.60, the option contract would cost \$360. After issuance, stock option contracts are adjusted for stock splits but not cash dividends. The primary U.S. exchanges for stock options are the Chicago Board Options Exchange (CBOE), Boston Options Exchange, NYSE Euronext, and the International Securities Exchange.

**Currency options.** Investors holding currency options receive the right to buy or sell an amount of foreign currency based on a domestic currency amount. For calls, a currency option is going to pay off only if the actual exchange rate is above a specified exercise rate. For puts, a currency option is going to pay off only if the actual exchange rate is below a specified exercise rate. The majority of currency options are traded on the over-the-counter market, while the remainder are exchange traded. The NASDAQ OMX trades European-style options for several currencies. Note that the unit size for currency options is considerably larger than stock options (i.e., 1 million units for yen and 10,000 units for other currencies).

**Index options.** Options on stock indices are typically European-style options and are cash settled. Index options can be found on both the over-the-counter markets and the exchange-traded markets. The payoff on an index call is the amount (if any) by which the index level at expiration exceeds the index level specified in the option (the strike price), multiplied by the contract multiplier (typically 100).

### Example: Index options

Assume you own a call option on an index with an exercise price equal to 950. The multiplier for this contract is 100. **Compute** the payoff on this option assuming that the index is 956 at expiration.

#### Answer:

The payoff on an index call (long) is the amount (if any) by which the index level at expiration exceeds the index level specified in the option (the exercise price), multiplied by the contract multiplier. An equal amount will be deducted from the account of the index call option writer. In this example, the expiration date payoff is  $(956 - 950) \times \$100 = \$600$ .



**Futures options.** American-style, exchange-traded options are most often utilized for futures contracts. Typically, the futures option expiration date is set to a date shortly before the expiration date of the futures contract. The market value of the underlying asset for futures options is the value of the underlying futures contract. The payoff for call options is calculated as the futures price less the strike price, while the payoff for put options is calculated as the strike price less the futures price.

## STOCK OPTIONS SPECIFICATIONS

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**LO 41.2: Explain the specification of exchange-traded stock option contracts, including that of nonstandard products.**

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### Expiration

Options can be either American or European style. As mentioned previously, American options can be exercised throughout the life of the option, while European options can only be exercised on the expiration date of the option. For this reason, American options are always at least as valuable as corresponding European options. Exchange-traded stock options are typically American-style options. The expiration dates of these options dictate how the option is named. For example, a June put option on Intel means that the option expires in June. The actual day of expiration is the Saturday following the third Friday of the expiration month. Different expiration cycles dictate the actual expiration months of a stock option over a given year. **Long-term equity anticipation securities (LEAPS®)** are simply long-dated options with expirations greater than one year. All LEAPS have January expirations.

### Strike Prices

Strike prices are dictated by the value of the stock. Low-value stocks have smaller strike increments than higher-value stocks. Typically, stocks that are priced around \$20 have increments of \$2.50, stocks that are priced around \$50 have increments of \$5.00, and so on. The strike price is usually denoted as  $X$  and the underlying stock as  $S$ .

### Moneyness, Time Value, and Intrinsic Value

An *option class* refers to all options of the same type, whether calls or puts. An *option series* refers to an option class with the same expiration. For a call (put), when the underlying asset price is less (greater) than the strike price, the option is said to be out of the money. For both a call and put, when the underlying asset price is equal to the strike price, the option is said to be at the money. For a call (put), when the underlying asset is greater (less) than the strike price, the option is said to be in the money. An option price (or premium) prior to expiration has two components: the time value and the intrinsic value. The *intrinsic value* is the maximum of zero or the difference between the underlying asset and the strike price [i.e., intrinsic value of a call option =  $\max(0, S - X)$  and intrinsic value of a put option =  $\max(0, X - S)$ ]. The *time value* is the difference between the option premium and the intrinsic value.

## Nonstandard Products

Nonstandard option products include flexible exchange (FLEX) options, exchange-traded fund (ETF) options, weekly options, binary options, credit event binary options (CEBOs), and deep out-of-the-money (DOOM) options.

**FLEX options.** FLEX options are exchange-traded options on equity indices and equities that allow some alteration of the options contract specifications. The nonstandard terms include alteration of the strike price, different expiration dates, or European-style (rather than the standard American-style). FLEX options were developed in order for the exchanges to better compete with the nonstandard options that trade over the counter. The minimum size for FLEX trades is typically 100 contracts.

**ETF options.** While similar to index options, ETF options are typically American-style options and utilize delivery of shares rather than cash at settlement.

**Weekly options.** *Weeklys* are short-term options that are created on a Thursday and have an expiration date on the Friday of the next week.

**Binary options.** Binary options generate discontinuous payoff profiles because they pay only one price (\$100) at expiration if the asset value is above the strike price. The term binary means the option payoff has one of two states: the option pays \$100 at expiration if the option is above the strike price or the option pays nothing if the price is below the strike price. Hence, a payoff discontinuity results from the fact that the payoff is only one value—it does not increase continuously with the price of the underlying asset as in the case of a traditional option.

**CEBOs.** A CEBO is a specific form of credit default swap. The payoff in a CEBO is triggered if the reference entity suffers a qualifying credit event (e.g., bankruptcy, missed debt payment, or debt restructuring) prior to the option's expiration date (which always occurs in December). Option payoff, if any, occurs on the expiration date. CEBOs are European options that are cash settled.

**DOOM options.** These put options are structured to only be in the money in the event of a large downward price movement in the underlying asset. Due to their structure, the strike price of these options is quite low. In terms of protection, DOOM options are similar to credit default swaps. Note that this option type is always structured as a put option.

## The Effect of Dividends and Stock Splits

In general, options are not adjusted for cash dividends. This will have option pricing consequences that will need to be incorporated into a valuation model. Options are adjusted for *stock splits*. For example, if a stock has a 2-for-1 stock split, then the strike price will be reduced by one-half and the number of shares underlying the option will double. In general, if a stock experiences a  $b$ -for- $a$  stock split, the strike price becomes  $(a/b)$  of its previous value and the number of shares underlying the option is increased by multiples of  $(b/a)$ . Stock dividends are dealt with in the same manner. For example, if a stock pays a 25% stock dividend, this is treated in the same manner as a 5-for-4 stock split.

## Position and Exercise Limits

The number of options a trader can have on one stock is limited by the exchange. This is called a position limit. Additionally, short calls and long puts are considered to be part of the same position. The exercise limit equals the position limit and specifies the maximum number of option contracts that can be exercised by an individual over any five consecutive business days.

## OPTION TRADING

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### LO 41.3: Describe how trading, commissions, margin requirements, and exercise typically work for exchange-traded options.

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As mentioned, options are quoted relative to one underlying stock. To compute the actual option cost, the quote needs to be multiplied by 100. This is because an options contract represents an option on 100 shares of the underlying stock. The quotes will also include the strike, expiration month, volume, and the option class.

Market makers will quote bid and offer (or ask) prices whenever necessary. They profit on the bid-offer spread and add liquidity to the market. Floor brokers represent a particular firm and execute trades for the general public. The order book official enters limit orders relayed from the floor broker. An offsetting trade takes place when a long (short) option position is offset with a sale (purchase) of the same option. If a trade is not an offsetting trade, then open interest increases by one contract.

## Commissions

Option investors must consider the commission costs associated with their trading activity. Commission costs often vary based on trade size and broker type (discount vs. full service). Brokers typically structure commission rates as a fixed amount plus a percentage of the trade amount. The following example provides an illustration on how commission costs affect an option trade's profitability.

**Example: Commission costs**

An investor buys a call contract with a strike price of \$260. The current price of the underlying stock is \$245. Assume the option price is \$10 and the contract is settled with shares rather than cash. Using the commission schedule for a discount broker below, **calculate** (1) the commission costs incurred by the investor based on the initial trade and (2) the investor's net profit if the stock price increases to \$280 prior to expiration. Assume the cost to exercise the option is 1% of the trade amount and the cost to sell stock is also 1% of the trade amount.

**Figure 5: Commission Schedule**

<i>Trade Amount</i>	<i>Commission Rate</i>
$\leq \$3,000$	$\$30 + 0.8\%$ of trade amount
$\$3,001$ to $\$14,999$	$\$30 + 0.6\%$ of trade amount
$\geq \$15,000$	$\$30 + 0.4\%$ of trade amount
<i>Other details:</i>	
Minimum charge per contract: \$4	
Maximum charge per contract: \$35	

**Answer:**

1. Contract cost =  $\$10 \times 100 = \$1,000$

Initial commission costs =  $\$30 + (\$1,000 \times 0.8\%) = \$38$ . Because this exceeds the maximum contract charge, \$35 is charged (i.e., the maximum contract charge).

2. Gross profit:  $\$280 - \$260 = \$20$  per share.  $\$20 \times 100$  shares = \$2,000

Additional commission costs =  $1\% \times 2 \times \$280 \times 100 = \$560$

Total commission costs =  $\$35 + \$560 = \$595$

Net profit =  $\$2,000 - \$1,000 - \$595 = \$405$

Due to the costs associated with exercising the option and then selling the stock, some retail investors may find it more efficient to simply sell the option to another investor.

One final note on option commission costs is that they fail to account for the cost embedded in the bid-offer spread. The cost associated with this spread for options can be calculated by multiplying the spread by 50%. For example, if the bid price is \$12 and the offer price is \$12.20, the associated cost for both the option buyer and option seller would be \$0.10 per contract  $[ (= \$12.20 - \$12.00) \times 50\% ]$ . This cost is also present in stock transactions.

## Margin Requirements

Options with maturities nine months or fewer cannot be purchased on margin. This is because the leverage would become too high. For options with longer maturities, investors can borrow a maximum of 25% of the option value.

Investors who engage in writing options must have a margin account due to the high potential losses and potential default. The required margin for option writers is dependent on the amount and position of option contracts written.

**Naked options** (or *uncovered options*) refers to options in which the writer does not also own a position in the underlying asset. The size of the initial and maintenance margin for naked option writing is equal to the option premium plus a percentage of the underlying share price. Writing *covered calls* (selling a call option on a stock that is owned by the seller of the option) is far less risky than naked call writing.

## The Options Clearing Corporation

Similar to a clearinghouse for futures, the **Options Clearing Corporation (OCC)** guarantees that buyers and sellers in the exchange-traded options market will honor their obligations and records all option positions. Exchange-traded options have no default risk because of the OCC, while over-the-counter options possess default risk. The OCC requires that all trades are cleared by one of its clearing members. OCC members must meet net capital requirements and help finance an emergency fund that is utilized in the event of a member default. Non-member brokers must contact a clearing member to clear their option trades. The OCC guarantees contract performance and therefore requires option writers to post margin as a means of supporting their obligation and option buyers to deposit required funds by the morning of the business day immediately following the day the option is purchased.

## Exercising an Option

When an investor decides to exercise an option prior to contract expiration, her broker contacts the assigned OCC member responsible for clearing that broker's trades. This OCC member then submits an exercise order to the OCC which matches it with a clearing member who identifies an investor who has written a stock option. This assigned investor then must sell (if a call option) or buy (if a put option) the underlying at the specified strike price on the third business day after the order to exercise is received. Exercising an option results in the open interest being reduced by one. At contract expiration, unexercised options that are in the money after accounting for transaction costs will be exercised by brokers.

## Other Option-Like Securities

Exchange-traded options are not issued by the company and delivery of shares associated with the exercise of exchange-traded options involves shares that are already outstanding. *Warrants* are often issued by a company to make a bond issue more attractive and will typically trade separately from the bond at some point. Warrants are like call options



except that, upon exercise, the company receives the strike price and may issue new shares to deliver. The same distinction applies to *employee stock options*, which are issued as an incentive to company employees and provide a benefit if the stock price rises above the exercise price. When an employee exercises incentive stock options, any shares issued by the company will increase the number of shares outstanding.

*Convertible bonds* contain a provision that gives the bondholder the option of exchanging the bond for a specified number of shares of the company's common stock. At exercise, the newly issued shares increase the number of shares outstanding and debt is retired based on the amount of bonds exchanged for the shares. There is a potential for dilution of the firm's common shares from newly issued shares with warrants, employee stock options, and convertible bonds that does not exist for exchange-traded options.

## KEY CONCEPTS

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### LO 41.1

A call (put) option gives the owner the right to purchase (sell) the underlying asset at a strike price. When the owner executes this right, the option is said to be exercised. Because long (buy, purchase) option positions give the owner the right to exercise, the seller (short, writer) of the option has the obligation to meet the terms of the option.

American options may be exercised at any time up to and including the contract's expiration date, while European options can be exercised only on the contract's expiration date. Exchange-traded options are typically American options.

Primary types of exchange-traded options include option on individual stocks, foreign currency, stock indices, and futures.

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### LO 41.2

For a call (put), when the underlying asset price is less (greater) than the strike price, the option is said to be out of the money. For both a call and put, when the underlying asset price is equal to the strike price, the option is said to be at the money. For a call (put), when the underlying asset price is greater (less) than the strike price, the option is said to be in the money. Options are not adjusted for cash dividends, but are adjusted for stock splits.

LEAPS are options with expiration dates greater than a year. Nonstandard option products include FLEX options, ETF options, weekly options, binary options, CEBOs, and DOOM options.

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### LO 41.3

Options with a maturity of nine months or fewer cannot be purchased on margin and must be paid in full due to the leverage effect of options. For options with longer maturities, investors can borrow up to 25% of the option value. Writers of options are required to have margin accounts with a broker.

Investors must account for commission costs when utilizing option. Commissions vary based on trade size and broker type. Commission rates typically are structured as a fixed dollar amount plus a percentage of the trade amount. In some instances, investors can earn higher profits by selling in-the-money options rather than exercising the options.

The Options Clearing Corporation (OCC) guarantees that buyers and sellers in the options market will honor their obligations and records all option positions. This minimizes default risk.

Warrants, employee stock options, and convertible bonds are option-like securities. Unlike options, these securities are issued by financial institutions or companies. The cost to the issuer of these securities is the possibility of increased dilution of the stock.



## CONCEPT CHECKERS

Use the following data to answer Questions 1 and 2.

An investor owns a stock option that currently has a strike price of \$100.

1. If the stock experiences a 4-to-1 split, the strike price becomes:
  - A. \$20.
  - B. \$25.
  - C. \$50.
  - D. \$100.
2. The number of shares now covered by each option contract is:
  - A. 100.
  - B. 200.
  - C. 300.
  - D. 400.
3. If an option is quoted at \$2.75, the cost of one contract to the potential buyer is closest to:
  - A. \$0.275.
  - B. \$2.75.
  - C. \$275.00.
  - D. \$2,750.00.
4. Which of the following statements regarding option value or expiration is correct?
  - I. American-style options are less valuable than European options.
  - II. All options expire on the third Wednesday of the expiration month.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
5. Which of the following option characteristics is correct?
  - I. A put option is in the money when the asset price is less than the strike price.
  - II. LEAPS are long-term (over one-year) options that expire in December of each year.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.

## CONCEPT CHECKER ANSWERS

1. B  $\frac{a}{b} = \frac{1}{4} \times \$100 = \$25$
2. D  $\frac{b}{a} = \frac{4}{1} \times \$100 = \$400$  (Each option contract is originally for 100 shares.)
3. C Multiply the quote by 100 because each option contract is for 100 shares.  $\$2.75 \times 100 = \$275.00$
4. D American-style options are at least as valuable as European-style options. Options expire on the Saturday after the third Friday.
5. A A put option is in the money when the asset price is less than the strike price. LEAPS expire in January.

# PROPERTIES OF STOCK OPTIONS

Topic 42

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## EXAM FOCUS

Stock options have several properties relating both to their value and to the factors that affect their price. Six factors affect option prices: the current value of the stock; the strike price; the time to expiration; the volatility of the stock price; the risk-free rate; and dividends. The value of stock options have upper and lower pricing bounds. Be familiar with these pricing bounds as well as the relationships that exist between the value of European and American options.

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## SIX FACTORS THAT AFFECT OPTION PRICES

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**LO 42.1: Identify the six factors that affect an option's price and describe how these six factors affect the price for both European and American options.**

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The following six factors will impact the value of an option:

1.  $S_0$  = current stock price.
2.  $X$  = strike price of the option.
3.  $T$  = time to expiration of the option.
4.  $r$  = short-term risk-free interest rate over  $T$ .
5.  $D$  = present value of the dividend of the underlying stock.
6.  $\sigma$  = expected volatility of stock prices over  $T$ .

When evaluating a change in any one of the factors, hold the other factors constant.

### Current Price of the Stock

For call options, as  $S$  increases (decreases), the value of the call increases (decreases). For put options, as  $S$  increases (decreases), the value of the put decreases (increases). This simply states that as an option becomes closer to or more in-the-money, its value increases.

### Strike Price of the Option

The effect of strike prices on option values will be exactly the opposite of the effect of the current price of the stock. For call options, as  $X$  increases (decreases), the value of the call decreases (increases). For put options, as  $X$  increases (decreases), the value of the put increases (decreases). This is the same as the logic for the current price of the stock: the option's value will increase as it becomes closer to or more in-the-money.

### The Time to Expiration

For American-style options, increasing time to expiration will increase the option value. With more time, the likelihood of being in-the-money increases. A general statement cannot be made for European-style options. Suppose we have a 1-month and 3-month call option on the same underlying with the same exercise price. Also suppose a large dividend is expected to be paid in two months. Because the stock price and 3-month option price will fall when the dividend is paid in two months, the 1-month option may be worth more than the 3-month option.

### The Risk-Free Rate Over the Life of the Option

As the risk-free rate increases, the value of the call (put) will increase (decrease). The intuition behind this property involves arbitrage arguments that require the use of synthetic securities.

### Dividends

The option owner does not have access to the cash flows of the underlying stock, and the stock price decreases when a dividend is paid. Thus, as the dividend increases, the value of the call (put) will decrease (increase).

### Volatility of the Stock Price Over the Life of the Option

Volatility is the friend of all options. As volatility increases, option values increase. This is due to the asymmetric payoff of options. Since long option positions have a maximum loss equal to the premium paid, increased volatility only increases the chances that the option will expire in-the-money. Many consider volatility to be the most important factor for option valuation.

Figure 1 summarizes the factors' effects on option prices: "+" indicates a positive effect on option price from an increase in the factor, and "-" is a negative effect on option price.

**Figure 1: Summary of Effects of Increasing a Factor on the Price of an Option**

<i>Factor</i>	<i>European Call</i>	<i>European Put</i>	<i>American Call</i>	<i>American Put</i>
S	+	–	+	–
X	–	+	–	+
T	?	?	+	+
$\sigma$	+	+	+	+
r	+	–	+	–
D	–	+	–	+

## UPPER AND LOWER PRICING BOUNDS

### LO 42.2: Identify and compute upper and lower bounds for option prices on non-dividend and dividend paying stocks.

In addition to those previously introduced, consider the following variables:

- $c$  = value of a European call option.
- $C$  = value of an American call option.
- $p$  = value of a European put option.
- $P$  = value of an American put option.
- $S_T$  = value of the stock at expiration.

Also, assume in the following examples that there are no transaction costs, all profits are taxed at the same rate, and borrowing and lending can be done at the risk-free rate.

#### Upper Pricing Bounds for European and American Options

A call option gives the right to purchase one share of stock at a certain price. Under no circumstance can the option be worth more than the stock. If it were, everyone would sell the option and buy the stock and realize an arbitrage profit. We express this as:

$$c \leq S_0 \text{ and } C \leq S_0$$

Similarly, a put option gives the right to sell one share of stock at a certain price. Under no circumstance can the put be worth more than the sale or strike price. If it were, everyone would sell the option and invest the proceeds at the risk-free rate over the life of the option. We express this as:

$$p \leq X \text{ and } P \leq X$$

For a European put option, we can further reduce the upper bound. Since it cannot be exercised early, it can never be worth more than the present value of the strike price:

$$p \leq Xe^{-rT}$$

#### Lower Pricing Bounds for European Calls on Nondividend-Paying Stocks

Consider the following two portfolios:

- Portfolio  $P_1$ : one European call,  $c$ , with exercise price  $X$  plus a zero-coupon risk-free bond that pays  $X$  at  $T$ .
- Portfolio  $P_2$ : one share of the underlying stock,  $S$ .

At expiration,  $T$ , Portfolio  $P_1$  will always be the greater of  $X$  (when the option expires out-of-the-money) or  $S_T$  (when the option expires in-the-money). Portfolio  $P_2$ , on the other hand, will always be worth  $S_T$ . Therefore,  $P_1$  is always worth at least as much as  $P_2$  at

expiration. If we know that at  $T$ ,  $P_1 \geq P_2$ , then it always has to be true because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$c + Xe^{-rT} \geq S_0$$

Since the value of a call option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European call on a nondividend-paying stock is:

$$c \geq \max (S_0 - Xe^{-rT}, 0)$$

### Lower Pricing Bounds for European Puts on Nondividend-Paying Stocks

Consider the following two portfolios:

- Portfolio  $P_3$ : one European put,  $p$ , plus one share of the underlying stock,  $S$ .
- Portfolio  $P_4$ : zero-coupon risk-free bond that pays  $X$  at  $T$ .

At expiration,  $T$ , Portfolio  $P_3$  will always be the greater of  $X$  (when the option expires in-the-money) or  $S_T$  (when the option expires out-of-the-money). Portfolio  $P_4$ , on the other hand, will always be worth  $X$ . Therefore,  $P_3$  is always worth at least as much as  $P_4$  at expiration. If we know that at  $T$ ,  $P_3 \geq P_4$ , it has to be true always because if it were not, arbitrage would be possible. Therefore, we can state the following:

$$p + S_0 \geq Xe^{-rT}$$

Since the value of a put option cannot be negative (if the option expires out-of-the-money, its value will be zero), the lower bound for a European put on a nondividend-paying stock is:

$$p \geq \max (Xe^{-rT} - S_0, 0)$$

### COMPUTING OPTION VALUES USING PUT-CALL PARITY

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#### LO 42.3: Explain put-call parity and apply it to the valuation of European and American stock options.

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The derivation of **put-call parity** is based on the payoffs of two portfolio combinations, a fiduciary call and a protective put.

A *fiduciary call* is a combination of a pure-discount (i.e., zero coupon), riskless bond that pays  $X$  at maturity and a call with exercise price  $X$ . The payoff for a fiduciary call at expiration is  $X$  when the call is out of the money, and  $X + (S - X) = S$  when the call is in the money.

A *protective put* is a share of stock together with a put option on the stock. The expiration date payoff for a protective put is  $(X - S) + S = X$  when the put is in the money, and  $S$  when the put is out of the money.



*Professor's Note: When working with put-call parity, it is important to note that the exercise prices on the put and the call and the face value of the riskless bond are all equal to  $X$ .*

When the put is in the money, the call is out of the money, and both portfolios pay  $X$  at expiration.

Similarly, when the put is out of the money and the call is in the money, both portfolios pay  $S$  at expiration.

Put-call parity holds that portfolios with identical payoffs must sell for the same price to prevent arbitrage. We can express the put-call parity relationship as:

$$c + Xe^{-rT} = S + p$$

Equivalencies for each of the individual securities in the put-call parity relationship can be expressed as:

$$\begin{aligned} S &= c - p + Xe^{-rT} \\ p &= c - S + Xe^{-rT} \\ c &= S + p - Xe^{-rT} \\ Xe^{-rT} &= S + p - c \end{aligned}$$

The single securities on the left-hand side of the equations all have exactly the same payoffs as the portfolios on the right-hand side. The portfolios on the right-hand side are the “synthetic” equivalents of the securities on the left. Note that the options must be European-style and the puts and calls must have the same exercise price for these relations to hold.

For example, to synthetically produce the payoff for a long position in a share of stock, you use the relationship:

$$S = c - p + Xe^{-rT}$$

This means that the payoff on a long stock can be synthetically created with a long call, a short put, and a long position in a risk-free discount bond.

The other securities in the put-call parity relationship can be constructed in a similar manner.



*Professor's Note: After expressing the put-call parity relationship in terms of the security you want to synthetically create, the sign on the individual securities will indicate whether you need a long position (+ sign) or a short position (– sign) in the respective securities.*



**Example: Call option valuation using put-call parity**

Suppose that the current stock price is \$52 and the risk-free rate is 5%. You have found a quote for a 3-month put option with an exercise price of \$50. The put price is \$1.50, but due to light trading in the call options, there was not a listed quote for the 3-month, \$50 call. Estimate the price of the 3-month call option.

**Answer:**

Rearranging put-call parity, we find that the call price is:

$$\text{call} = \text{put} + \text{stock} - Xe^{-rT}$$

$$\text{call} = \$1.50 + \$52 - \$50e^{-0.0125} = \$4.12$$

This means that if a 3-month, \$50 call is available, it should be priced at \$4.12 per share.

### LOWER PRICING BOUNDS FOR AN AMERICAN CALL OPTION ON A NONDIVIDEND-PAYING STOCK

#### LO 42.4: Explain the early exercise features of American call and put options.

Recall the following equation from our earlier discussion of the lower pricing bounds for a *European* call option:

$$c \geq \max (S_0 - Xe^{-rT}, 0)$$

Since the only difference between an American option and a European option is that the American option can be exercised early, American options can always be used to replicate their corresponding European options simply by choosing not to exercise them until expiration. Therefore, it follows that:

$$C \geq c \geq \max (S_0 - Xe^{-rT}, 0)$$

Note that when an American call is exercised, it is only worth  $S_0 - X$ . Since this value is never larger than  $S_0 - Xe^{-rT}$  for any  $r$  and  $T > 0$ , it is never optimal to exercise early. In other words, the investor can keep the cash equal to  $X$ , which would be used to exercise the option early, and invest that cash to earn interest until expiration. Since exercising the American call early means that the investor would have to forgo this interest, it is never optimal to exercise an American call on a nondividend-paying stock before the expiration date (i.e.,  $c = C$ ).

## LOWER PRICING BOUNDS FOR AN AMERICAN PUT OPTION ON A NONDIVIDEND-PAYING STOCK

While it is never optimal to exercise an American call on a nondividend-paying stock, American puts are optimally exercised early if they are sufficiently in-the-money. If an option is sufficiently in-the-money, it can be exercised, and the payoff ( $X - S_0$ ) can be invested to earn interest. In the extreme case when  $S_0$  is close to zero, the future value of the exercised cash value,  $Xe^{rT}$ , is always worth more than a later exercise,  $X$ . We know that:

$$P \geq p \geq \max (Xe^{-rT} - S_0, 0) \text{ for the same reasons that } C \geq c$$

However, we can place an even stronger bound on an American put since it can always be exercised early:

$$P \geq \max (X - S_0, 0)$$

Figure 2 summarizes what we now know regarding the boundary prices for American and European options.

**Figure 2: Lower and Upper Bounds for Options**

<i>Option</i>	<i>Minimum Value</i>	<i>Maximum Value</i>
European call	$c \geq \max (0, S_0 - Xe^{-rT})$	$S_0$
American call	$C \geq \max (0, S_0 - Xe^{-rT})$	$S_0$
European put	$p \geq \max (0, Xe^{-rT} - S_0)$	$Xe^{-rT}$
American put	$P \geq \max (0, X - S_0)$	$X$



*Professor's Note: For the exam, know the price limits in Figure 2. You will not be asked to derive them, but you may be expected to use them.*

### Example: Minimum prices for American vs. European puts

**Compute** the lowest possible price for 4-month American and European 65 puts on a stock that is trading at 63 when the risk-free rate is 5%.

**Answer:**

$$P \geq \max (0, X - S_0) = \max (0, 2) = \$2$$

$$p \geq \max (0, Xe^{-rT} - S_0) = \max (0, 65e^{-0.0167} - 63) = \$0.92$$

**Example: Minimum prices for American vs. European calls**

Compute the lowest possible price for 3-month American and European 65 calls on a stock that is trading at 68 when the risk-free rate is 5%.

**Answer:**

$$C \geq \max(0, S_0 - Xe^{-rT}) = \max(0, 68 - 65e^{-0.0125}) = \$3.81$$

$$c \geq \max(0, S_0 - Xe^{-rT}) = \max(0, 68 - 65e^{-0.0125}) = \$3.81$$

**RELATIONSHIP BETWEEN AMERICAN CALL OPTIONS AND PUT OPTIONS**

Put-call parity only holds for European options. For American options, we have an inequality. This inequality places upper and lower bounds on the difference between the American call and put options.

$$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$$

**Example: American put option bounds**

Consider an American call and put option on stock XYZ. Both options have the same 1-year expiration and a strike price of \$20. The stock is currently priced at \$22, and the annual interest rate is 6%. What are the upper and lower bounds on the American put option if the American call option is priced at \$4?

**Answer:**

The upper and lower bounds on the difference between the American call and American put options are:

$$S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$$

$$S_0 - X = 22 - 20 = \$2$$

$$S_0 - Xe^{-rT} = 22 - 20e^{-0.06(1)} = 22 - 18.84 = \$3.16$$

$$\$2 \leq C - P \leq \$3.16$$

or

$$-\$2 \geq P - C \geq -\$3.16$$

Therefore, when the American call is valued at \$4, the upper and lower bounds on the American put option will be:

$$\$2 \geq P \geq \$0.84$$

## THE IMPACT OF DIVIDENDS ON OPTION PRICING BOUNDS

Since most stock options have an expiration of less than a year, dividends can be estimated fairly accurately. Recall that to prevent arbitrage, when a stock pays a dividend, its value must decrease by the amount of the dividend. This increases the value of a put option and decreases the value of a call option.

Consider the following portfolios:

- Portfolio  $P_6$ : one European call option,  $c$ , plus cash equal to  $D + Xe^{-rT}$ .
- Portfolio  $P_7$ : one share of the underlying stock,  $S$ .

Similar to the development of the  $c \geq \max(S_0 - Xe^{-rT}, 0)$  equation, Portfolio  $P_6$  is always at least as large as  $P_7$ , or:

$$c \geq S_0 - D - Xe^{-rT}$$

All else equal, the payment of a dividend will reduce the lower pricing bound for a call option.

For put options:

- Portfolio  $P_8$ : one European put,  $p$ , plus one share of the underlying stock,  $S$ .
- Portfolio  $P_9$ : cash equal to  $D + Xe^{-rT}$ .

Using the same development as the  $p \geq \max(Xe^{-rT} - S_0, 0)$  equation:

$$p \geq D + Xe^{-rT} - S_0$$

All else equal, the payment of a dividend will increase the lower pricing bound for a put option.

## IMPACT OF DIVIDENDS ON EARLY EXERCISE FOR AMERICAN CALLS AND PUT-CALL PARITY

When the dividend is large enough, American calls might be optimally exercised early. This will be the case if the amount of the dividend exceeds the amount of interest that is forgone as a result of the early exercise. Note that if a large dividend makes early exercise optimal, exercise should take place immediately before the ex-dividend date. Put-call parity is adjusted for dividends in the following manner:

$$p + S_0 = c + D + Xe^{-rT}$$

This equation is verified using the same development as was used to derive the  $p + S_0 = c + Xe^{-rT}$  equation. The  $S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$  equation that we used to show the relationship between American call and put options is modified as follows:

$$S_0 - X - D \leq C - P \leq S_0 - Xe^{-rT}$$

## KEY CONCEPTS

### LO 42.1

Six factors influence the value of an option: current value of the underlying asset (stock); the strike price; the time to expiration of the option; the volatility of the stock price; the risk-free rate; and dividends.

With the exception of time to expiration, all of these factors affect European- and American-style options in the same way.

### LO 42.2

Call options cannot be worth more than the underlying security, and put options cannot be worth more than the strike price.

When the stock does not pay a dividend, European call options cannot be worth less than the difference between the current stock price and the present value of the strike price. European put options cannot be worth less than the difference between the present value of the strike price and the current stock price.

### LO 42.3

Put-call parity is a no-arbitrage relationship for European-style options with the same characteristics. It states that a portfolio consisting of a call option and a zero-coupon bond with a face value equal to the strike must have the same value as a portfolio consisting of the corresponding put option and the stock:

$$p + S_0 = c + Xe^{-rT}$$

### LO 42.4

It is never optimal to exercise an American call option on nondividend-paying stock prior to expiration.

American put options on nondividend-paying stocks can be optimally exercised prior to expiration if the put is sufficiently in-the-money.

Call options are always worth more than corresponding put options prior to expiration when both are at-the-money.

The difference between prices of an American call and corresponding put is bounded below by the difference between the current stock price and strike price, and above by the difference between the current stock price and the present value of the strike price.

## CONCEPT CHECKERS

1. Which of the following will not cause a decrease in the value of a European call option position on XYZ stock?
  - A. XYZ declares a 3-for-1 stock split.
  - B. XYZ raises its quarterly dividend from \$0.15 per share to \$0.17 per share.
  - C. The Federal Reserve lowers interest rates by 0.25% in an effort to stimulate the economy.
  - D. Investors believe the volatility of XYZ stock has declined.
2. Consider a European put option on a stock trading at \$50. The put option has an expiration of six months, a strike price of \$40, and a risk-free rate of 5%. The lower bound and upper bound on the put are:
  - A. \$10, \$40.00.
  - B. \$10, \$39.01.
  - C. \$0, \$40.00.
  - D. \$0, \$39.01.
3. Consider a 1-year European put option that is currently valued at \$5 on a \$25 stock and a strike of \$27.50. The 1-year risk-free rate is 6%. Which of the following is closest to the value of the corresponding call option?
  - A. \$0.00.
  - B. \$3.89.
  - C. \$4.10.
  - D. \$5.00.
4. Consider an American call and put option on the same stock. Both options have the same 1-year expiration and a strike price of \$45. The stock is currently priced at \$50, and the annual interest rate is 10%. Which of the following could be the difference in the two option values?
  - A. \$4.95.
  - B. \$7.95.
  - C. \$9.35.
  - D. \$12.50.
5. According to put-call parity for European options, purchasing a put option on ABC stock would be equivalent to:
  - A. buying a call, buying ABC stock, and buying a zero-coupon bond.
  - B. buying a call, selling ABC stock, and buying a zero-coupon bond.
  - C. selling a call, selling ABC stock, and buying a zero-coupon bond.
  - D. buying a call, selling ABC stock, and selling a zero-coupon bond.

**CONCEPT CHECKER ANSWERS**

1. A After a stock split, both the price of the stock and the strike price of the option will be adjusted, so the value of the option position will be the same. An increase in the dividend, a lower risk-free interest rate, and lower volatility of the price of the underlying stock, will all decrease the value of a European call option.
2. D The upper bound is the present value of the exercise price:  $\$40 \times e^{-0.05 \times 0.5} = \$39.01$ . Since the put is out-of-the-money, the lower bound is zero.
3. C  $c = p - Xe^{-rT} + S_0 = \$5 - \$27.50e^{-0.06 \times 1} + \$25 = \$4.10$
4. B The upper and lower bounds are:  $S_0 - X \leq C - P \leq S_0 - Xe^{-rT}$  or  $\$5 \leq C - P \leq \$9.28$ . Only \$7.95 falls within the bounds.
5. B The formula for put-call parity is  $p + S_0 = c + Xe^{-rT}$ . Rearranging to solve for the price of a put, we have  $p = c - S_0 + Xe^{-rT}$ .



# TRADING STRATEGIES INVOLVING OPTIONS

Topic 43

## EXAM FOCUS

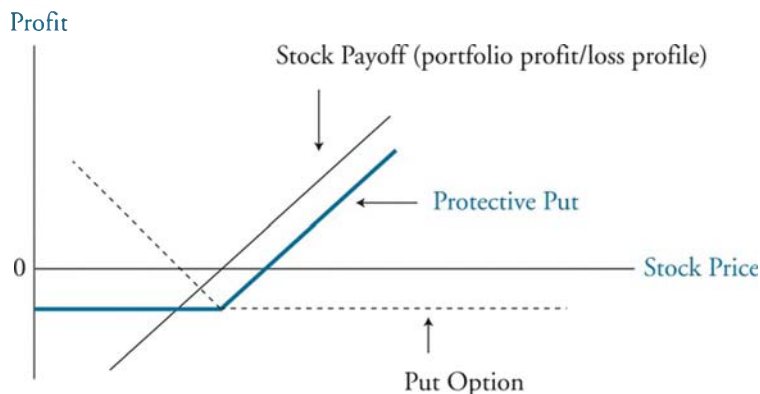
Traders and investors use option-based trading strategies to create an extraordinary spectrum of payoff profiles. This enables investors to take positions based on almost any possible expectation of the underlying stock over the life of the options. This topic describes the common option trading strategies and implementation. For the exam, know the general payoff graphs for each strategy discussed. In addition, know how to calculate the payoff for some of the more popular strategies including protective put, covered call, bull call spread, butterfly spread, and straddle.

## COVERED CALLS AND PROTECTIVE PUTS

**LO 43.1: Explain the motivation to initiate a covered call or a protective put strategy.**

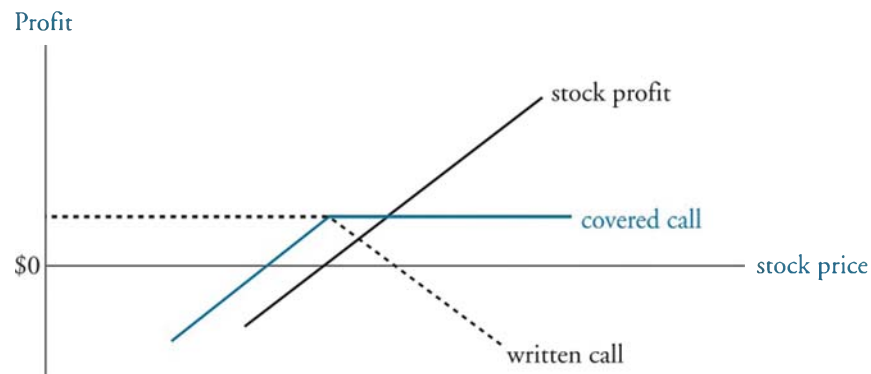
When an at-the-money long put position is combined with the underlying stock, we have created a **protective put** strategy. A protective put (also called *portfolio insurance* or a *hedged portfolio*) is constructed by holding a long position in the underlying security and buying a put option. You can use a protective put to limit the downside risk at the cost of the put premium,  $P_0$ . You will see by the diagram in Figure 1 that the investor will still be able to benefit from increases in the stock's price, but it will be lower by the amount paid for the put,  $P_0$ . Notice that the combined strategy looks very much like a call option. This should not be surprising since put-call parity requires that  $p + S_0$  be the same as  $c + Xe^{-rT}$ . Figure 1 illustrates this property.

**Figure 1: Protective Put Strategy**



Another common strategy is to sell a call option on a stock that is owned by the option writer. This is called a **covered call** position. By writing an out-of-the-money call option, the combined position caps the upside potential at the strike price. In return for giving up any potential gain beyond the strike price, the writer receives the option premium. This strategy is used to generate cash on a stock that is not expected to increase above the exercise price over the life of the option.

Figure 2: Profit Profile for a Covered Call



## SPREAD STRATEGIES

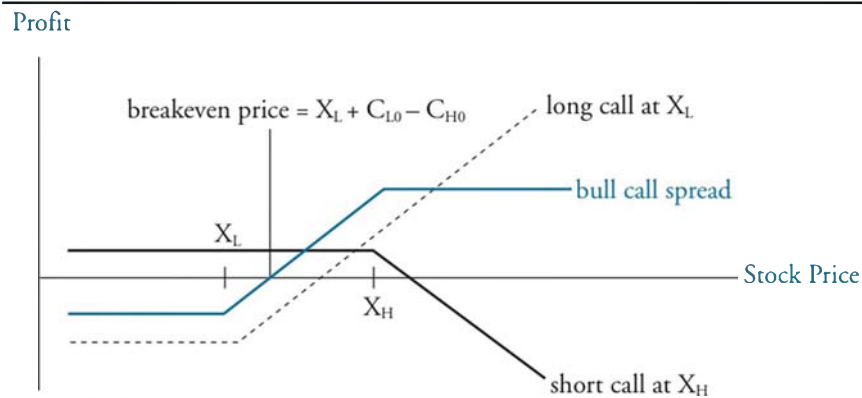
### LO 43.2: Describe the use and calculate the payoffs of various spread strategies.

Several spread strategies exist. These strategies combine options positions to create a desired payoff profile. The differences between the options are either the strike prices and/or the time to expiration. We will discuss bull and bear spreads, butterfly spreads, calendar spreads, and diagonal spreads.

### Bull and Bear Spreads

In a *bull call spread*, the buyer of the spread purchases a call option with a low exercise price,  $X_L$ , and subsidizes the purchase price of the call by selling a call with a higher exercise price,  $X_H$ . The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.

Figure 3: Bull Call Spread

**Example: Bull call spread**

An investor purchases a call for  $C_{L0} = \$3.00$  with a strike of  $X = \$40$  and sells a call for  $C_{H0} = \$1.00$  with a strike price of  $\$50$ . **Compute** the payoff of a bull call spread strategy when the price of the stock is at  $\$45$ .

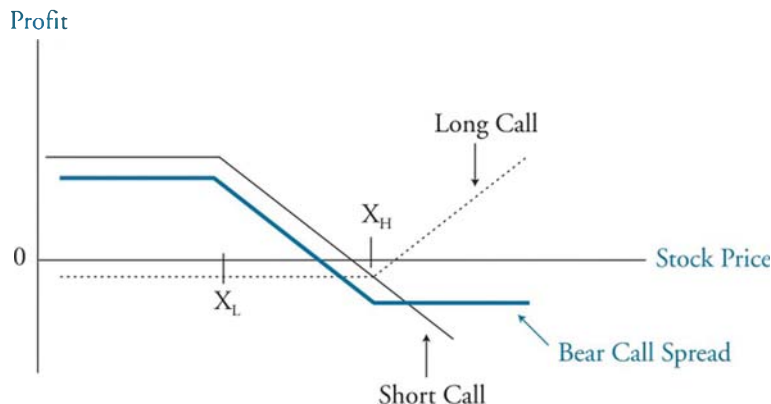
**Answer:**

$$\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$$

$$\text{profit} = \max(0, 45 - 40) - \max(0, 45 - 50) - 3 + 1 = \$3.00$$

A *bear call spread* is the sale of a bull spread. That is, the bear spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a “bear” strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call’s cost. The purpose of the long call is to protect from sharp increases in stock prices. The payoff is the opposite (mirror image) of the bull call spread and is shown in Figure 4.

Figure 4: Bear Call Spread



Puts can also be used to replicate the payoffs for both a bull call spread and a bear call spread. In a *bear put spread* the investor buys a put with a higher exercise price and sells a put with a lower exercise price.

**Example: Bear put spread**

An investor sells a put for  $P_{L0} = \$3.00$  with a strike of  $X = \$20$  and purchases a put for  $P_{H0} = \$4.50$  with a strike price of  $\$40$ . Compute the payoff of a bear put spread strategy when the price of the stock is at  $\$35$ .

**Answer:**

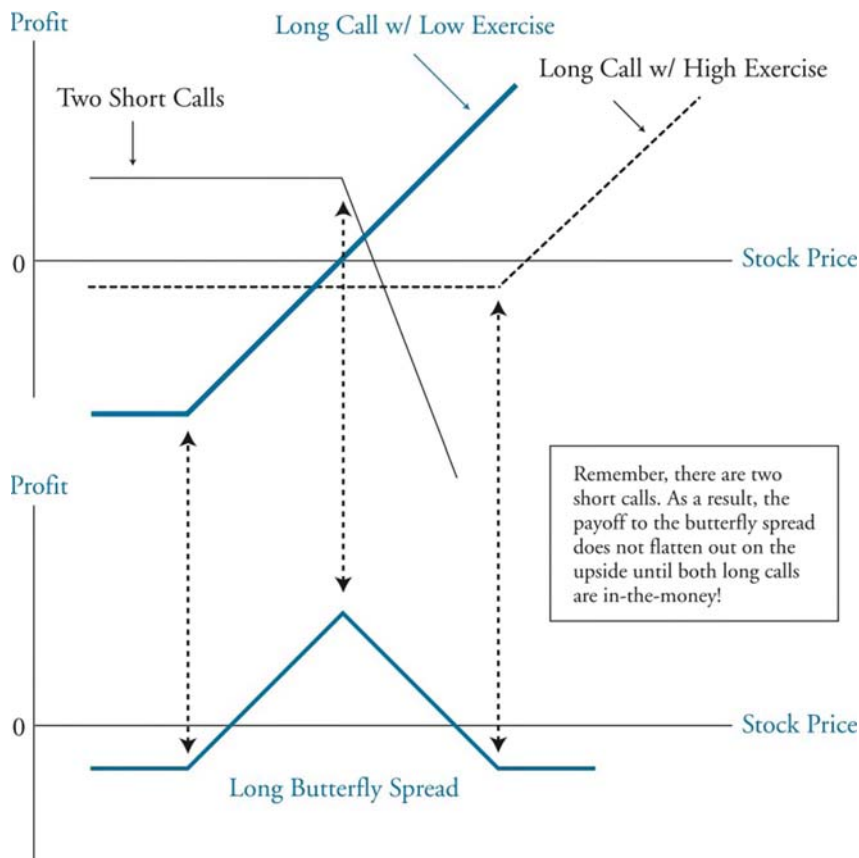
$$\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$$

$$\text{profit} = \max(0, 40 - 35) - \max(0, 20 - 35) - 4.50 + 3 = \$3.50$$

**Butterfly Spreads**

A *butterfly spread* involves the purchase or sale of *three* different call options. Here, the investor buys one call with a low exercise price, buys another call with a high exercise price, and sells *two* calls with an exercise price in between. The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is limited. The two graphs in Figure 5 illustrate the construction and payoffs of a butterfly spread.

Figure 5: Butterfly Spread Construction and Behavior

**Example: Butterfly spread with calls**

An investor makes the following transactions in calls on a stock:

- Buys one call defined by  $C_{L0} = \$7.00$  and  $X_L = \$55$ .
- Buys one call defined by  $C_{H0} = \$2.00$  and  $X_H = \$65$ .
- Sell two calls defined by  $C_{M0} = \$4.00$  and  $X_M = \$60$ .

Compute the payoff of a butterfly spread strategy with calls when the stock is at \$60.

**Answer:**

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

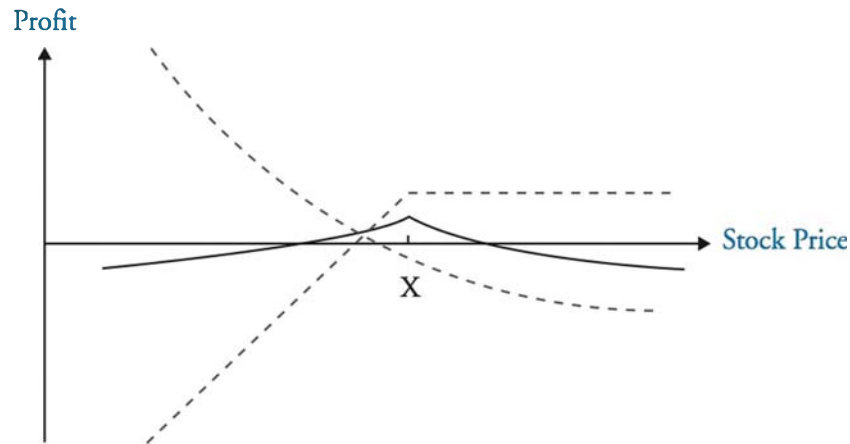
$$\text{profit} = \max(0, 60 - 55) - 2\max(0, 60 - 60) + \max(0, 60 - 65) - 7 + 2(4) - 2 = \$4.00$$

To create a butterfly spread with put options, the investor would buy a low and high strike put option and sell two puts with an intermediate strike price. Again, the combined position is constructed by summing the payoffs of the individual options at each stock price.

## Calendar Spreads

A *calendar spread* is created by transacting in two options that have the same strike price but different expirations. Figure 6 shows a calendar spread using put options. The strategy sells the short-dated option and buys the long-dated option. Notice that the payoff here is similar to the butterfly spread. The investor profits only if the stock remains in a narrow range, but losses are limited. In this case, the losses are not symmetrical as they are in the butterfly spread. A calendar spread based on calls is created in similar fashion.

Figure 6: Calendar Spread (Using Two Put Options)



Calendar spreads are categorized differently depending on the relationship between the strike price and the current stock price. The strategy is referred to as a **neutral calendar spread** if the strike price is close to the current stock price. A **bullish calendar spread** has a strike price above the current stock price, and a **bearish calendar spread** has a strike price below the current stock price.

A **reverse calendar spread** produces a payoff that is opposite of the graph shown in Figure 6. Instead of selling a short-dated option and buying a long-dated option, the investor of a reverse calendar spread will buy a short-dated option and sell a long-dated option. The investor will profit when the stock is well above or below the strike price and will suffer a loss if the stock is near the strike price.

## Diagonal Spreads

A *diagonal spread* is similar to a calendar spread except that instead of using options with the same strike price and different expirations, the options in a diagonal spread can have different strike prices in addition to different expirations.

## Box Spreads

A *box spread* is a combination of a bull call spread and a bear put spread on the same asset. This strategy will produce a constant payoff that is equal to the high exercise price ( $X_H$ ) minus the low exercise price ( $X_L$ ). Under a no arbitrage assumption, the present value of the payoff will equal the net premium paid (i.e., profit will equal zero).

When the profit from this strategy is different than zero, an investor can capitalize on the arbitrage opportunity by either buying or selling the box. If the profit is positive, the investor will create a long box spread by buying a call at  $X_L$ , selling a call at  $X_H$ , buying a put at  $X_H$ , and selling a put at  $X_L$ . If the profit is negative, the investor will create a short box spread by buying a call at  $X_H$ , selling a call at  $X_L$ , buying a put at  $X_L$ , and selling a put at  $X_H$ . Note that box spread arbitrage is only successful with European options.

## COMBINATION STRATEGIES

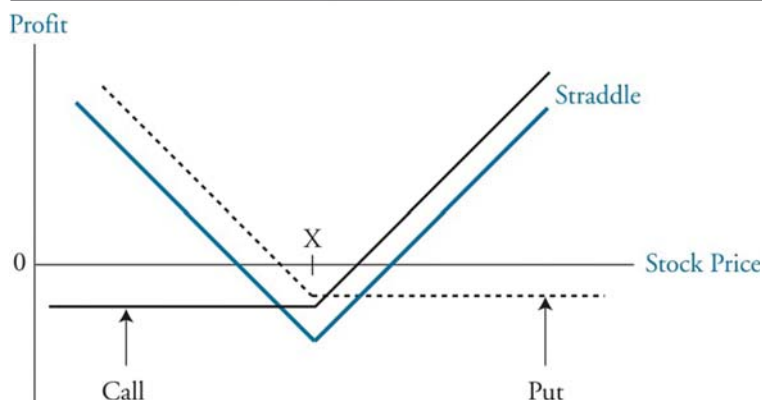
**LO 43.3: Describe the use and explain the payoff functions of combination strategies.**

Combinations are option strategies involving both puts and calls. We will discuss straddles, strangles, strips, and straps.

### Straddle

A long *straddle* (bottom straddle or straddle purchase) is created by purchasing a call and a put with the same strike price and expiration. Figure 7 illustrates the payoff for a long straddle position. Both options have the same exercise price and expiration. Note that this strategy is profitable when the stock price moves strongly in either direction. This strategy bets on volatility. A short straddle (top straddle or straddle write) sells both options and bets on little movement in the stock. A short straddle bets on the same thing as the butterfly spread or the calendar spread, except the losses are not limited. It is a bet that will profit more if correct but also lose more if it is incorrect. Straddles are symmetric around the strike price.

**Figure 7: Long Straddle Profit/Loss**





**Example: Straddle**

An investor purchases a call on a stock, with an exercise price of \$45 and a premium of \$3, and purchases a put option with the same maturity that has an exercise price of \$45 and a premium of \$2. **Compute** the payoff of a straddle strategy if the stock is at \$35.

**Answer:**

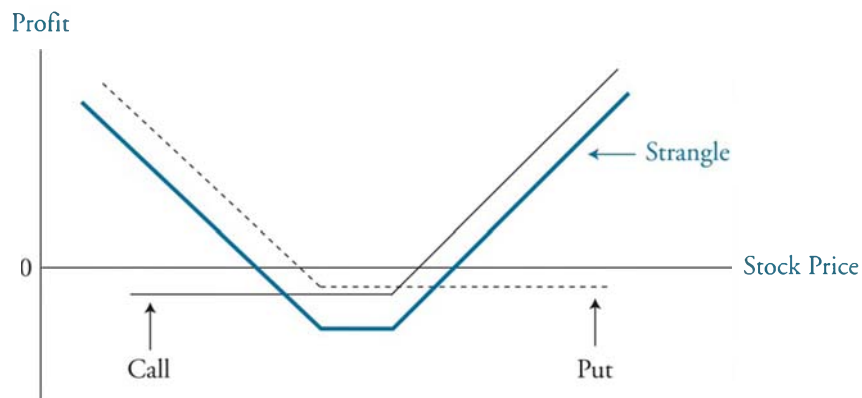
$$\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$

$$\text{profit} = \max(0, 35 - 45) + \max(0, 45 - 35) - 3 - 2 = \$5$$

**Strangle**

A *strangle* (or bottom vertical combination) is similar to a straddle except that the options purchased are slightly out-of-the-money, so it is cheaper to implement than the straddle. The payoff is similar to the straddle except for a flat section between the strike prices, as shown in Figure 8. Because it is cheaper, the stock will have to move more relative to the straddle before the strangle pays off. Strangles are also symmetric around the strikes.

**Figure 8: Long Strangle Profit/Loss**

**Example: Strangle**

An investor purchases a call on a stock, with an exercise price of \$50 and a premium of \$1.50, and purchases a put option with the same maturity that has an exercise price of \$45 and a premium of \$2. **Compute** the payoff of a strangle strategy if the stock is at \$40.

**Answer:**

$$\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$

$$\text{profit} = \max(0, 40 - 50) + \max(0, 45 - 40) - 1.50 - 2 = \$1.50$$

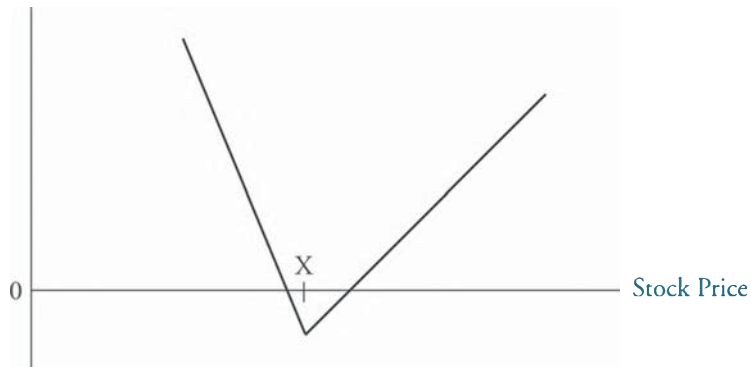
A short strangle (or a top vertical combination) is similar to the short straddle.

## Strips and Straps

A *strip* involves purchasing two puts and one call with the same strike price and expiration. Figure 9 illustrates a strip. Notice the asymmetry of the payoff. A strip is betting on volatility but is more bearish since it pays off more on the downside.

**Figure 9: Strip Profit/Loss**

Profit/Loss

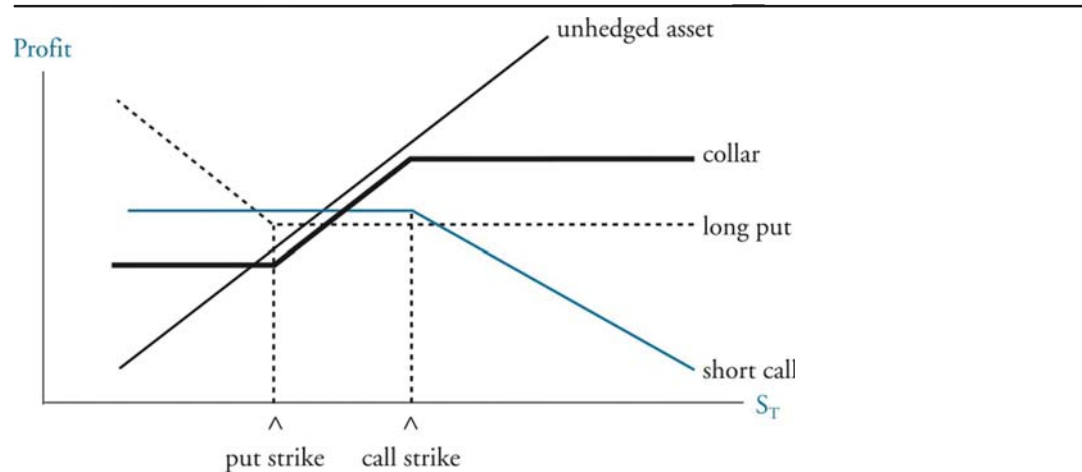


A *strap* involves purchasing two calls and one put with the same strike price and expiration. A strap is betting on volatility but is more bullish since it pays off more on the upside.

## Collar

A *collar* is the combination of a protective put and covered call. The usual goal is for the owner of the underlying asset to buy a protective put and then sell a call to pay for the put. If the premiums of the two are equal, it is called a **zero-cost collar**.

Figure 10: Collar



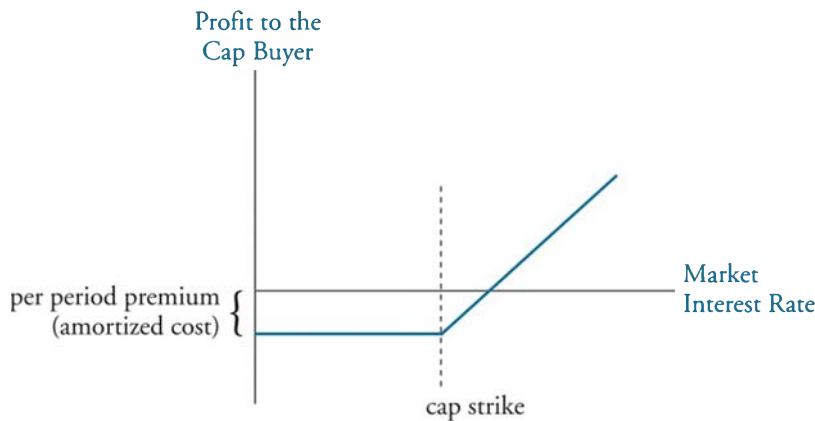
## INTEREST RATE CAPS AND FLOORS

An **interest rate cap** is an agreement in which one party agrees to pay the other at regular intervals over a certain period of time when the benchmark interest rate (e.g., LIBOR) exceeds the strike rate specified in the contract. This strike rate is called the **cap rate**. For example, the seller of a cap might agree to pay the buyer at the end of any quarter over the next two years if LIBOR is greater than a cap rate of 6%.

The buyer of a cap has a position similar to that of a buyer of a call on LIBOR, both of whom benefit when interest rates rise. Because an interest rate cap is a multi-period agreement, a cap is actually a portfolio of call options on LIBOR called **caplets**. For example, the 2-year cap discussed above is actually a portfolio of eight interest rate options with different maturity dates.

The cap buyer pays a premium to the seller and exercises the cap if the market rate of interest rises above the cap strike. The diagram in Figure 11 illustrates the profits of an interest rate cap at the end of one particular settlement period. It has the familiar shape of a long position in a call option.

Figure 11: Profit to a Long Cap

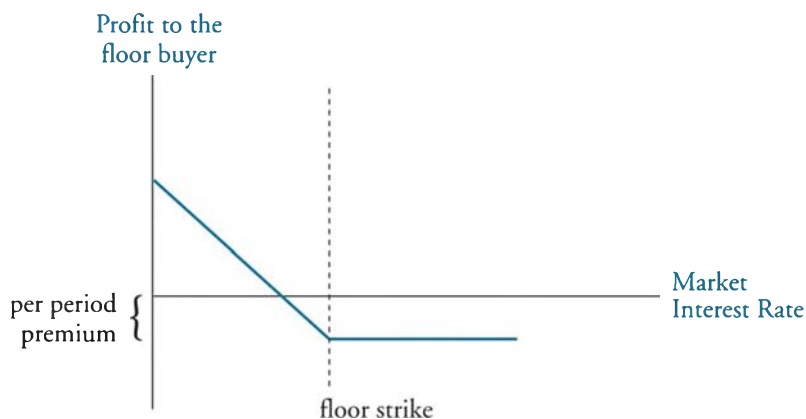


An **interest rate floor** is an agreement in which one party agrees to pay the other at regular intervals over a certain time period when the benchmark interest rate (e.g., LIBOR) falls below the strike rate specified in the contract. This strike rate is called the **floor rate**. For example, the seller of a floor might agree to pay the buyer at the end of any quarter over the next two years if LIBOR is less than a floor rate of 4%.

The buyer of a floor benefits from an interest rate decrease and, therefore, has a position that is similar to that of a buyer of a put on LIBOR, who benefits when interest rates fall and the price of the instrument rises. Once again, because a floor is a multi-period agreement, a floor is actually a portfolio of put options on LIBOR called **floorlets**.

The floor buyer pays a premium and exercises the floor if the market rate of interest falls below the floor strike. The diagram in Figure 12 illustrates the profits of an interest rate floor at the end of one particular settlement period. It has the same shape as a long put option.

Figure 12: Profit to a Long Floor



Options are traded both on *interest rates* and on *prices* of fixed-income securities. So far we've talked about options on interest rates. The values of comparable options on rates and prices respond differently to changes in interest rates because of the inverse relationship

between bond yields and bond prices. Figure 13 outlines how each type of option responds to changes in yields and bond prices.

**Figure 13: Options on Rate vs. Options on Prices**

<i>Option</i>	<i>If Rates Increase and Bond Prices Decrease</i>	<i>If Rates Decrease and Bond Prices Increase</i>
Value of call on LIBOR	Increases	Decreases
Value of call on bond price	Decreases	Increases
Value of put on LIBOR	Decreases	Increases
Value of put on bond price	Increases	Decreases

We can also interpret caps and floors in terms of options on the prices of fixed-income securities:

- A long cap is equivalent to a portfolio of long put options on fixed-income security prices.
- A long floor is equivalent to a portfolio of long call options on fixed-income security prices.

An **interest rate collar** is a simultaneous position in a floor and a cap on the same benchmark rate over the same period with the same settlement dates. There are two types of collars:

- The first type of collar is to purchase a cap and sell a floor. For example, an investor with a LIBOR-based liability could purchase a cap on LIBOR at 8% and simultaneously sell a floor on LIBOR at 4% over the next year. The investor has now hedged the liability so that the borrowing costs will stay within the “collar” of 4% to 8%. If the cap and floor rates are set so that the premium paid from buying the cap is exactly offset by the premium received from selling the floor, the collar is called a “zero-cost” collar.
- The second type of collar is to purchase a floor and sell a cap. For example, an investor with a LIBOR-based asset could purchase a floor on LIBOR at 3% and simultaneously sell a cap at 7% over the next year. The investor has now hedged the asset so the returns will stay within the collar of 3% to 7%. The investor can create a zero-cost collar by choosing the cap and floor rates so that the premium paid on the floor offsets the premium received on the cap.

## KEY CONCEPTS

### LO 43.1

Stock options can be combined with their underlying stock to generate various payoff profiles. A protective put combines an at-the-money long put position with the underlying stock. A covered call involves selling a call option on a stock that is owned by the option writer.

### LO 43.2

Spread strategies combine options in the same option class to generate various payoff profiles.

The buyer of a bull call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for the out-of-the-money written call.

The bear call spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices (i.e., a “bear” strategy). As stock prices fall, the investor keeps the premium from the written call, net of the long call’s cost.

A box spread is an extreme method of locking in value. The dollar return for a box spread is fixed. It is a combination of a bull call spread and a bear put spread.

A calendar spread is created by transacting in two options that have the same strike price but different expirations.

The buyer of a butterfly spread is essentially betting that the stock price will stay near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains if the stock price strays from this level is not large.

In a diagonal spread, options can have different strike prices and different expirations.

Bull call spread payoff:

$$\text{profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$$

Bear put spread payoff:

$$\text{profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$$

Butterfly spread payoff:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

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**LO 43.3**

Combination strategies combine puts and calls to generate various payoff strategies.

A long straddle (bottom straddle or straddle purchase) is created by purchasing a call and a put with the same strike price and expiration. Note that this strategy only pays off when the stock moves in either direction.

A strangle (or bottom vertical combination) is similar to a straddle except that the option purchased is slightly out-of-the-money, so it is cheaper to implement than the straddle.

A strip is betting on volatility but is more bearish since it pays off more on the down side.

A strap is betting on volatility but is more bullish since it pays off more on the up side.

Straddle payoff:

$$\text{profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$

Strangle payoff:

$$\text{profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$



## CONCEPT CHECKERS

1. An investor is very confident that a stock will change significantly over the next few months; however, the direction of the price change is unknown. Which strategies will most likely produce a profit if the stock price moves as expected?
  - I. Short butterfly spread.
  - II. Bearish calendar spread.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
  
2. Which of the following will create a bear spread?
  - A. Buy a call with a strike price of  $X = 45$  and sell a call with a strike price of  $X = 50$ .
  - B. Buy a call with a strike price of  $X = 50$  and buy a put with a strike price of  $X = 55$ .
  - C. Buy a put with a strike price of  $X = 45$  and sell a put with a strike price of  $X = 50$ .
  - D. Buy a call with a strike price of  $X = 50$  and sell a call with a strike price of  $X = 45$ .
  
3. An investor believes that a stock will either increase or decrease greatly in value over the next few months, but believes a down move is more likely. Which of the following strategies will be the best for this investor?
  - A. A protective put.
  - B. An at-the-money strip.
  - C. An at-the-money strap.
  - D. A top vertical combination.
  
4. An investor constructs a long straddle by buying an April \$30 call for \$4 and buying an April \$30 put for \$3. If the price of the underlying shares is \$27 at expiration, what is the profit on the position?
  - A. -\$4.
  - B. -\$2.
  - C. \$2.
  - D. \$3.
  
5. Consider an option strategy where an investor buys one call option with an exercise price of \$55 for \$7, sells two call options with an exercise price of \$60 for \$4, and buys one call option with an exercise price of \$65 for \$2. If the stock price declines to \$25, what will be the profit or loss on the strategy?
  - A. -\$3.
  - B. -\$1.
  - C. \$1.
  - D. \$2.

**CONCEPT CHECKER ANSWERS**

1. A A short butterfly spread will produce a modest profit if there is a large amount of volatility in the price of the stock. A bearish calendar spread is a play using options with different expiration dates.
2. D Spread strategies involve purchasing and selling an option of the same type. A bear spread with calls involves buying a call with a high strike price and selling a call with a low strike price. The investor profits if stock prices fall by keeping the premium from the written call, net of the premium from the purchased call. Note that a bear spread can also be constructed with put options by buying a put with a high strike price and selling a put with a low strike price. With a bear put spread, if the stock price declines and both puts are exercised, the investor receives the difference between the strike prices less the net premium paid.
3. B An at-the-money strip bets on volatility but is more bearish since it pays off more on the downside.
4. A The sum of the premiums paid for the position is \$7. With the underlying stock at \$27, the put will be worth \$3, while the call option will be worthless. The value of the position is  $(-\$7 + \$3) = -\$4$ .
5. B The strategy described is a butterfly spread where the investor buys a call with a low exercise price, buys another call with a high exercise price, and sell two calls with a price in between. In this case, if the option moves to \$25, none of the call options will be in the money, so the profit is equal to the net premium paid, which is  $-\$7 + (2 \times \$4) - \$2 = -\$1$ .

# EXOTIC OPTIONS

Topic 44

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## EXAM FOCUS

In this topic, we define and discuss the important characteristics of a variety of exotic options. The difference between exotic options and more traditional exchange-traded instruments is also highlighted. Be familiar with the payoff structures for the various exotic options discussed.

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## EVALUATING EXOTIC OPTIONS

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**LO 44.1: Define and contrast exotic derivatives and plain vanilla derivatives.**

**LO 44.2: Describe some of the factors that drive the development of exotic products.**

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Plain vanilla derivatives include listed futures contracts and commonly used forwards and other over-the-counter (OTC) derivatives that are traded in fairly liquid markets. Exotic derivatives are customized to fit a specific firm need for hedging that cannot be met by plain vanilla derivatives. With plain vanilla derivatives, there is little uncertainty about the cost, the current market value, when they will pay, how much they will pay, and the cost of exiting the position. With exotic derivatives, some or all of these may be in question.

Exotic derivatives are developed for several reasons. The main purpose is to provide a unique hedge for a firm's underlying assets. Other reasons include addressing tax and regulatory concerns as well as speculating on the expected future direction of market prices.

Four questions that should be considered when evaluating exotic derivative strategies are:

- Will the strategy pay in the right circumstances to provide an effective hedge? Problems with understanding the payoff of the exotic derivative and credit risk of the derivative strategy can lead to a difference between the payoff the user expects and the actual payoff received.
- What is the cost of the exotic derivative hedging strategy?
- Is a pricing model needed, and does the user have the appropriate pricing model to estimate dealer cost and monitor the value of non-traded derivatives over time?
- How is a derivative position reversed? Note that the costs of exiting a position or strategy may involve penalties and large bid-ask spreads or require a pricing model to evaluate alternatives.

## USING PACKAGES TO FORMULATE A ZERO-COST PRODUCT

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### LO 44.3: Explain how any derivative can be converted into a zero-cost product.

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A package is defined as some combination of standard European options, forwards, cash, and the underlying asset. Bull, bear, and calendar spreads, as well as straddles and strangles, are examples of packages. Packages usually consist of selling one instrument with certain characteristics and buying another with somewhat different characteristics. Because packages often consist of a long position and a short position, they can be constructed so that the initial cost to the investor is zero.

For example, consider a zero-cost short collar. A short collar combines a long standard put option with an exercise price  $X_L$  and a short standard call option with exercise price  $X_H$  (where  $X_L < X_H$ ). If the premium the investor pays for the put option is exactly offset by the premium the investor receives for the short call position, the investor's net cost for implementing the short collar strategy is zero. In any case where the investor's cash outflows from long positions are offset by cash inflows from short positions, the investor can use a package to create a zero-cost product.

## TRANSFORMING STANDARD AMERICAN OPTIONS INTO NONSTANDARD AMERICAN OPTIONS

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### LO 44.4: Describe how standard American options can be transformed into nonstandard American options.

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Recall that standard exchange-traded American options can be exercised at any time prior to expiration. If some of the available expiration periods are restricted, or changes are made to other standard features, standard options become what we refer to as **nonstandard options**. Nonstandard options are common in the over-the-counter (OTC) market.

There are three common features that transform standard American options into nonstandard options:

- The most common transformation can be made to restrict early exercise to certain dates (e.g., a three month call option may only be exercised on the last day of each month.) This type of transformation results in a **Bermudan option**.
- Early exercise can be limited to a certain portion of the life of the option (e.g., there is a "lock out" period that does not allow a 6-month call option to be exercised in the first three months of the call's life).
- The option's strike price may change (e.g., the strike price of a 3-year call option with a strike price of 40 at initiation may rise to 44 in year 2 and 48 in year 3).

## EXOTIC OPTION PAYOFF STRUCTURES

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**LO 44.5: Identify and describe the characteristics and pay-off structure of the following exotic options: gap, forward start, compound, chooser, barrier, binary, lookback, shout, Asian, exchange, rainbow, and basket options.**

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### Gap Options

A gap option has two strike prices,  $X_1$  and  $X_2$ . ( $X_2$  is sometimes referred to as the trigger price.) If these two strike prices are equal, the gap option payoff will be the same as an ordinary option. If the two strike prices differ and the payoff for a gap option is non-zero, there will be a gap in the payoff graph that is either increased or decreased by the difference between the strike prices. Gap options can be valued with a slight modification to the Black-Scholes-Merton option pricing model, which will be discussed in Book 4.

For a *gap call option*, if  $X_2$  is greater than  $X_1$ , and the stock price at maturity,  $S_T$ , is greater than the trigger price,  $X_2$ , then the payoff for the call option will be equal to  $S_T - X_1$ . If the stock price is less than or equal to  $X_2$ , the payoff will be zero. Note that a negative payoff can occur if the stock price is greater than  $X_2$  and  $X_2$  is less than  $X_1$ . In this case, the payoff will be reduced by  $X_2 - X_1$ .

For a *gap put option*, if  $X_2$  is less than  $X_1$ , and the stock price at maturity,  $S_T$ , is less than the trigger price,  $X_2$ , then the payoff for the put option will be equal to  $X_1 - S_T$ . If the stock price is greater than or equal to  $X_2$ , the payoff will be zero. A negative payoff can occur if the stock price is less than  $X_2$  and  $X_2$  is greater than  $X_1$ . Like with a gap call option, if this is the case, the payoff will be reduced by  $X_2 - X_1$ .

### Forward Start Options

Forward start options are options that begin their existence at some time in the future. For example, today an investor may purchase a 3-month call option that will not come into existence until six months from today. Employee incentive plans commonly incorporate forward start options in which at-the-money options will be created after some period of employment has passed. Note that when the underlying asset is a nondividend paying stock, the value of a forward start option will be identical to the value of a European at-the-money option with the same time to expiration as the forward start option.

### Compound Options

Compound options are options on options. There are four key types of compound options:

- A *call on a call* gives the investor the right to buy a call option at a set price for a set period of time.
- A *call on a put* gives the investor the right to buy a put option at a set price for a set period of time.

- A *put on a call* gives the investor the right to sell a call option at a set price for a set period of time.
- A *put on a put* gives the investor the right to sell a put option at a set price for a set period of time.

Compound options have two levels of the underlying that determine their value—the value of the underlying option, which in turn is determined by the value of the underlying asset.

Compound options consist of two strike prices and two exercise dates. The first strike price and exercise date are used by the holder to evaluate whether to exercise the first option to receive the second option, where the second option is an option on the underlying asset, or just let the compound option expire. For example, a call on a call would be exercised if the price of the call on the underlying for the second call option were greater than the strike price of the initial option. The strike price and exercise date on the second call, however, are related to the value of the underlying asset.

### Chooser Options

This interesting option allows the owner, after a certain period of time has elapsed, to choose whether the option is a call or a put. The option with the greater value after the requisite time has elapsed will determine whether the owner will choose the option to be a put or a call.

### Barrier Options

Barrier options are options whose payoffs (and existence) depend on whether the underlying's asset price reaches a certain barrier level over the life of the option. These options are usually less expensive than standard options, and essentially come in either *knock-out* or *knock-in* flavors. Specific types of barrier options are:

- *Down-and-out call (put)*. A standard call (put) option that ceases to exist if the underlying asset price hits the barrier level, which is set below the current stock value.
- *Down-and-in call (put)*. A standard call (put) option that only comes into existence if the underlying asset price hits the barrier level, which is set below the current stock value.
- *Up-and-out call (put)*. A standard call (put) option that ceases to exist if the underlying asset price hits a barrier level, which is set above the current stock value.
- *Up-and-in call (put)*. A standard call (put) option that only comes into existence if the underlying asset price hits the above-current stock-price barrier level.

Barrier options have characteristics that can be very different from those of standard options. For example, vega, the sensitivity of an option's price to changes in volatility, is always positive for a standard option but may be negative for a barrier option. Increased volatility on a down-and-out option and an up-and-out option does not increase value because the closer the underlying gets to the barrier price, the greater the chance the option will expire.

Note that the value of a down-and-out call combined with the value of a down-and-in call is equal to the value of a standard call option. In other words, by knowing the value of two of these three options you can calculate the value of the remaining option (e.g., down-and-out call = standard call – down-and-in call). Similarly, the value of a standard put option is equal to the value of an up-and-out put plus the value of an up-and-in put.



## Binary Options

Binary options generate discontinuous payoff profiles because they pay only one price at expiration if the asset value is above the strike price. The term binary means that the option payoff has one of two states: the option pays a set dollar amount at expiration if the option is above the strike price, or the option pays nothing if the price is below the strike price. Hence, a payoff discontinuity results from the fact that the payoff is only one value—it does not increase continuously with the price of the underlying asset as in the case of a traditional option.

In the case of a **cash-or-nothing call**, a fixed amount,  $Q$ , is paid if the asset ends up above the strike price. Since the Black-Scholes-Merton formula denotes  $N(d_2)$  as the probability of the asset price being above the strike price, the value of a cash-or-nothing call is equal to  $Qe^{-rT}N(d_2)$ .

An **asset-or-nothing call** pays the value of the stock when the contract is initiated if the stock price ends up above the strike price at expiration. The corresponding value for this option is  $S_0e^{-qT}N(d_1)$ , where  $q$  is the continuous dividend yield.

## Lookback Options

Lookback options are options whose payoffs depend on the maximum or minimum price of the underlying asset during the life of the option. A **floating lookback call** pays the difference between the expiration price and the minimum price of the stock over the horizon of the option. This essentially allows the owner to purchase the security at its lowest price over the option's life. On the other hand, a **floating lookback put** pays the difference between the expiration and maximum price of the stock over the time period of the option. This translates into allowing the owner of the option to sell the security at its highest price over the life of the option.

Lookback options can also be fixed when an exercise price is specified. A **fixed lookback call** has a payoff function that is identical to a European call option. However, for this exotic option, the final stock price (or expiration price) in the European call option payoff is replaced by the maximum price during the option's life. Similarly, a **fixed lookback put** has a payoff like a European put option but replaces the final stock price with the minimum price during the option's life.

## Shout Options

A shout option allows the owner to pick a date when he “shouts” to the option seller, which then translates into an intrinsic value of the option at the time of the shout. At option expiration, the owner receives the maximum of the shout intrinsic value or the option expiration intrinsic value. In other words, for a shout call option, even if the price of the stock falls after the shout, the investor has locked in the difference between the price of the stock and the shout price. If the stock continues to rise, the shout option will have a payoff consistent with a standard call option. Note that most shout options allow for one “shout” during the option's life.



## Asian Options

Asian options have payoff profiles based on the average price of the security over the life of the option. *Average price* calls and puts pay off the difference between the average stock price and the strike price. Note that the average price will be much less volatile than the actual price. This means that the price for an Asian average price option will be lower than the price of a comparable standard option. *Average strike* calls and average strike puts pay off the difference between the stock expiration price and average price, which essentially represents the strike price in a typical intrinsic value calculation. If the average price or strike price for an Asian option is based on a geometric average, then using an option pricing model is not a problem because a geometric average is lognormal. However, most Asian options base their average calculations on arithmetic averages, which complicates the pricing process. In this case, a lognormal distribution of prices is assumed, which provides an adequate approximation.

## Exchange Options

A common use of an option to exchange one asset for another, often called an exchange option, is to exchange one currency with another. For example, consider a U.S. investor who holds an option to purchase euros with yen at a specified exchange rate. In this particular case, the option will be exercised if euros are more valuable to the U.S. investor than yen. Other applications, such as tender offers to exchange one stock for another, also arise in certain situations.

## Basket Options

Basket options are simply options to purchase or sell baskets of securities. These baskets may be defined specifically for the individual investor and may be composed of specific stocks, indices, or currencies. Any exotic options that involve several different assets are more generally referred to as **rainbow options**.

## Volatility and Variance Swaps

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### LO 44.6: Describe and contrast volatility and variance swaps.

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A **volatility swap** involves the exchange of volatility based on a notional principal. One side of the swap pays based on a pre-specified fixed volatility while the other side pays based on realized volatility. Unlike the exotic options we have discussed thus far, volatility swaps are a bet on volatility alone as opposed to a bet on volatility and the price of the underlying asset.

Much like a volatility swap, a **variance swap** involves exchanging a pre-specified fixed variance rate for a realized variance rate. The variance rate being exchanged is simply the square of the volatility rate. However, unlike volatility swaps, variance swaps are easier to price and hedge since they can be replicated using a collection of call and put options.

## ISSUES IN HEDGING EXOTIC OPTIONS

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**LO 44.7: Explain the basic premise of static option replication and how it can be applied to hedging exotic options.**

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The typical dynamic option-hedging situation uses option Greeks to measure sensitivity of the option value to changes in underlying asset characteristics (i.e., creating a delta-neutral portfolio). Hedging is simpler with some exotic options than it is with plain vanilla options. Asian options, for instance, depend on the average price of the underlying. Through time, the uncertainty of the average value gets smaller. Hence, the option begins to become less sensitive to changes in the value of the security because the payoff can be estimated more accurately.

Hedging positions in barrier and other exotic options are not so straightforward. This type of hedging requires the replication of a portfolio that is exactly opposite to the option position. When the replication portfolio requires frequent adjustments to the holdings in the underlying assets, the hedging procedure is referred to as dynamic options replication. **Dynamic options replication** requires frequent trading, which makes it costly to implement.

As an alternative, a **static options replication** approach may be used to hedge positions in exotic options. In this case, a short portfolio of actively traded options that approximates the option position to be hedged is constructed. This short replication options portfolio is created once, which drastically reduces the transaction costs associated with dynamic rebalancing.

## KEY CONCEPTS

### LO 44.1

Plain vanilla derivatives include listed futures contracts and commonly used forwards and other OTC derivatives that are traded in fairly liquid markets. Exotic derivatives are customized to fit a specific firm need.

### LO 44.2

The main purpose for the development of exotic derivatives is to provide a unique hedge for a firm's underlying assets. Additional reasons include addressing tax and regulatory concerns as well as speculating on the expected future direction of market prices.

### LO 44.3

Packages are portfolios of European options, forwards, cash, and the underlying asset. Given that packages often consist of a long position and a short position, they can be constructed so that the initial cost to the investor is zero.

### LO 44.4

Restricting exercise dates and changing strike prices can transform standard options into nonstandard options.

### LO 44.5

A gap option has two strike prices. If the two strike prices differ and the payoff is non-zero, there will be a gap in the payoff graph that is either increased or decreased by the difference between the strike prices.

Forward start options are options that commence in the future.

A compound option is defined as an option on another option.

Chooser options allow the owner to choose whether the option is a call or a put, after option initiation.

Barrier options are options whose payoffs (and existence) depend on whether the underlying's asset price reaches a certain barrier level over the life of the option.

Binary options either pay nothing (if price is below strike price) or a fixed amount at expiration.

Lookback options depend on the maximum or minimum value of the underlying asset during the life of the option.

Shout options allow the owner to receive either the intrinsic value of the option at the shout date or at expiration, whichever is greater.

Asian options have payoff profiles that depend on the average underlying asset price over the life of the option.

An exchange option is an option to exchange one asset for another.

Basket options allow the owner to buy or sell portfolios of assets. Exotic options that involve several different assets are more generally referred to as rainbow options.

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#### LO 44.6

A volatility swap involves the exchange of volatility based on a notional principal. A variance swap involves exchanging a pre-specified fixed variance rate for a realized variance rate.

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#### LO 44.7

Exotic options can be hedged in either a dynamic or static context, depending on the characteristics of the option.

**CONCEPT CHECKERS**

1. A down-and-in call option is an option that comes into existence only when the underlying asset price:
  - A. rises to a set barrier level.
  - B. falls to a set barrier level.
  - C. falls to a set average barrier level.
  - D. rises to a set average barrier level.
2. A cash-or-nothing put option has a payout profile equivalent to zero or:
  - A. the underlying asset price if the value of the asset ends below the strike price.
  - B. the underlying asset price if the value of the asset ends above the strike price.
  - C. a set amount if the value of the asset ends below the strike price.
  - D. a set amount if the value of the asset ends above the strike price.
3. An Asian option can be hedged dynamically because the:
  - A. average value of the underlying asset price decreases uncertainty the closer the option gets to expiration.
  - B. average value of the underlying asset price increases uncertainty the closer the option gets to expiration.
  - C. maximum value of the underlying asset price decreases uncertainty the closer the option gets to expiration.
  - D. minimum value of the underlying asset price increases uncertainty the closer the option gets to expiration.
4. Which of the following options is most likely to have a negative vega?
  - A. A chooser option close to expiration.
  - B. A forward start put option before the start date.
  - C. An Asian put option close to the beginning of the option's life.
  - D. An up and out put when the stock price is close to the barrier.
5. Under which of the following circumstances would the value of an up and out call option be zero?
  - A. The strike price is above the barrier price.
  - B. The stock price is below the barrier price.
  - C. The stock price is above the strike price.
  - D. The stock price is below the strike price.

## CONCEPT CHECKER ANSWERS

1. B Down-and-in call options are standard options that come into existence only if the asset price falls to a set barrier price level, which is set below the current stock price.
2. C Cash-or-nothing put options pay only a set amount if the stock price ends below the strike price. These options differ from standard put options because the payment is a set amount that does not continuously increase with the decrease in stock price.
3. A Dynamic hedging can be used to hedge Asian options because uncertainty in the expiration value is decreased the closer one gets to expiration. This occurs because the intrinsic value becomes “set” due to the averaging effect over the life of the option.
4. D Vega is the sensitivity of the price of an option to changes in volatility of the underlying stock. For most options, vega is always positive—as volatility of the underlying stock increases, the price of the option also increases. An exception would be a knockout barrier option when the stock price is close to the barrier. Higher volatility means the barrier is more likely to be reached and the option will cease to exist.
5. A With an up and out call, if the stock price rises beyond the barrier price, the option ceases to exist. It therefore follows that if the strike price is above the barrier price, the option will never come into the money because the option will cease to exist before the option will ever come into the money.

# COMMODITY FORWARDS AND FUTURES

Topic 45

## EXAM FOCUS

This topic on commodity forwards and futures focuses on the pricing relationships that exist when commodities have characteristics such as lease rates, storage costs, and/or convenience yields. Before you begin this topic, recall the no-arbitrage pricing relationships for futures contracts that were discussed in Topic 38 (Determination of Forward and Futures Prices). You should understand the basic futures pricing equation and how it is adjusted for lease rates, storage costs, and/or convenience yields.

## PRICING COMMODITY FORWARDS AND FUTURES

**LO 45.1: Apply commodity concepts such as storage costs, carry markets, lease rate, and convenience yield.**

**LO 45.2: Explain the basic equilibrium formula for pricing commodity forwards.**

**LO 45.13: Explain how to create a synthetic commodity position, and use it to explain the relationship between the forward price and the expected future spot price.**



*Professor's Note: LO 45.1 is addressed throughout this topic.*

Commodity and financial forward contracts are similar in some regards. For example, the prices of both are logically based upon expected spot prices. Some financial forwards (e.g., S&P 500 Index) are based upon the expected future spot price minus dividends received during the holding period. The price of a commodity forward must also be based upon expectations, but there are several factors to consider. For example, based upon their physical qualities, some commodities are *storable* (e.g., metals) and the associated costs depend upon the physical characteristics of the commodity. Also, due to their physical nature, others are not storable (e.g., electricity, perishable foods).

Some commodities are also appropriate for *leasing*. That is, an investor without a current need purchases the commodity and then lends it out to others who do have a current need. Just as with the loan of any asset the lender requires a return, so a **lease rate** (i.e., required return) is established. For example, assume an investor uses cash and purchases a commodity. If a viable lease market exists for the commodity, the investor might lend it to someone. Since the investor used cash to acquire the commodity, he must charge a lease rate. Failing to do so would amount to an interest-free loan of the money tied up in the commodity.



Since commodity forward prices are based upon expected spot prices and expected spot prices are, in turn, dependent upon expected supply and demand forces, forward prices for commodities need not be constant from period to period. There are factors such as weather that can affect expected supply. For example, severe weather might be expected to reduce future coffee supplies, so the forward coffee price might incorporate the expected shortage into an increased forward price. Demand for a commodity can also be subject to change. For example, demand for electricity is not constant during the day nor is it constant across different seasons of the year or in different locations across the country. Estimating the expected spot price for a commodity, therefore, must utilize forecasts of all relevant factors.

For a given commodity on any trading day, several futures contracts will exist with varying maturity dates. The prices of the commodity futures contracts will differ with the different contract expiration dates. The set of futures prices for a given commodity is known as a **forward curve** or a **forward strip** on that particular day.

Assume that we do not know the forward price of the commodity and wish to derive it. A synthetic commodity forward price can be derived by combining a long position on a commodity forward,  $F_{0,T}$ , and a long zero-coupon bond that pays  $F_{0,T}$  at time  $T$ .

The total cost at time 0 is equivalent to the cost of the bond,  $e^{-rT}F_{0,T}$ , where  $r$  represents the risk-free rate of return. The forward contract does not have any initial cash flows at time 0. The payoff at time  $T$  will be the payoff from the forward contract ( $S_T - F_{0,T}$ ) plus the payoff from the bond ( $F_{0,T}$ ):

$$S_T - F_{0,T} + F_{0,T} = S_T$$

where:

$S_T$  = spot price of the commodity at time  $T$

The present value of the expected spot price at time  $T$  is  $E(S_T)e^{-\alpha T}$ , where  $\alpha$  represents the discount rate for the  $S_T$  cash flow at time  $T$ . This amount is equivalent to the cost of the bond,  $e^{-rT}F_{0,T}$ , because both represent the amount you would pay today to receive the commodity at time  $T$ . This equality is expressed in the following equation:

$$e^{-rT}F_{0,T} = E(S_T)e^{-\alpha T}$$

This equation illustrates that when using a risk-free discount rate, the discounted commodity forward price at time  $T$  is equivalent to the present value of a unit of commodity received at time  $T$ .

Multiplying each side of the equation by  $e^{rT}$  allows us to express the commodity forward price as follows:

$$F_{0,T} = E(S_T)e^{(r-\alpha)T}$$

Thus, the forward price today is a biased estimate of the expected commodity spot price at time  $T$ . The bias is a function of the risk premium on the commodity,  $r - \alpha$ . This equation is used to calculate the net present value (NPV) of commodities with available forward prices.

## COMMODITY ARBITRAGE

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**LO 45.3: Describe an arbitrage transaction in commodity forwards, and compute the potential arbitrage profit.**

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A **cash-and-carry arbitrage** consists of buying the commodity, storing/holding the commodity, and selling the commodity at the futures price when the contract expires. The steps in a cash-and-carry arbitrage are as follows:

At the initiation of the contract:

- Borrow money for the term of the contract at market interest rates.
- Buy the underlying commodity at the spot price.
- Sell a futures contract at the current futures price.

At contract expiration:

- Deliver the commodity and receive the futures contract price.
- Repay the loan plus interest.

If the futures contract is overpriced, this 5-step transaction will generate a riskless profit. The futures contract is overpriced if the actual market price is greater than the no-arbitrage price.

**Example: Futures cash-and-carry arbitrage**

Assume the spot price of gold is \$900/oz., that the 1-year futures price is \$975/oz., and that an investor can borrow or lend funds at 7%. Ignore transaction and storage costs. Calculate the arbitrage profit.

**Answer:**

The futures price, according to the no-arbitrage principle, should be:

$$F_{0,T} = \$900e^{0.07} = \$965$$

Instead, it's trading at \$975. That means the futures contract is overpriced, so we should conduct cash and carry arbitrage by going short in the futures contract, buying gold in the spot market, and borrowing money to pay for the purchase. If we borrow \$900 to fund the purchase of gold, we must repay \$965.

<i>Today</i>		<i>1 year from today</i>	
Spot price of gold	\$900		
Futures price of gold	\$975		
<i>Transaction</i>	<i>Cash flow</i>	<i>Transaction</i>	<i>Cash flow</i>
Short futures	\$0	Settle short position by delivering gold	+\$975
Buy gold in spot market	–\$900		
Borrow at 7%	<u>+\$900</u>	Repay loan	<u>–\$965</u>
Total cash flow	\$0	Total cash flow = arbitrage profit	+\$10

The riskless profit is equal to the difference between the futures contract proceeds and the loan payoff, or \$975 – \$965 = \$10. Notice that this profit is equal to the difference between the actual futures price of \$975 and the no-arbitrage price of \$965.

If the futures price is too low (which presents a profitable arbitrage opportunity), the opposite of each step should be executed to earn a riskless profit.

This is **reverse cash-and-carry arbitrage**. The steps in reverse cash-and-carry arbitrage are as follows.

At the initiation of the contract:

- Sell commodity short.
- Lend short sale proceeds at market interest rates.
- Buy futures contract at market price.

At contract expiration:

- Collect loan proceeds.
- Take delivery of the commodity for the futures price and cover the short sale commitment.

**Example: Futures reverse cash-and-carry arbitrage**

Assume gold is priced at \$900/oz., that the 1-year futures price is \$950/oz., and that an investor can borrow or lend funds at 7%. Ignore transaction and storage costs. **Calculate** the profits from arbitrage.

**Answer:**

The futures price, according to the no-arbitrage principle, should be:

$$F_{0,T} = \$900e^{0.07} = \$965$$

Instead, it's trading at \$950. That means the futures contract is underpriced, so we should conduct reverse cash and carry arbitrage by going long in the futures contract, shorting gold, and investing the short-sale proceeds:

<i>Today</i>		<i>1 year from today</i>	
Spot price of gold	\$900		
Futures price of gold	\$950		
<i>Transaction</i>	<i>Cash flow</i>	<i>Transaction</i>	<i>Cash flow</i>
Long futures	\$0	Settle long position by buying gold	–\$950
Short gold	+\$900	Deliver gold to close short position	
Invest short-sale proceeds at 7%	–\$900	Receive investments proceeds	+\$965
Total cash flow	\$0	Total cash flow = arbitrage profit	+\$15

The riskless profit is equal to the loan proceeds less the futures contract payment, or \$965 – \$950 = \$15.



*Professor's Note: It may help to remember "buy low, sell high." If the futures price is "too high," sell the future and buy the spot. If the futures price is "too low," buy the future and sell the spot.*

## LEASE RATES

**LO 45.4: Define the lease rate and explain how it determines the no-arbitrage values for commodity forwards and futures.**

A **lease rate** is the amount of interest a lender of a commodity requires. The lease rate is defined as the amount of return the investor requires to buy and then lend a commodity. From the borrower's perspective, the lease rate represents the cost of borrowing the commodity. The lease rate and risk-free rate are important inputs to determine the commodity forward price. The lease rate in the pricing of a commodity forward is very similar to the dividend payment in a financial forward.

A no-arbitrage price can be established if there is an active lending market for a commodity. A commodity lender can earn a return, the lease rate, by buying a commodity and immediately selling it forward. The amount a commodity borrower is willing to pay must equal the amount the lender requires in return for lending out the commodity for time  $T$ . This interest or lease amount is an important factor in establishing the forward price for the commodity.

The commodity forward price for time  $T$  with an active lease market is expressed as:

$$F_{0,T} = S_0 e^{(r - \delta_1)T}$$

where:

$S_0$  = commodity current spot price

$r - \delta_1$  = risk-free rate less the lease rate

The lease rate,  $\delta_1$ , is income earned only if the commodity is loaned out.

### Example: Pricing a commodity forward with a lease payment

**Calculate** the 12-month forward price for a bushel of corn that has a spot price of \$5 and an annual lease rate of 7%. The appropriate continuously compounding annual risk-free rate for the commodity is equivalent to 9%.

**Answer:**

We can determine the 12-month forward price as follows:

$$F_{0,T} = (S_0) e^{(r - \delta_1)T} = \$5 \times e^{(0.09 - 0.07)} = \$5.101$$

To further illustrate that this relationship must hold, consider the following no-arbitrage example.

**Example: No-arbitrage for a commodity forward**

Assume there is an active lending market for a bushel of corn. If no-arbitrage positions exist, calculate the forward price of a bushel of corn in one year if the lease rate is equal to 9%, the effective annual risk-free rate is equal to 9%, and the expected spot price in one year is equal to \$2/bushel of corn.

**Answer:**

Figure 1 represents a no-arbitrage opportunity for a bushel of corn. An investor could borrow money at the risk-free rate of 9% to purchase a bushel of corn and short sell it forward. The investor immediately lends the bushel of corn out at a lease rate of 9%. At the end of the lease period,  $T_1$ , the individual would pay back the loan with interest at \$2.18, sell the corn at \$2.00, and receive the lease payment of \$0.18. In order for a no-arbitrage position to exist, the forward price,  $F_{0,1}$ , must be equal to the expected spot price of \$2.00.

**Figure 1: No-Arbitrage Opportunity on Bushel of Corn**

<i>Transaction</i>	<i>Time = <math>T_0</math></i>	<i>Time = <math>T_1</math></i>
Borrow @ 9%	\$2.00	\$(2.18)
Buy a bushel of corn	\$(2.00)	\$2.00
Lend bushel of corn	\$0	\$0.18
Short forward @ \$2	\$0	$F_{0,1} - \$2$
Total	\$0	$F_{0,1} - \$2$

**CONTANGO AND BACKWARDATION**

An upward-sloping forward curve indicates that forward prices more distant in time are higher than current forward prices. The market is described as being in **contango** with an upward-sloping forward curve. A contango commodity market occurs when the lease rate is less than the risk-free rate. Based on the commodity forward formula,  $F_{0,T} = S_0 e^{(r - \delta_1)T}$ , if  $r > \delta_1$ , the forward price must be greater than the spot price.

The market is described as being in **backwardation** with a downward-sloping forward curve. A backwardation commodity market occurs when the lease rate is greater than the risk-free rate. Based on the commodity forward formula,  $F_{0,T} = S_0 e^{(r - \delta_1)T}$ , if  $r < \delta_1$ , the forward price must be less than the spot price.

## STORAGE COSTS

**LO 45.5: Define carry markets, and illustrate the impact of storage costs and convenience yields on commodity forward prices and no-arbitrage bounds.**

**LO 45.6: Compute the forward price of a commodity with storage costs.**

When holding a commodity requires storage costs, *the forward price must be greater than the spot price* to compensate for the physical storage costs (i.e., costs associated with constructing and maintaining a storage facility) and financial storage costs (i.e., interest). The owner of a commodity can either sell it today for a price of  $S_0$  or for delivery at time  $T$  at the forward price. If the owner sells it at a forward price, this is known as *cash-and-carry* (as we saw in LO 45.3) because the seller receives the cash but must store (i.e., carry) the commodity until the delivery date. The market in which a commodity is stored is referred to as a **carry market**. The owner will only store the commodity if the forward price is greater than or equal to the expected spot price plus storage costs. This is represented mathematically as:

$$F_{0,T} \geq S_0 e^{rT} + \lambda(0,T)$$

where:

$\lambda(0,T)$  = FV of storage costs for one unit of the commodity from time 0 to  $T$

If storage costs are paid continuously and are proportional to the value of the commodity, the no-arbitrage forward price becomes:

$$F_{0,T} = S_0 e^{(r + \lambda)T}$$

where:

$\lambda$  = continuous annual storage cost proportional to the value of the commodity

### Example: Commodity forward pricing with storage costs and effective interest

**Calculate** the 3-month forward price for a bushel of soybeans if the current spot price is \$3/bushel, the effective monthly interest rate is 1%, and the monthly storage costs are \$0.04/bushel.

**Answer:**

First, calculate the future cost of storage for three months,  $\lambda(0,T)$ , as follows:

$$\$0.04 + \$0.04(1.01) + \$0.04(1.01)^2 = \$0.1212$$

The amount of \$0.1212 represents the three months storage costs plus interest. Next, add the cost of storage to the spot price plus interest.

$$F_{0,T} = S_0 e^{rT} + \lambda(0,T) \approx \$3.00(1.01^3) + \$0.1212 = \$3.0909 + \$0.1212 = \$3.2121$$





*Professor's Note: Notice the approximation used in the previous example:  $F_{0,T} = S_0 e^{rT} \approx S_0 \times (1 + r)^T$ . Using either approach will produce similar results.*

## CONVENIENCE YIELD

If the owners of the commodity need the commodity for their business, holding physical inventory of the commodity creates value. For example, assume a manufacturer requires a specific commodity as a raw material. To reduce the risk of running out of inventory and slowing down production, excess inventory is held by the manufacturer. This reduces the risk of idle machines and workers. In the event that the excess inventory is not needed, it can always be sold. Holding an excess amount of a commodity for a non-monetary return is referred to as **convenience yield**.

A convenience yield *cannot* be earned by the average investor who does not have a business reason for holding the commodity. The forward price including a convenience yield is calculated as follows:

$$F_{0,T} \geq S_0 e^{(r + \lambda - c)T}$$

where:

$c$  = continuously compounded convenience yield, proportional to the value of the commodity

For the investor who does not earn the convenience yield, cash-and-carry arbitrage implies that:

$$F_{0,T} \leq S_0 e^{(r + \lambda)T}$$

### Example: Impact of convenience yield on the no-arbitrage cash-and-carry commodity forward pricing range

Suppose the owner of a commodity decides to lend out the commodity. The commodity has a continuously compounded convenience yield of  $c$ , proportional to the value of the commodity. **Determine** which range of prices must represent the no-arbitrage cash-and-carry opportunity for an investor who recognizes a convenience yield.

**Answer:**

The owner of a commodity is able to create a range of no-arbitrage prices as follows:

$$S_0 e^{(r + \lambda - c)T} \leq F_{0,T} \leq S_0 e^{(r + \lambda)T}$$

The upper bound depends on storage costs but not on the convenience yield. The lower bound adjusts for the convenience yield and therefore explains why forward prices may appear lower at times when the convenience yield is accounted for.

## COMPARING LEASE RATES, STORAGE COSTS, AND CONVENIENCE YIELD

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### LO 45.7: Compare the lease rate with the convenience yield.

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Here is a handy guide for relating forward and spot commodity prices on the exam. Start with the basic expression relating forward and spot prices:

$$F_{0,T} = S_0 e^{rT}$$

This expression says that if there are no costs or benefits associated with buying and holding the commodity, the forward price is just the spot price compounded at the risk-free rate over the holding period.

If there are benefits (e.g., lease rates, convenience yield) to buying the commodity today, the holder is willing to accept a lower forward price. The forward price is reduced by the benefit, either the lease rate or convenience yield:

$$F_{0,T} = S_0 e^{(r - c)T} < S_0 e^{rT}$$

where  $c$  = the convenience yield, or

$$F_{0,T} = S_0 e^{(r - \delta)T} < S_0 e^{rT}$$

where  $\delta$  = the lease rate

If there are costs, such as storage costs, associated with purchasing the commodity today, the forward price is increased by the cost:

$$F_{0,T} = S_0 e^{(r + \lambda)T} > S_0 e^{rT}$$

where  $\lambda$  = the storage costs

Of course, there can be combinations of costs and benefits, so be sure to increase the exponent for costs and reduce it for benefits:

$$F_{0,T} = S_0 e^{(r + \lambda - c)T}$$

In the equation above, the lease rate is equal to storage costs minus the convenience yield.

## COMMODITY CHARACTERISTICS

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### LO 45.8: Identify factors that impact gold, corn, electricity, natural gas, and oil forward prices.

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Certain commodities exhibit unique properties that impact their forward price. For example, gold, corn, electricity, natural gas, and oil are all commodities with characteristics that differ with respect to storage costs, the ability to store, production costs, and seasonal

demand. These differences are reflected in lease rates, storage costs, and convenience yields that influence the commodity forward prices and the shape of the forward curves.

### Gold Forward Price Factors

Because gold can earn a return by being loaned out, strategies for holding synthetic gold offer a higher return than holding just the physical gold without lending it out. When a positive lease rate is present, the synthetic gold is preferred to physically holding the gold because the lease rate represents the cost of holding the gold without lending it.

The value of gold is also influenced by the cost of production. The present value of gold received in the future is simply the present value of the forward price computed at the risk-free rate of return. The present value of gold production is calculated as follows:

$$\text{PV of gold production} = \sum_{i=1}^n n_{t_i} [F_{0,t_i} - x(t_i)] e^{-r(0,t_i)t_i}$$

where:

$n_{t_i}$  = amount of ounces of gold we expect to extract, with an extraction cost of  $x(t_i)$

Under this framework, the gold mine is assumed to operate the entire time, and production is known with certainty.

### Corn Forward Price Factors

Corn is an example of a commodity with seasonal production and a constant demand. Corn is produced every fall, but it is consumed throughout the year. In order to meet consumption needs, corn must be stored. Thus, interest and storage costs need to be considered. The price of the corn will fall as it is being harvested and then rise to reflect the cost of storage over the next 12 months until it is harvested again. Thus, the forward curve is increasing until harvest time, and it drops sharply and slopes upward again after harvest time is over.

#### Example: Corn commodity pricing with storage costs

Suppose the spot price today for a bushel of corn is \$2.25, the continuously compounded interest rate is 5.5%, and the storage cost is 2.0% per month. Calculate the 6-month forward price.

**Answer:**

$$F_{0,0.5} = \$2.25 \times e^{(0.00458 + 0.02)6} = \$2.25 \times 1.15893 = \$2.61$$



*Professor's Note: The 0.458% used for the monthly interest rate is the annual rate divided by 12.*

## Electricity Price Factors

As previously mentioned, electricity is not a storable commodity. Once it is produced, it must be used or it will likely go to waste. In addition, demand for electricity is not constant and will vary with time of day, day of the week, and season. Given the non-storability characteristic of electricity, its price is set by demand and supply at a given point in time. Since arbitrage opportunities do not exist with electricity (i.e., the inability to buy electricity during one season and sell it during another season) futures prices on electricity will vary much more during the trading day than financial futures.

## Natural Gas Forward Price Factors

Natural gas is an example of a commodity with constant production but seasonal demand. Natural gas is expensive to store, and demand in the United States peaks during high periods of use in the winter months. In addition, the price of natural gas is different for various regions due to high international transportation costs. Storage is at its peak in the fall just prior to the peak demand. Therefore, the forward curve rises steadily in the fall.

### Example: Calculation of natural gas forward price with storage costs

Calculate the natural gas implied storage cost for the month of October if the October 2005 spot price is 4.071, the annual risk-free rate of interest is 6%, and the November forward price is 4.157.

**Answer:**

$$\$4.157 = \$4.071e^{0.005} + \lambda_{\text{Oct2005}}$$

$$\$4.157 = \$4.091 + \lambda_{\text{Oct2005}}$$

$$\$4.157 - \$4.091 = \lambda_{\text{Oct2005}}$$

$$\$0.066 = \lambda_{\text{Oct2005}}$$

## Oil Forward Price Factors

The physical characteristics of oil make it easier to transport than natural gas. Therefore, the price of oil is comparable worldwide. In addition, demand is high in one hemisphere when it is low in the other. Lower transportation costs and more constant worldwide demand causes the long-run forward price to be more stable. In the short-run, supply and demand shocks cause more volatile prices because supply is fixed. For example, the Organization of Petroleum Exporting Countries (OPEC) may decrease supply to increase prices by causing a shortage in the short run. Supply and demand adjust to price changes in the long run.

## COMMODITY SPREAD

### LO 45.9: Compute a commodity spread.

A **commodity spread** results from a commodity that is an input in the production process of other commodities. For example, soybeans are used in the production of soybean meal and soybean oil. A trader creates a **crush spread** by holding a long (short) position in soybeans and a short (long) position in soybean meal and soybean oil.

Similarly, oil can be refined to produce different types of petroleum products such as heating oil, kerosene, or gasoline. This process is known as “cracking,” and thus the difference in prices of crude oil, heating oil, and gasoline is known as a **crack spread**. For example, seven gallons of crude oil may be used to produce four gallons of gasoline and three gallons of heating oil. Commodity traders refer to the crack spread as 7-4-3, reflecting the seven gallons of crude oil, four gallons of gasoline, and three gallons of heating oil. Thus, an oil refiner could lock in the price of the crude oil input and the finished good outputs by an appropriate crack spread reflecting the refining process. However, this is not a perfect hedge because there are other outputs that can be produced such as jet fuel and kerosene.

#### Example: Pricing a crack (commodity) spread

Suppose we plan on buying crude oil in one month to produce gasoline and kerosene for sale in two months. The 1-month futures price for crude oil is currently \$30/barrel. The 2-month future prices for gasoline and heating oil are \$41/barrel and \$31.50/barrel, respectively. Calculate the 5-3-2 crack (commodity) spread.

#### Answer:

The 5-3-2 spread tells us the amount of profit that can be locked in by buying five barrels of oil and producing three barrels of gasoline and two barrels of heating oil.

$$\begin{aligned} \text{profit for a 5-3-2 spread} &= \\ (3 \times \$41) + (2 \times \$31.50) - (5 \times \$30) &= \$123 + \$63 - \$150 = \$36 \text{ for five barrels, or} \\ \$36 / 5 \text{ barrels} &= \$7.20/\text{barrel} \end{aligned}$$



*Professor's Note: There is no calculation for interest adjustment in this example.*

## BASIS RISK

### LO 45.10: Explain how basis risk can occur when hedging commodity price exposure.

As you may recall, **basis** is the difference between the spot price (or rate) and the price (or rate) of the futures contract used to hedge. If the values of both move together perfectly, an

investor long or short the asset can lock in a return or value by selling or buying futures, respectively.



*Professor's Note: When you expect to receive the commodity in the future, we say you are long the commodity and you will hedge the value of the expected commodity by selling the corresponding futures contracts. If you will deliver the commodity in the future without first owning the commodity, you are short, and you will hedge by taking a long position in the corresponding futures contracts.*

Any time the values of the spot and futures contracts do not move together perfectly, the hedger faces **basis risk**. An example with financial futures is using a basket currency futures contract to hedge the value of a transaction in an emerging market. Since the hedged asset (i.e., the emerging market currency) and the underlying in the futures contract are not identical, there is risk associated with changes in their relative values. Also, if the financial futures contract must be rolled over, or if it matures after the delivery date, this adds to the basis risk.

Since there are storage and transportation costs associated with commodities, hedgers face more concerns. As with financial futures, every commodity futures contract specifies a delivery amount and a delivery date. In addition, however, every commodity futures contract specifies a delivery *location* and the deliverable *grade* (i.e., quality). For example, an investor planning to receive oil in New York City might use NYMEX futures, which specify delivery in Oklahoma. At the producer level, an Iowa corn farmer might use CBOT corn futures, which specify delivery in Chicago.

## STRIP HEDGE VS. STACK HEDGE

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**LO 45.11: Evaluate the differences between a strip hedge and a stack hedge and explain how these differences impact risk management.**

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An oil producer may enter into a contract to supply a fixed amount of barrels of oil per month at a fixed price. The oil producer could set up a **strip hedge** by buying futures contracts that match the maturity and quantity for every month of the obligation.

To help reduce transaction costs, the oil producer might instead utilize a **stack hedge**. To form a stack hedge, the oil producer would enter into a one-month futures contract equaling the total value of the year's promised deliveries. As transactions costs are less for short-term (e.g., one-month) contracts, the total cost of implementing this strategy is less than for a comparable strip hedge. At the end of the first month, the producer rolls into the next one-month contract, and so forth, each month setting the total amount of the contract equal to the remaining promised deliveries. This strategy of continually rolling into the next near-term contract is referred to as **stack and roll**.

A stack hedge has the advantage when near-term contracts are more readily available due to heavier volume and more liquidity. Another advantage of near-term contracts is that distant futures on commodities often have wider bid-ask spreads and therefore larger transaction costs. In addition, an oil producer may prefer a stack hedge in order to speculate on the shape of the forward curve. For example, assume the forward curve looks unusually steep.



The oil producer would then enter into a stacked hedge with a large near-term contract. If the forward curve later flattens, the oil producer locks in all the oil at a relatively cheap near-term price compared to the more expensive futures using the strip strategy.

**Example: Creation of a strip or stack hedge**

**Determine** how an oil producer could hedge the risk of an agreement to supply 150,000 barrels of oil each month for a year at a fixed price.

**Answer:**

The oil producer could enter into a strip hedge by obtaining a long futures contract position for every month of the year for 150,000 barrels.

Alternatively, the oil producer could create a long position of a near-term futures contract for a little less than 1,800,000 barrels. At the end of the month, the oil producer would enter into a new near-term futures contract for a smaller amount representing the present value of future deliveries.

## CROSS HEDGING

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### LO 45.12: Provide examples of cross-hedging, specifically the process of hedging jet fuel with crude oil and using weather derivatives.

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In some cases, a futures contract with an underlying instrument that is exactly the same as the position to be hedged will not exist. For example, there are no contracts for jet fuel futures in the United States. Therefore, hedging jet fuel requires a **cross hedge**. Some firms hedge the cost of jet fuel with crude oil futures while others hedge using a combination of crude oil and heating oil futures. Three factors are relevant when making a cross hedge decision:

- The liquidity of the futures contract (since delivery may not be an option).
- The correlation between the underlying for the futures contract and the asset(s) being hedged.
- The maturity of the futures contract.

Each of these factors has an impact on the effectiveness of the hedge. The liquidity of the cross hedge is important in order for the portfolio manager to quickly unwind the futures obligation. Thus, the manager should try to choose among liquid instruments to find the futures contract whose maturity most closely matches that of the horizon of the hedged position.

To illustrate the concept of cross hedging, consider a firm that uses crude oil futures to hedge jet fuel prices. The payoff from this type of hedge will depend on both the change in jet fuel prices and the change in oil futures prices. Thus, the number of crude oil futures contracts required is estimated using regression analysis, where the change in jet fuel prices is dependent on the change in oil futures prices. The slope coefficient from the regression



results will provide the portfolio manager with hedge ratio information regarding the degree that crude oil price changes affect the price of jet fuel.

A cross hedge is also applied when firms use **weather derivatives**. Weather risk is a business risk that is faced by agricultural firms as well as many firms involved with providing recreational services. It refers to any financial losses, explicit and implicit, that a firm faces from changes in the weather.

Utility companies use weather derivatives, which are based on “degree days,” to hedge the cost of energy purchases. Much of the energy supplied by utilities is used for heating or cooling with variations in demand directly correlated with weather patterns. Demand can rise and fall dramatically in conjunction with the weather experienced in the areas that the utilities service.

Utilities can use derivatives with payoffs based on the weather experienced at weather stations that are representative of the areas that they serve. For example, a utility located in the northeast U.S. contracts for energy needs based on average weather experienced over previous years and predictions for the coming year. Unhedged, the utility would leave itself exposed to rising prices from energy producers in the event that the coming winter is far worse than predicted.

If hedging with weather derivatives (specifically weather options), and the winter were worse than expected (have more heating degree days than the strike value of the contract), the utility would receive the specified payment. If the winter were milder than expected, the contract would expire worthless. The actual measurements are from specified U.S. government sites in the areas specified by the contract.

The use of weather derivatives by other investors is growing, but one of the biggest problems is basis risk. That is, it is difficult to accurately match up the exposure of other assets to the weather with that specified by the contracts. Other than large-scale exposure, such as that experienced by utilities, many producers are much more susceptible to more local variations. For instance, a large farming operation has exposure to the rain falling on its own fields and may suffer losses from too much or too little rain. The rain on its fields may not have a high correlation with the rain experienced at the weather station 50 miles away.

## KEY CONCEPTS

### LO 45.1

When holding a commodity requires storage costs, the forward price must be greater than the spot price to compensate for the physical storage costs and financial storage costs.

The market in which a commodity is stored is referred to as a carry market.

A lease rate is the amount of interest a lender of a commodity requires.

Holding an excess amount of a commodity for a non-monetary return is referred to as convenience yield.

### LO 45.2

The commodity forward price today is defined as a biased estimate of the expected spot commodity price at time T as follows:

$$F_{0,T} = E(S_T)e^{(\text{risk-free rate} - \text{discount rate})T}$$

### LO 45.3

The steps in a cash-and-carry arbitrage are as follows:

At the initiation of the contract:

*Step 1:* Borrow money for the term of the contract at market interest rates.

*Step 2:* Buy the underlying commodity at the spot price.

*Step 3:* Sell a futures contract at the current futures price.

At contract expiration:

*Step 1:* Deliver the commodity and receive the futures contract price.

*Step 2:* Repay the loan plus interest.

### LO 45.4

The lease rate is defined as the amount of return the investor requires to buy and then lend a commodity. If an active lease market exists for a commodity, a commodity lender can earn the lease rate by buying a commodity and immediately selling it forward.

The commodity market is in contango with an upward-sloping forward curve when the lease rate is less than the risk-free rate. The market is in backwardation with a downward-sloping forward curve when the lease rate is greater than the risk-free rate.

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**LO 45.5**

Holding an excess physical inventory of the commodity creates non-monetary value for commodity owners who require the commodity as a production input. This is referred to as convenience yield, and the forward price including a convenience yield is calculated as:

$$F_{0,T} \geq S_0 e^{(r+\lambda-c)T}, \text{ where } c \text{ is the continuously compounded convenience yield, proportional to the value of the commodity}$$

The owner of a commodity who uses the commodity in production is able to create a range of no-arbitrage prices as follows:

$$S_0 e^{(r+\lambda-c)T} \leq F_{0,T} \leq S_0 e^{(r+\lambda)T}$$

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**LO 45.6**

A commodity owner will only store the commodity if the forward price is greater than or equal to the spot price plus the future storage costs as follows:

$$F_{0,T} \geq S_0 e^{rT} + \lambda(0,T), \text{ where } \lambda(0,T) \text{ represents the future value of storage costs for one unit of the commodity from time 0 to } T.$$

If storage costs are paid continuously and are proportional to the value of the commodity, the no-arbitrage forward price becomes:

$$F_{0,T} = S_0 e^{(r+\lambda)T}$$

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**LO 45.7**

If there are benefits to buying the commodity, the holder is willing to accept a lower forward price. The forward price is reduced by the benefit, either the lease rate or convenience yield.

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**LO 45.8**

Since gold can earn a return by being loaned out, strategies for holding synthetic gold offer a higher return than holding just the physical gold without lending it out.

Corn is an example of a commodity with seasonal production and a constant demand.

Electricity is not a storable commodity. In addition, demand for electricity is not constant and will vary with time of day, day of the week, and season.

Natural gas is an example of a commodity with constant production but seasonal demand.

Oil is easier to transport than natural gas. Therefore, the price of oil is comparable worldwide. Supply and demand adjust to price changes in the long run.

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**LO 45.9**

A commodity spread results from a commodity that is an input in the production process of other commodities. For example, a 7-4-3 crack spread refers to the profit for holding four gasoline futures plus three heating oil futures less seven crude oil futures.

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**LO 45.10**

Basis risk results from the inability to create a perfect hedge due to differences in the commodities with respect to timing, grade, storage costs, and/or transportation costs.

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**LO 45.11**

A strip hedge is created by buying futures contracts that match the maturity and quantity for every month of the obligation. A stack hedge is created by buying a futures contract with a single maturity based on the present value of the future obligations. Advantages of the stack hedge are the availability and liquidity of near-term contracts and narrower bid-ask spreads for near-term contracts.

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**LO 45.12**

There are no contracts for jet fuel futures in the United States. Therefore, hedging jet fuel costs requires a cross hedge (e.g., hedge with crude oil futures). A cross hedge is also applied when firms use weather derivatives.

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**LO 45.13**

A synthetic commodity forward price can be derived by combining a long position on a commodity forward,  $F_{0,T}$ , and a long zero-coupon bond that pays  $F_{0,T}$  at time  $T$ .

## CONCEPT CHECKERS

1. Which of the following statements regarding lease rates is(are) true? The lease rate is:
  - I. the amount of return the investor requires to buy and then lend a commodity.
  - II. very similar to the dividend payment in a financial forward.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
2. Suppose there is an active lending market for a bushel of soybeans (which has a current spot price of \$4/bushel). If the annual lease rate is equal to 7%, the effective annual risk-free rate is equal to 7%, and the expected spot price in one year is equal to \$4/bushel of soybeans, how could an investor create an arbitrage opportunity (assuming the forward contract is overpriced)? An individual could:
  - A. borrow money at 7% and purchase a bushel of soybeans and sell it forward.
  - B. borrow a bushel of soybeans and sell a bushel of soybeans at the spot price and buy a long forward.
  - C. sell a bushel of soybeans at the forward price and lend the money at the risk-free rate.
  - D. go long in soybean forward contracts, short in soybean spot prices, and lend the excess proceeds at the risk-free rate.
3. What is the 3-month forward price for a bushel of corn if the current spot price for corn is \$3/bushel, the effective monthly interest rate is 1.5%, and the monthly storage costs are \$0.03/bushel?
  - A. \$3.18.
  - B. \$3.23.
  - C. \$3.29.
  - D. \$3.31.
4. Suppose we plan on buying crude oil in one month to produce gasoline and heating oil for sale in two months. The 1-month future price for crude oil is currently \$42.5/barrel. The 2-month future prices for gasoline and heating oil are \$45/barrel and \$43.50/barrel, respectively. What is the 7-5-2 crack (commodity) spread?
  - A. \$2.07/barrel.
  - B. \$6.00/barrel.
  - C. \$14.50/barrel.
  - D. \$22.09/barrel.
5. Which of the following statements is an example of basis risk? Purchasing:
  - A. an oil contract with delivery in a different geographical region.
  - B. a commodity with a desired distant delivery with near-term contracts.
  - C. a eurodollar contract, due to lack of commodity futures.
  - D. All of the above statements are correct.

## CONCEPT CHECKER ANSWERS

1. C A *lease rate* is the amount of interest a lender of a commodity requires. From the borrower's perspective, the lease rate represents the cost of borrowing the commodity. The lease rate in the pricing of a commodity future is very similar to the dividend payment in a financial forward.
2. A An individual could borrow money at the risk-free rate of 7% to purchase a bushel of soybeans and sell it forward. The individual immediately lends the bushel of soybeans out at a lease rate of 7%. At the end of the lease period,  $T_1$ , the individual would pay back the loan with interest at \$4.28, sell the soybeans at \$4.00, and receive the lease payment of \$0.28. In order for a no-arbitrage position to exist, the forward price,  $F_{0,1}$ , must be equal to the expected spot price of \$4.00. An arbitrage position exists if the forward price is not equivalent to the expected spot price.

### No-Arbitrage Opportunity on Bushel of Soybeans

<i>Transaction</i>	<i>Time = <math>T_0</math></i>	<i>Time = <math>T_1</math></i>
Borrow @ 7%	\$4.00	(\$4.28)
Buy a bushel of soybeans	(\$4.00)	\$4.00
Lend bushel of soybeans	\$0	\$0.28
Short forward @ \$4	\$0	$F_{0,1} - \$4$
Total	\$0	$F_{0,1} - \$4$

3. B First calculate the future cost of storage for three months,  $\lambda(0,T)$ , as follows:

$$\$0.03 + \$0.03(1.015) + \$0.03(1.015)^2 = \$0.0914$$

The amount of \$0.0914 represents the 3-month storage costs plus interest. Next, add the cost of storage to the spot price plus interest.

$$F_{0,T} = S_0 e^{rT} + \lambda(0,T) \approx \$3.00(1.015^3) + \$0.0914 = \$3.1370 + \$0.0914 = \$3.23$$

4. A The 7-5-2 spread tells us the amount of profit that can be locked in by buying seven barrels of oil and producing five barrels of gasoline and two barrels of heating oil.

Profit for a 7-5-2 spread =

$$(5 \times \$45) + (2 \times \$43.50) - (7 \times \$42.5) = \$225 + \$87 - \$297.5 = \$14.50 \text{ for seven barrels, or } \$14.5 / 7 \text{ barrels} = \$2.07/\text{barrel}.$$

5. D All are examples of basis risk, which results from the inability of commodities to create a perfect hedge. Differences due to timing, grade, storage costs, or transportation costs create basis risk.

# EXCHANGES, OTC DERIVATIVES, DPCs AND SPVs

Topic 46

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## EXAM FOCUS

In this topic, we look at the role of exchanges and the differences between exchange-traded derivatives and over-the-counter (OTC) derivatives trading. We then examine the three forms of clearing and the role of the central counterparty (CCP) in clearing and mitigating counterparty risk. For the exam, be able to compare and contrast exchange-traded and OTC derivatives. Also, be familiar with the development of central clearing, including the various mechanisms that exist to manage risks, including special purpose vehicles (SPVs), derivatives product companies (DPCs), monolines, and credit derivative product companies (CDPCs). In addition, be able to contrast these mechanisms with CCPs.

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## EXCHANGE FUNCTIONS

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### LO 46.1: Describe how exchanges can be used to alleviate counterparty risk.

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Market participants can trade derivatives bilaterally or through exchanges. An **exchange** is a central market where standardized futures, options, and other derivatives contracts can be traded. Exchanges have a long history dating back to at least the 19th century. They have evolved from simple trading forums without risk management functions to sophisticated financial centers with settlement and counterparty risk management functions.

Exchange functions fall into three primary categories: product standardization, trading venue, and reporting services.

- *Product standardization.* Exchanges set the terms of traded, standardized products. Terms include maturity dates, trading price increments, and delivery grades and locations.
- *Trading venue.* Exchanges may be physical locations or electronic platforms that provide a central location for trading, which then facilitates price discovery. Entities trading on an exchange must accept the exchange's rules and conditions.
- *Reporting services.* Exchanges report transaction prices to various entities, including trading participants, vendors, and subscribers.

## FORMS OF CLEARING

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### LO 46.2: Explain the developments in clearing that reduce risk.

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**Clearing** is the process of reconciling and matching contracts between counterparties from the time the commitments are made until settlement. Clearing, along with the mechanisms of margining and netting, are important counterparty risk mitigants. *Margining* involves



posting both initial and variation margins. *Initial margin* represents upfront funds posted to mitigate against counterparty default, while *variation margin* represents the daily transfer of funds (cash or other assets) to cover position gains and losses. *Netting* refers to consolidating multiple offsetting positions between counterparties into a single payment.

The three forms of clearing that we look at in this section include: direct clearing, clearing rings, and complete clearing (i.e., central clearing).

**Direct clearing** is a mechanism for bilaterally reconciling commitments between two counterparties. For example, consider a scenario where counterparty X has an agreement to sell 10 contracts at \$50 to counterparty Y, and Y has an agreement to sell 10 contracts to X at \$55. Instead of exchanging the full 10 contracts and associated payments of \$500 and \$550, under a direct netting scenario only the net payment of the \$50 difference is paid by X to Y. This type of direct clearing for OTC derivatives is typically called netting, or payment of difference.

A **clearing ring** is a mechanism to reduce counterparty exposure between three or more exchange members. A clearing ring is voluntary for exchange members. Once members join, however, they must accept the rules of the exchange and must accept each other's contracts and allow for counterparties to be substituted. For example, if counterparty X is long a contract with counterparty Y, and Y is long the same number of contracts with Z, then Y may be removed from the ring and the two separate obligations would be replaced with a single obligation between X and Z. Clearing rings are designed to mitigate counterparty risk, improve liquidity and facilitate the close-out process. Not all exchange members would benefit from joining a clearing ring. Members that have a single position with another counterparty would not benefit from the ring.

**Complete clearing** refers to clearing through a central counterparty (CCP). The CCP, which can be either operated directly by an exchange or provided as a service by an independent third party, assumes the contractual obligations of clearing exchange members and acts as a buyer to sellers and a seller to buyers. By doing so, the CCP reduces counterparty risk and facilitates both clearing and settlement. Complete clearing can be seen as an improvement to a clearing ring since it reduces the risk of member failure and any resulting contagion effect.

CCPs also use margining rules to help protect against counterparty risk. Initial margin involves members pledging upfront funds to offset closeout costs in a member default scenario. Variation margin involves settlement of daily profit and loss of derivatives contracts (mark-to-market process).

## EXCHANGE-TRADED VS. OTC DERIVATIVES

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### LO 46.3: Compare exchange-traded and OTC markets and describe their uses.

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Exchange-traded derivatives are standardized contracts with a liquid, active, and regulated market, with the exchange or CCP acting as the central counterparty to trades. In contrast, OTC derivatives are privately negotiated bilateral contracts transacted in a market with little or no regulation. OTC derivatives have historically been traded between an end user and a dealer. The terms, settlement, and documentation are bilaterally negotiated. This allows for

contracts to be tailored to the specific needs of counterparties and includes a high level of customization. Customization is beneficial for hedging since it reduces basis risk, or the risk of term mismatches, which is a concern when using standardized exchange-traded contracts.

Given that OTC derivatives are bilaterally transacted between two counterparties, counterparty credit risk is a concern. In addition, due to the highly customized nature of the contracts, unwinding a trade may only be done at unfavorable terms (with the original counterparty or a new one). Novation of contracts (i.e., replacement of contracts) may also be problematic given the lack of fungibility in the OTC markets. In other words, contracts typically cannot be easily closed out given a lack of substitution between contracts.

The following table provides a comparison of the differences between exchange-traded and OTC derivatives.

	<i>Exchange-Traded Derivatives</i>	<i>OTC Derivatives</i>
<b>Terms</b>	Standardized	Custom, negotiable
<b>Maturity</b>	Standardized	Negotiable, non-standard
<b>Liquidity</b>	Strong	Weak
<b>Credit risk</b>	Little (CCP guarantee)	High (bilateral)

The clearing process is also different for exchange-traded and OTC derivatives. Whereas exchange-traded derivatives are typically shorter term and are settled within a few days, OTC derivatives are longer term with later settlements. This makes clearing more challenging for OTC derivatives.

Clearing and settlement on exchanges are functions carried out centrally by the CCP. For OTC derivatives, clearing and settlement are done bilaterally. To benefit from central clearing, an OTC derivative does not have to become exchange traded, and instead could be cleared by CCPs (e.g., LCH.Clearnet's SwapClear service offers clearing of bilaterally traded OTC derivatives).

## CLASSES OF OTC DERIVATIVES

**LO 46.4: Identify the classes of derivatives securities and explain the risk associated with them.**

OTC derivatives comprise of five broad classes: interest rate, foreign exchange, equity, commodity, and credit derivatives. Interest rate derivatives dominate the five classes and comprise nearly three quarters of the total gross notional outstanding. As of June 2013, interest rate derivatives had a gross notional value outstanding of \$561.3 trillion, which represented 84% of the total gross notional value of OTC derivatives of \$668 trillion. The second and third largest categories were foreign exchange derivatives and credit default swaps, respectively, followed by equity and commodity derivatives. It is important to note, however, that although interest rate derivatives comprise the majority of the OTC derivatives market, counterparty risk is particularly a concern for certain foreign exchange derivatives (including cross-currency swaps), which typically have long-dated maturities and require the exchange of notional principal. Furthermore, credit default swaps tend to

be more volatile and can carry significant wrong-way risk (when the credit quality of the counterparty is inversely related to the level of exposure to the counterparty).

As mentioned, interest rate derivatives dominate the market by gross notional value outstanding of contracts. However, measuring OTC derivatives exposure through gross notional value can be misleading. A basic fixed-for-floating coupon interest rate swap, for example, does not have principal risk because only the coupon cash flows are exchanged at each settlement. Furthermore, even coupon risk is lower, because only the net cash flows are exchanged. When considering cash flows, the swap may have a negative value to a party when its counterparty defaults. As a result, gross market value is often seen as a more useful measure for OTC derivatives, including the ratio of gross market value to gross notional value. The ratio is typically relatively small, and was close to 3% (at June 2013) for interest rate, foreign exchange, and credit default swaps.

## MITIGATING RISKS OF OTC DERIVATIVES

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### LO 46.5: Identify risks associated with OTC markets and explain how these risks can be mitigated.

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The default of a large market participant can create a ripple effect leading to systemic risk through the failure of many counterparties. Various mitigants exist to contain or reduce the risk of the initial default, including capital requirements, regulation, netting, and margining. While these mitigants have been widely utilized, they can create additional complexity which in turn may increase risk in the market.

In addition to these mitigants, other mechanisms also exist for controlling counterparty risk. These include special purpose vehicles (SPVs), derivatives product companies (DPCs), monolines, and credit derivative product companies (CDPCs). While these mechanisms have largely lost their relevance today, they provide important historical lessons for managing counterparty risk. Ultimately, rather than relying on these credit mitigants, systemic risk may be best managed by central clearing through CCPs, which act to manage systemic failure in a controlled way. Losses are spread across all exchange members through a loss sharing mechanism, thereby containing systemic risk.

### Special Purpose Vehicles (SPVs)

SPVs are bankruptcy remote legal entities set up by a parent firm to shield the SPV from any financial distress of the firm. The firm transfers assets to the SPV, which in turn issues structured products to investors to finance a particular project. The primary benefit of using an SPV is to obtain a strong credit rating, typically AAA. The SPV's rating is therefore stronger than the firm's credit rating. As a result, issuing securities through the SPV is more beneficial (i.e., lower cost of funding) than if the firm issued securities directly in the market.

One of the main benefits of SPVs is that they alter bankruptcy rules and transform counterparty risk into legal risk. The specific legal risk is that the courts may view the SPV and the originating firm as a single legal entity. This is referred to as *consolidation*, and it would effectively negate the intended benefits of the SPV as a separate, bankruptcy remote

legal entity. In the United States, there is a history of consolidation rulings by courts, whereas United Kingdom courts have been more reluctant to issue consolidation rulings. Differences in U.S. and U.K. court rulings were particularly evident during the bankruptcy of Lehman Brothers, which used SPVs to protect investors from its own counterparty risk. Legal risk is therefore an important consideration, especially in central clearing where a CCP must be certain of its legal authorities.

### **Derivatives Product Companies (DPCs)**

DPCs are set up by firms as bankruptcy remote subsidiaries to originate derivatives products and sell them to investors. Unlike SPVs, however, in order to receive a strong (e.g., AAA) rating they are separately capitalized and have restrictions on their activities and margin. DPCs calculate their internal quantitative risk assessment to quantify credit risk and to make sure they are benchmarked similarly relative to the desired AAA ratings criteria.

A DPC's AAA rating depends on three criteria: (1) market risk minimization through participating on both sides of the market, (2) parent support, with the bankruptcy remote status shielding against the parent's potential distress, and (3) credit risk and operational risk management through restrictions like limits, margin, and daily mark-to-market.

DPCs used defined triggers for their own failure through a “pre-packaged bankruptcy” process, which lays out the bankruptcy process and is intended to provide a simpler alternative to the standard bankruptcy process. Once a firm enters bankruptcy, DPCs could either continue on as part of another firm, or be terminated.

The advent of alternative AAA-rated entities, the perception that DPCs were inextricably linked to their parents and the loss of credibility in their AAA rating following the global financial crisis essentially rendered DPCs obsolete mechanisms.

### **Monolines and Credit Derivative Product Companies (CDPCs)**

**Monolines** are highly-rated insurance companies that provide financial guarantees, called “credit wraps” to investors. CDPCs are similar to the DPCs, discussed earlier, but have a business model more similar to that of a monoline.

Monolines and CDPCs are well-capitalized entities with their AAA ratings supported by capitalization requirements based on possible losses and related to the assets for which they provided guarantees. They are generally highly leveraged entities that do not have to post margin. During the recent global financial crisis, several monolines failed (including XL Financial Assurance Ltd., AMBAC Insurance Corporation, MBNA Insurance Corporation), and both monolines and CDPCs, which are considered similar to monolines, fell out of favor.

### Lessons Learned from Risk Mitigation

The history of SPVs, DPCs, monolines, and CDPCs provide the following valuable lessons for CCPs in a central clearing setting:

1. CCPs give priority to OTC derivatives counterparties to the detriment of other parties, including bondholders. This increases the risk in other markets.
2. Relying on a solid legal framework exposes CCPs and exchange members to legal risk. For example, as seen in the case of SPVs and DPCs, courts may change the priority of claims in a bankruptcy scenario, or courts in different jurisdictions may rule in contradictory ways.
3. Although CCPs share similarities with monolines and CDPCs in that they are highly-rated entities set up to manage counterparty risk, CCPs do not take residual risk in the market given that they maintain a matched book of trades. This is in contrast to monolines and CDPCs, which typically have one-way market exposures.
4. In contrast to monolines and CDPCs, which post no variation margin and often no initial margin, CCPs require members to post both initial and variation margin.



## KEY CONCEPTS

### LO 46.1

Trading derivatives can be done bilaterally or through exchanges. An exchange is a central market where standardized contracts can be traded. Exchanges perform three primary functions: product standardization, trading venue, and reporting services.

### LO 46.2

Clearing, margining, and netting are important counterparty risk mitigants. Clearing is the process of reconciling and matching contracts between counterparties. Margining represents both upfront funds posted to mitigate against counterparty default (initial margin), and daily transfer of funds to cover position gains and losses (variation margin). Netting refers to consolidating multiple offsetting positions between counterparties into a single payment.

The three forms of clearing include direct clearing, clearing rings, and complete clearing (i.e., central clearing). Direct clearing is a mechanism for bilaterally reconciling commitments between two counterparties. A clearing ring is a mechanism to reduce counterparty exposure between members by allowing for counterparty substitution. Complete clearing is clearing through a CCP, where the CCP assumes the obligations of clearing exchange members.

### LO 46.3

The main benefits of OTC derivatives include customization of terms, settlement, and documentation, which are negotiated bilaterally between two parties. Customization can be beneficial since it reduces basis risk (i.e., risk of term mismatches).

Disadvantages of OTC derivatives include counterparty risk, difficulty in unwinding trades, and novation of contracts.

Clearing is more challenging for OTC derivatives compared to exchange-traded derivatives given the generally longer maturities. OTC derivatives trades could be cleared by CCPs.

### LO 46.4

OTC derivatives comprise of five broad classes of derivatives: interest rate, foreign exchange, equity, commodity, and credit derivatives. Interest rate derivatives comprise the largest class, followed by foreign exchange derivatives and credit derivatives.

### LO 46.5

Mechanisms for controlling counterparty risk include: special purpose vehicles (SPVs), derivatives product companies (DPCs), monolines, and credit derivative product companies (CDPCs).

SPVs are bankruptcy remote legal entities set up by a parent firm to shield the SPV from any financial distress of the firm. SPVs essentially alter bankruptcy rules and transform counterparty risk into legal risk. The legal risk is consolidation, or the risk that the courts view the SPV and the originating firm as the same legal entity.

DPCs are bankruptcy remote subsidiaries of firms set up to originate derivatives products sold to investors. DPCs are separately capitalized and have restrictions on their activities and margin. They are generally AAA rated where the rating depends on three criteria: (1) market risk minimization (2) parent support, and (3) credit risk and operational risk management.

Monolines are highly-rated insurance companies that provide financial guarantees, or “credit wraps” to investors. CDPCs are akin to DPCs, but with a business model that is closer to that of a monoline.



## CONCEPT CHECKERS

1. Which of the following functions is least likely performed by an exchange?
  - A. Derivatives contract design and specifying contract terms.
  - B. Price negotiation through a bilateral process.
  - C. Limiting access to approved firms and individuals.
  - D. Reporting transaction prices to trading participants and data vendors.
  
2. Consider counterparties *A*, *B*, and *C*, which are members of a derivatives exchange. *A* is short a derivatives position with *B*, and *B* is short the same derivatives position with *C*. Replacing these two positions with a single position between *A* and *C* is an example of:
  - A. direct clearing.
  - B. bilateral clearing.
  - C. complete clearing.
  - D. clearing ring.
  
3. When contrasting exchange-traded derivatives and over-the-counter (OTC) derivatives, basis risk and credit risk are generally a concern for:
 

<u>Basis Risk</u>	<u>Credit Risk</u>
A. exchange-traded derivatives	exchange-traded derivatives
B. exchange-traded derivatives	OTC derivatives
C. OTC derivatives	exchange-traded derivatives
D. OTC derivatives	OTC derivatives
  
4. A credit default swap (CDS) most likely has higher counterparty risk than an interest rate swap (IRS) given that it has:
 

<u>Wrong-Way Risk</u>	<u>Reduced Volatility</u>
A. Yes	No
B. Yes	Yes
C. No	No
D. No	Yes
  
5. Which of the following entities best describe a mechanism that transforms counterparty risk into legal risk?
  - A. Derivatives product companies (DPCs).
  - B. Credit derivative product companies (CDPCs).
  - C. Special purpose vehicles (SPVs).
  - D. Monolines.

## CONCEPT CHECKER ANSWERS

1. B Exchanges set specific prices and standardize contracts. They do not negotiate prices bilaterally. Price negotiation through a bilateral process is a feature of the OTC derivatives market.
2. D Clearing rings allow members to substitute counterparties and replace multiple positions with a single netted position. They are designed to mitigate counterparty risk, improve liquidity, and facilitate the close-out process.
3. B Basis risk refers to the risk of term mismatches, and is a concern when using standardized exchange-traded contracts that may not precisely match the term or size of position to be hedged. Basis risk is a less of a concern with OTC derivatives, which can be customized to meet specific investor hedging needs.

Counterparty (credit) risk refers to the risk of counterparty default or non-payment. It is a risk specific to OTC derivatives trading, and is less of a concern when trading with central counterparties through exchanges.

4. A CDSs typically have *higher* volatility and exhibit considerable *wrong-way risk* (a risk that arises when the exposure to a counterparty is inversely related to the credit quality of the counterparty).
5. C SPVs are bankruptcy remote legal entities set up by a firm to shield the SPV from financial distress of the parent firm. SPVs essentially alter bankruptcy rules and transform counterparty risk into legal risk.

# BASIC PRINCIPLES OF CENTRAL CLEARING

Topic 47

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## EXAM FOCUS

This topic covers the principles of central clearing, including the functions and mechanics of a central counterparty (CCP). For the exam, be able to describe these functions and understand the key related terminology. Clearing, novation, netting, and offset are also important concepts for the exam. In addition, understand the advantages and disadvantages of CCPs, and be able to discuss common terms, including offsetting, loss mutualization, moral hazard, adverse selection, and procyclicality.

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## THE ROLE OF A CENTRAL COUNTERPARTY

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**LO 47.1: Provide examples of the mechanics of a central counterparty (CCP).**

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### Clearing and Settlement

A **central counterparty (CCP)** plays an important role in the clearing and settlement of transactions following the initial trade execution. **Clearing** refers to the processes (including margining and netting) between the period from trade execution until settlement. This period is typically short (a few days or months) for classically cleared non over-the-counter (OTC) derivatives. In contrast, for OTC derivatives this time period could extend to years or even decades. **Settlement** of a trade occurs when the trade is completed and all payments have been made and legal obligations satisfied.

A CCP's primary function is to simplify the operational processes and reduce counterparty risk that exists in the bilateral market. When a CCP interjects itself as the central counterparty between OTC trades and acts as the seller to each buyer and the buyer to each seller, it reduces the interconnectedness of trades and of participants, and reduces the risk of default or non-payment by a counterparty. At the same time, the process improves trade liquidity and transparency.

### Auctions and Loss Mutualization

Key functions of a CCP related to the clearing process include: margining, novation, netting, managing the auction process, and loss mutualization. Margining will be discussed in LO 47.3, and novation and netting will be discussed in LO 47.4.

When a central clearing member defaults, rather than closing out the trades at market value, the CCP typically auctions off the trades to the surviving members through an **auctioning** process. Participating in the auctioning process is in the best interest of the members in order to minimize their losses through lower market prices or through default funds.

**Loss mutualization** is a form of insurance and refers to members' contributions to a default fund to cover future losses from member defaults. Since all members must contribute to the fund, the potential losses from the default of any given member are contained. When a member does default, any amounts that cannot be covered from the member's own resources are covered from the fund. Given that losses are spread among surviving members, it is possible that a member will suffer losses even if it never traded with the defaulting counterparty or had no positions with the CCP.

### Other Mechanics of a CCP

#### *Products*

The OTC derivatives markets include a wide range of products ranging from standard to non-standard and exotic derivatives. There are currently four categories of OTC derivatives according to their stages of central clearing history:

1. Products with a long history of central clearing (e.g., interest rate swaps).
2. Products with a short history of central clearing (e.g., index credit default swaps).
3. Products that may soon be centrally cleared (e.g., interest rate swaptions, credit default swaps).
4. Products that are not suitable for central clearing (e.g., exotic derivatives including Asian options, and derivatives with illiquid reference assets).

The following conditions are important for a product to be centrally cleared:

- *Standardization*: Legal and economic terms should be standard.
- *Complexity*: Transactions need to be easily valued for trading and margin purposes, therefore only less complex (i.e., vanilla, not exotic) trades can be cleared.
- *Liquidity*: Cleared products are typically more liquid than OTC products. Liquidity is important for determining market price for initial margin and default fund contributions, and for the auctioning process. CCPs are also reluctant to develop clearing capability for products that could not be properly cleared due to thin trading. Liquidity also allows for easier close out of trades in a default scenario.

#### *Participants*

Transacting with CCPs is restricted to clearing members only. Becoming a member includes a number of requirements, including:

- *Admission criteria*: CCPs set different criteria for admission, including restrictions on credit quality (e.g., investment grade only) and size (e.g., minimum \$50 million).

- *Financial commitment:* The primary financial commitment by a member is to contribute to the CCP's default fund.
- *Operational criteria:* Members' operational requirements include posting margin, and participating in "fire drills" to simulate member default and in auctions if default does occur.

These criteria can be onerous, and as a result, only large banks or global financial institutions typically become clearing members. Smaller entities including small banks and financial institutions and some non-financial end users would likely not participate as direct clearing members, but would participate in the clearing process through transacting with a member on a principal-to-principal basis, or on an agency basis. These players would therefore have a bilateral relationship with the clearing member but not the CCP. This clearing process may be similar to the clearing between the member and the CCP, with some differences, including no default fund commitment by the non-member players.

### *Number of CCPs*

A single, large CCP may be optimal given the benefits of offsetting trades and cost minimization through economies of scale. However, it is generally not feasible to have a single CCP for the following reasons:

- *Regional differences:* Regional CCPs may be beneficial to centrally clear trades in the region's currency and under the laws and regulations of the region.
- *Product types:* CCPs often specialize in clearing certain derivatives products.
- *Regulatory reasons:* Regulations may dictate that products be cleared by local CCPs. However, CCPs need not operate in isolation, and CCPs working together may be necessary. It is important to recognize, however, that this may increase systemic risk and the risk of contagion during stress times.

### *Types of CCPs*

Arguments exist for both a utility-driven CCP and a profit-driven CCP. A utility-driven CCP would be focused on long-term stability rather than short-term profits. A profit-driven CCP would be focused on the bottom line in order to attract personnel and build the best systems. Currently, there are stronger arguments in favor of profit-driven CCPs.

### *Failure of a CCP*

While it is an infrequent event, CCPs do fail. However, the failure of a large and systemically important CCP could lead to potentially catastrophic events. As a result, CCPs must maintain sufficient loss absorption methods to withhold large member defaults. The financial trouble of a CCP may ultimately result in liquidity support from a central bank.

## CENTRAL CLEARING

### LO 47.2: Describe advantages and disadvantages of central clearing of OTC derivatives.

The following table illustrates the primary differences between the OTC derivatives markets and CCPs and exchanges. It should be noted that while the OTC market provides the greater breadth of participants and products and offers customization, the CCP/exchange-based market offers stronger margining and loss buffers and reduced counterparty and systemic risk.

	<i>OTC Derivatives</i>	<i>CCP/Exchanges</i>
Trading	Bilateral	Bilateral / Centralized
Counterparty	Original trade counterparty	CCP (replaces counterparty)
Participants	All	Clearing members (dealers)
Products	All (including non-standard, exotic)	Standard, vanilla
Margining	Bilateral, custom	Full margining set by CCP (initial, variation)
Loss buffers	Margin, regulatory capital	Initial margin, default fund, CCP capital

### Advantages of Central Clearing

Central clearing through CCPs has the following advantages:

- *Transparency:* In OTC markets, parties typically do not see all outstanding trades between the various counterparties. CCPs have a consolidated view of trading positions and can therefore better react to extreme events.
- *Offsetting:* By transacting through a CCP, duplicate bilateral contracts can be offset, which improves flexibility for new transactions and reduces costs.
- *Loss mutualization:* A member's losses are distributed among all surviving members, which spread the impact of losses, reduce costs, and minimize market impact and systemic risk.
- *Legal and operational efficiency:* The centralized role of CCPs in the clearing (margining, netting) and settlement process improves operational efficiency while reducing costs.
- *Liquidity:* The daily margining of products in a centrally-cleared market ensures greater transparency in product valuation, which increases product liquidity.
- *Default management (counterparty risk):* CCPs act as the counterparty to each trade, which reduces counterparty risk. Member defaults are centrally managed through the auction process which minimizes price disruptions.

### Disadvantages of Central Clearing

While we noted loss mutualization as an advantage of the central clearing process, it can lead to potential problems, including moral hazard and adverse selection.

- *Moral hazard:* Moral hazard is the risk that one party will take on higher risk knowing that another party bears the costs of this risk. In central clearing, the risk is that members will have less incentive to monitor risk knowing that the CCP takes on most of the risks.



- *Adverse selection:* Adverse selection is the risk that participants with a better understanding of product risks and pricing will trade more products whose risks the CCP underprices, and will trade fewer products whose risks the CCP overprices.
- *Bifurcation:* The separation of trading into cleared and non-cleared products can increase cash flow volatility even for hedged products.
- *Procyclicality:* Procyclicality essentially reflects the downside of margining. It reflects a scenario where a CCP increases margin requirements (initial margin) in volatile markets or during a crisis, which may aggravate systemic risk.

## MARGINING

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### LO 47.3: Compare margin requirements in centrally cleared and bilateral markets, and explain how margin can mitigate risk.

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One of the risk mitigation tools employed by CCPs to minimize counterparty and market risk is margining. **Margining** by CCPs is stricter than in the OTC derivatives markets and it involves posting cash or marketable security collateral for initial margin and variation margin requirements. Initial margin represents cash or liquid assets transferred by a member at trade inception to cover a worst-case loss in the event of a member default. Variation margin is typically cash posted by a member to cover the daily net change of the member's position.

CCPs normally set margin requirements based only on the risks of the members' transactions. For initial margin, the credit quality of the member is typically not a consideration and therefore members with different credit risk may be posting the same amount of initial margin.

## NOVATION AND NETTING

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### LO 47.4: Compare and contrast bilateral markets to the use of novation and netting.

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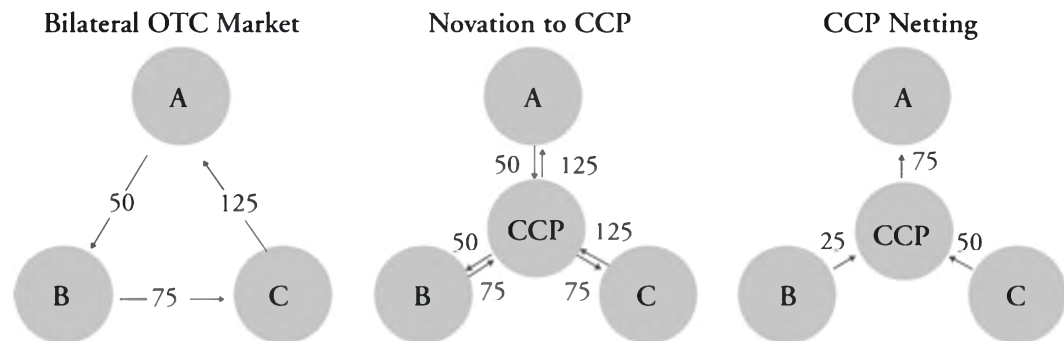
The legal process of interposing the CCP between the seller and the buyer is called **novation**. Through novation, one contract (the bilateral contract between OTC participants) is replaced with another contract (or contracts) with the CCP. This is important because novation transforms the process from bilateral trading to trading with a CCP, where the CCP is the insurer of counterparty risk. Assuming the legal enforceability of novation, the old contracts cease to exist and the original bilateral parties have no further obligations to each other. At the same time, because all trades are centralized, the CCP maintains a "matched book" of trades with no net market risk. The CCP does have conditional credit risk from a member's potential default.

Market participants often prefer to offset rather than to terminate trades, which creates redundant trades. When trades are novated to a CCP, these redundant trades become a single net obligation between each participant and the CCP. This process is called **multilateral offsetting**, or **netting**. Netting reduces total risk and minimizes the potential of a domino effect stemming from the default of a participant.



The process of moving from bilateral to novation to netted positions is illustrated in Figure 1.

**Figure 1: Multilateral Offsetting**



### IMPACT OF CENTRAL CLEARING

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#### LO 47.5: Assess the impact of central clearing on the broader financial markets.

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By now it should be evident that central clearing through a CCP has significant advantages, but it is not without its challenges. When a CCP is included in the clearing process, systemic risk in the financial markets is reduced, but can be increased at the same time. Systemic risk is reduced because CCPs reduce counterparty risk by offsetting positions (novation and netting), they provide transparency for the market, and improve liquidity. However, the potential requirement that members post higher initial margin during times of increased market volatility could increase systemic risk. In addition, concentrating all trades in a single place exposes the market to the risk of CCP failure and heightened systemic risk.

It is worth noting that the protection that CCPs offer for OTC derivatives may come at the expense of other groups. For example, netting and margining protects OTC derivatives participants, but may not benefit creditors. In addition, the long-term maturity of OTC derivatives contracts, often years or decades, also poses challenges for CCPs. It is not yet evident that they are effective in clearing these long-dated, more complex and illiquid trades. Central clearing of exotic or non-standard derivatives may also be problematic.

## KEY CONCEPTS

### LO 47.1

CCPs play important roles in the clearing and settlement of transactions. Clearing refers to the processes between the period from trade execution until settlement. Settlement refers to the satisfaction of legal obligations and trade completion.

Functions of a CCP include novation, netting, margining, managing the auction process, and loss mutualization. Auctioning refers to selling off the defaulted member's trades to the surviving members through an auctioning process. Loss mutualization refers to members' contributions to a default fund to cover future losses from member defaults.

Other aspects and mechanics of CCPs include:

- *Categories of OTC derivatives products:* (1) long history of central clearing (e.g., interest rate swaps), (2) short history of central clearing (e.g., index credit default swaps), (3) soon to be centrally cleared (e.g., interest rate swaptions, credit default swaps), and (4) not suitable for central clearing (e.g., exotic derivatives).
- *Conditions needed for central clearing:* product standardization, lower complexity, and high liquidity.
- *Participants:* Transacting with CCPs is restricted to clearing members only. Member criteria include admission criteria, financial commitment, and operational criteria.
- *Number of CCPs:* It is generally not feasible to have a single CCP due to regional differences in trades and requirements, differences in product types, and regulatory reasons.
- *Types of CCPs:* CCPs could be utility-driven CCP (i.e., focused on long-term stability) or profit-driven CCP (i.e., focused on bottom line). Arguments generally support profit-driven CCPs.
- *Failure of a CCP:* The potential failure of a large CCP could create a catastrophic event. CCPs must therefore ensure sufficient loss absorption capacity.

### LO 47.2

Advantages of CCPs include: transparency, offsetting, loss mutualization, legal and operational efficiency, liquidity, and default management.

Disadvantages of CCPs include: moral hazard, adverse selection, separation of cleared and non-cleared products, and procyclicality of margin requirements.

### LO 47.3

Margining includes posting both initial margin and variation margin. Margining tends to be more stringent in central clearing than in OTC markets. CCPs set margin requirements based only on the risks of the members' transactions, and the credit quality of the member is typically not a consideration for initial margin.

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**LO 47.4**

Novation refers to replacing a bilateral OTC contract with another contract (or contracts) with the CCP, where the CCP is the insurer of counterparty risk. The CCP maintains a “matched book” of trades with no net market risk.

Multilateral offset, or netting, refers to creating a single net obligation between each participant and the CCP from the various bilateral OTC trades (which typically include redundant trades). Netting reduces total risk and minimizes contagion from a member default.

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**LO 47.5**

By including a CCP in the clearing process, systemic risk can be both reduced and increased. Systemic risk is reduced because counterparty risk is reduced, and transparency and liquidity improve. Systemic risk is increased because higher initial margin during times of stress would heighten market risk, and the failure of a CCP may lead to a catastrophic event.

## CONCEPT CHECKERS

1. Which of the following statements on central clearing is accurate?
  - I. The composition of clearing members typically includes a combination of large global banks and smaller banks and non-financial institutions.
  - II. In the auction process, a CCP normally does not close out trades at their market value.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
  
2. Alex Dell, a derivatives trader, has some reservations about the central clearing of OTC derivatives with a central counterparty (CCP). Specifically, he is worried that clearing members' willingness to monitor credit risk may decline since the CCP assumes most of the risks, and that CCPs may increase margin requirements during a period of market stress. Which of the following concepts best describe Dell's reservations?
 

<u>Decline in Willingness</u>	<u>Higher Margin Requirements</u>
A. Moral hazard	Procyclicality
B. Adverse selection	Offsetting
C. Moral hazard	Offsetting
D. Adverse selection	Procyclicality
  
3. In a recently released report to management, a credit analyst indicates that the level of initial margin set by a central counterparty (CCP) is dependent on the risk of the member that is required to post it, and on the risks of the specific derivatives transactions. The analyst is correct with respect to:
 

<u>Risk of Member</u>	<u>Risk of Transactions</u>
A. Yes	Yes
B. Yes	No
C. No	Yes
D. No	No
  
4. Alpha Bank recently noted that its bilateral over-the-counter (OTC) trade obligations with Beta Bank ceased to exist and the bank now directly faces a central counterparty (CCP) for its trade obligations. Which of the following concepts best identify this scenario?
  - A. Netting.
  - B. Novation.
  - C. Margining.
  - D. Multilateral offsetting.

5. Erin Parker and Nate James are analysts at a large financial institution. During one of their recent discussions on OTC derivatives and central clearing with a central counterparty (CCP), Parker states that: “CCPs are beneficial because they convert operational and legal risk into counterparty risk.” James adds to that statement by suggesting: “When requiring higher margin in turbulent times, CCPs reduce systemic risk.”

With respect to the statements made:

- A. Only Parker is correct.
- B. Only James is correct.
- C. Both Parker and James are correct.
- D. Neither Parker nor James is correct.

## CONCEPT CHECKER ANSWERS

1. B Clearing members typically include large players only, including large banks and global financial institutions. In the auction process, a CCP normally does not close out trades at their market value. Instead, trades are auctioned to existing members.
2. A Dell's reservations describe moral hazard and procyclicality, respectively. In central clearing, moral hazard is the risk that members have less incentive to monitor risk knowing that the CCP assumes most of the risks of the transactions. Procyclicality describes a scenario where a CCP increases margin requirements (initial margin) in volatile markets or during a crisis, which may aggravate systemic risk.

Offsetting describes the elimination of duplicate bilateral contracts by transacting through a CCP, which improves flexibility and reduces costs. Adverse selection is the risk that participants with a better understanding of product risks and pricing will trade more products whose risks the CCP underprices, and fewer products whose risks the CCP overprices.

3. C CCPs set initial margin requirements based on the risk of the transactions, but not on the risk of the members.
4. B Novation describes the process where one contract (the bilateral contract between OTC participants) is replaced with another contract (or contracts) with the CCP. As a result, counterparties' bilateral obligations with each other cease to exist.

Multilateral offsetting, also called netting, refers to creating, from the various bilateral OTC trades, a single net obligation between each participant and the CCP.

Margining is the process of posting some form of collateral, typically cash or marketable securities, to cover member defaults (initial margin) or security mark-to-market movements (variation margin).

5. D Neither Parker nor James is correct. CCPs convert counterparty risk into operational and legal risk. Also, when CCPs require higher margin in turbulent times, CCPs can increase systemic risk. This risk is known as procyclicality.

# RISKS CAUSED BY CCPs

Topic 48

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## EXAM FOCUS

This short and qualitative topic deals with the many risks specific to central counterparties (CCPs). Some of the types of risks such as default, liquidity, and operational risks are common to most entities while others such as settlement and payment risk, and custody risk are more specific to CCPs. For the exam, it is important to have an understanding of the full range of risks faced by CCPs as well as the methods to prevent CCP failures.

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## RISKS FACED BY CENTRAL COUNTERPARTIES

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### LO 48.1: Identify and explain the types of risks faced by CCPs.

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#### Default Risk

The default of a clearing member and its flow through effects is the most significant risk for a CCP. Because of a default, there may be the default or distress of other clearing members given that default correlation is likely to be high among over-the-counter (OTC) derivatives market participants.

In the event of a failed auction or an insufficient number of bids, the CCP will be required to pass on the defaulting member's losses through rights of assessment, loss allocation methods, or both. Passing on losses to other clearing members may result in defaults by those members. The loss allocation methods may be considered unfair because some of them, such as variation margin gains haircutting (VMGH) and tear-ups, impose losses on "winning positions." With VMGH, members whose positions increased in value (i.e., they are owed variation margin) will likely not receive the full amount for their gains (i.e., haircutting). Members who instead owe money to the CCP will still be required to pay the full margin amount to the CCP. In a tear-up, the CCP terminates the unmatched position, and may balance resources by drawing from both the defaulter's initial margin and the default fund.

Some clearing members may resign from the CCP after the default of another clearing member. In such instances, the applicable initial margins and default funds must be returned to the resigning clearing member. The initial resignation may result in a negative reputational impact to the CCP as witnessed by further resignations of clearing members.

#### Model Risk

OTC derivatives are not priced by the market but are instead priced using valuation models that perform the mark-to-market function, which subjects CCPs to model risk. Especially



sensitive to model risk would be a CCP's determination of initial margins. In that context, model risk could arise due to errors pertaining to volatility, tail risk, complex dependencies, and wrong-way risk.

Many models are linear in nature, which means that an initial margin will be adjusted in proportion to the increase in the size of the position. However, for large or concentrated positions, the margin may be too low. The use of a supplement to the computation, such as a margin multiplier, may assist in sufficient coverage of the risk.

## Liquidity Risk

There are large amounts of cash inflows and outflows flowing through the CCP due to initial margins and margin calls. As a result, CCPs are exposed to liquidity risk. The CCP attempts to earn the greatest return possible on the funds it holds without incurring too much credit or liquidity risk, thereby most commonly investing in short-term deposits, repos, and reverse repos. Should there be a default by one or more members, the CCP is still required to meet the obligations of the other members.

There is the risk that the CCP's investments are not always quickly and easily convertible to cash, which may require some liquidity support from a central bank. In this regard, a CCP is required to have sufficient liquid resources to meet its obligations in the event of the failure of one or two of its largest clearing members. The Basel III leverage ratio (calculated as the bank's tier 1 capital divided by its exposure) requirements serve to minimize the amount of risk taking. Exposure would include the gross notional amount of centrally cleared OTC derivatives transactions. Overall, regulations have attempted to address a CCP's potential liquidity risks; however, they may reduce the availability of clearing services.

## Operational Risk

Due to the centralization of some functions within a CCP to increase efficiency, additional risks arise that affect counterparties due to concentration at the CCP. CCPs face operational risks that are common to all entities such as business interruption due to information systems failures and internal or external fraud. However, a systems failure within a CCP could have a disastrous impact on many counterparties, especially if they hold large positions.

## Legal Risk

Legal risks in the form of litigation or claims may arise due to differing laws in different jurisdictions or laws that are inconsistent with the CCP's regulations. A good example would involve the segregation and movement of margin and positions (i.e., netting) through a CCP.

## Other Risks

*Investment risk* refers to the risk of losses of margin funds resulting from investment actions performed within or outside of the stated investment policy.

*Settlement and payment risk* refers to the risk that a bank no longer provides cash settlement services between a CCP and its members.

*Foreign exchange risk* refers to the risk of mismatches between margin payments and cash inflows or outflows in different currencies.

*Custody risk* refers to the risk of loss of securities, margins, or both by a custodian due to its failure, fraud, or negligence.

*Concentration risk* refers to the risk of clearing members, margins, or both that are located in a single geographic area. Essentially, it is a lack of diversification.

*Sovereign risk* refers to the risk that a foreign government could default on its debt obligations, thereby causing members to fail. It also refers to any potential loss in the value of sovereign bonds held as margin.

*Wrong-way risk* refers to the risk that exposure to a counterparty is negatively correlated with the credit quality of the counterparty. In other words, it occurs when credit exposure to a counterparty and the default risk of the counterparty increase together.

Overall, it is probable that the various loss events will be correlated and will impact the CCP at the same time. In the case of a default, there will probably be a major market impact that increases the probability of operational and investment issues. Additionally, in a default scenario, there is usually a wide spread between gain and loss positions that increase legal and fraud risks.

## RISKS TO CLEARING MEMBERS AND NON-MEMBERS

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### LO 48.2: Identify and distinguish between the risks to clearing members as well as non-members.

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Non-members face exposure from CCPs, clearing members, and other non-members. If a CCP fails, a non-member may be able to avoid losses so long as its counterparty (a clearing member) is solvent. Unlike clearing members, non-members are not required to contribute to default funds so, therefore, non-members are not exposed to losses that result from CCP failures.

Furthermore, the extent of non-members' losses due to defaults of CCPs and clearing members lies with the initial margins and whether they are segregated, guaranteed, or both. In addition, non-members face the risk of not being able to port their trades should the counterparty member default. As a result, such trades may have to be closed out at a loss.

Finally, one has to consider non-members' liability with respect to CCP loss allocation rules. It is possible that clearing members are able to pass on losses to non-members through VMGH or tear-up, which would reduce the gains of non-members. Clearing members are unable to pass on losses resulting from default fund utilization, rights of assessment, and forced allocation.

## LESSONS LEARNED FROM CCP FAILURES

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### LO 48.3: Identify and evaluate lessons learned from prior CCP failures.

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There are five key lessons learned from prior CCP failures:

1. Operational risk must be controlled to the maximum extent possible. For example, information systems should be updated sufficiently to be robust enough to handle unusually high trading volumes and to detect significant price changes.
2. Variation margins should be recalculated often and collected quickly (i.e., multiple times a day in certain cases). Having an information system that allows for automated payments could assist in preventing liquidity shortfalls. In addition, having cross-margining linkage arrangements (offsetting of hedged positions) between CCPs may avoid liquidity problems due to the hedging activities of the various CCPs.
3. Initial margins and default funds should be sufficiently large in order to withstand significant negative asset value declines as well as increased return correlations during a crises. The assumptions behind the initial margin computations need to be amended to account for significant changes in the market.
4. CCPs must actively monitor positions, penalize overly concentrated positions, and promptly liquidate or hedge extremely large positions.
5. CCPs must have one or more external sources of liquidity to avoid default due to illiquidity (even though it is still solvent).

## KEY CONCEPTS

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### LO 48.1

CCPs face five major risks: default risk, model risk, liquidity risk, operational risk, and legal risk. Other risks they may face include investment risk, settlement and payment risk, foreign exchange risk, custody risk, concentration risk, sovereign risk, and wrong-way risk.

The default of a clearing member and its flow through effects is the most significant risk for a CCP. Because of a default, there may be the default or distress of other clearing members given that default correlation is likely to be high among OTC derivatives market participants.

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### LO 48.2

Non-members face exposure from CCPs, clearing members, and other non-members.

If a CCP fails, a non-member may be able to avoid losses so long as its counterparty is solvent. Non-members are not required to contribute to default funds so they are not exposed to losses that result from CCP failures. The extent of non-members' losses lies with the initial margins and whether they are segregated, guaranteed, or both. Non-members face the risk of not being able to port their trades should the counterparty member default.

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### LO 48.3

Lessons learned from prior CCP failures include:

- Operational risk must be controlled to the maximum extent possible.
- Variation margins should be recalculated often and collected quickly.
- CCPs should have an information system that allows for automated payments.
- There should be cross-margining linkage arrangements between CCPs.
- Initial margins and default funds should be sufficiently large.
- CCPs must actively monitor positions.
- CCPs must have one or more external sources of liquidity.

## CONCEPT CHECKERS

1. Which of the following risks facing a central counterparty (CCP) is most likely to be introduced during a market crisis?
  - A. Default risk.
  - B. Liquidity risk.
  - C. Operational risk.
  - D. Settlement and payment risk.
2. Which of the following statements regarding risks facing a CCP is correct?
  - A. A good example of legal risk would involve netting arrangements.
  - B. Default correlations tend to be low among OTC derivatives market participants.
  - C. Many models for pricing OTC derivatives are linear in nature, which may result in excessive margins for large positions.
  - D. Investment risk refers to the risk of losses of margin funds resulting from investment actions performed outside of the stated investment policy.
3. A non-clearing member would face exposure from defaults by which of the following parties?
  - I. Clearing members.
  - II. Other non-clearing members.
  - A. I only.
  - B. II only.
  - C. Both I and II.
  - D. Neither I nor II.
4. Which of the following losses may be borne by non-members of CCPs?
  - A. Rights of assessment.
  - B. Tear-up.
  - C. Default fund utilization.
  - D. Forced allocation.
5. Which of the following statements regarding lessons learned from prior CCP failures is correct?
  - A. CCPs face the risk of default through insolvency even though they may be liquid.
  - B. In extreme cases, variation margins should be recalculated and collected on a daily basis.
  - C. Initial margins must be sufficient enough to withstand situations of extreme dependency.
  - D. CCPs should actively monitor positions but they need not penalize unintended concentrated positions.

## CONCEPT CHECKER ANSWERS

1. D Settlement and payment risk refers to the risk that a bank no longer provides cash settlement services between a CCP and its members. Such risk is not likely to be present during normal periods but is much more likely to be present during crisis periods.

The other risks mentioned are present in both normal and crisis periods. For example, liquidity and default risks are always present but would be exacerbated during a crisis period. Operational risks such as inadequate information systems that give rise to business interruption could be present in a normal period.

2. A Legal risks in the form of litigation or claims may arise due to differing laws in different jurisdictions or laws that are inconsistent with the CCP's regulations. A good example would involve the segregation and movement of margin and positions (i.e., netting) through a CCP.

Response B is not correct, because default correlation is likely to be *high* among OTC derivatives market participants. Response C is not correct, because many models are linear in nature, which means that an initial margin will be adjusted in proportion to the increase in the size of the position. However, for large and/or concentrated positions, the margin may be too low. Response D is not correct, because investment risk refers to the risk of losses of margin funds resulting from investment actions performed *within or outside* of the stated investment policy.

3. C Non-clearing members face exposure from CCPs, clearing members, and other non-clearing members.
4. B Clearing members may be able to pass on losses to non-members through a “tear-up,” which would reduce the gains of non-members. Clearing members are unable to pass on losses resulting from default fund utilization, rights of assessment, and forced allocation.
5. C An example of extreme dependency would be the increase in correlation of returns during a market crisis.

Response A is not correct, because CCPs face the risk of default through *illiquidity* even though they may be *solvent*. Response B is not correct, because in extreme cases, variation margins should be recalculated and collected up to several times a day. Response D is not correct, because CCPs should actively monitor positions and penalize overly concentrated positions regardless of whether they were intended or unintended.



# FOREIGN EXCHANGE RISK

Topic 49

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## EXAM FOCUS

Exposure to foreign exchange risks is a natural result of the globalization of financial institutions. These risks arise when foreign currency trading and/or foreign asset-liability positions are mismatched in individual currencies. Unexpected volatility can generate significant losses for the firm, which could, in turn, threaten profitability or even solvency. These risks can be mitigated by direct hedging through matching foreign asset-liability books of business, hedging through forward contracts, and through foreign asset and liability portfolio diversification.

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## SOURCES OF FOREIGN EXCHANGE RISK

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**LO 49.1: Calculate a financial institution's overall foreign exchange exposure.**

**LO 49.2: Explain how a financial institution could alter its net position exposure to reduce foreign exchange risk.**

**LO 49.3: Calculate a financial institution's potential dollar gain or loss exposure to a particular currency.**

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Large financial institutions (banks) frequently take significant positions in foreign currency assets and liabilities as a result of their foreign exchange trading activities. When looking at such financial institutions' currency trading activities, the aggregate position size in a particular currency may look extremely large; however, since buys and sells will offset one another in terms of exposure, the net exposure to the currency may actually be quite small.

A bank's actual exposure to any given currency can be measured by the **net position exposure**. Net exposure is the extent to which a bank is net long (or *positive*) or net short (or *negative*) in a given currency. For example, a bank's net euro (EUR) exposure would be:

$$\text{net EUR exposure} = (\text{EUR assets} - \text{EUR liabilities}) + (\text{EUR bought} - \text{EUR sold})$$

$$\text{net EUR exposure} = \text{net EUR assets} + \text{net EUR bought}$$

A **positive net exposure** position means that we are *net long in a currency*. In other words, we hold more assets than liabilities in a given currency. In this instance, the financial institution faces the risk that the foreign currency will *fall* in value against the domestic currency.

A **negative net exposure** position means that we are *net short in a currency*. The financial institution faces the risk that the foreign currency will *rise* in value against the domestic currency.



Therefore, if a U.S. financial institution fails to maintain a balanced position in a currency where assets (purchases) are exactly offset by liabilities (sales), the institution will be exposed to variations in the foreign exchange (FX) rate of that currency against the U.S. dollar. The more volatile the FX rate, the more potential impact a net exposure (either long or short) will have on the value of a bank's foreign currency portfolio.

## FOREIGN TRADING ACTIVITIES

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### LO 49.4: Identify and describe the different types of foreign exchange trading activities.

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A financial institution's buying and selling of foreign currencies, and hence the institution's position in the FX market, reflects four key trading activities:

1. Enabling customers to participate in international commercial business transactions.
2. Enabling customers to take positions in real or financial foreign investments. Note that a financial institution may also transact in foreign currencies to take positions in real or financial foreign investments for its own portfolio.
3. Offsetting exposure in a given currency for hedging purposes.
4. Speculating on foreign currencies in search of profit by forecasting and/or anticipating futures FX rate movements.

When a bank is buying or selling a foreign currency for the purpose of either allowing its customers to participate in international commercial business transactions or investing in real or financial foreign investments, the bank typically serves as an agent for the customers (receives a fee) and does not assume the FX risk itself.

When a bank is buying or selling a currency for hedging purposes, this will reduce FX exposure.

The fourth activity, trading foreign currencies with the intent to profit by anticipating future foreign currency rate movements, relates to open positions that are taken for speculative purposes and represents an unhedged position in a given currency. These speculative trades are usually made directly with other financial institutions or arranged through FX specialist brokers.

Currency spot trades are the most frequently executed speculative trades. The financial institution seeks to earn a profit on the difference between the buy and sell prices or on movements in the bid-ask spreads over time. Speculative positions can also be taken in FX forward contracts, futures, and options.

## SOURCES OF PROFITS AND LOSSES ON FOREIGN EXCHANGE TRADING

**LO 49.5: Identify the sources of foreign exchange trading gains and losses.**

**LO 49.6: Calculate the potential gain or loss from a foreign currency denominated investment.**

Most returns on FX trading arise from speculation in currencies or taking an unhedged position in a particular currency. Financial institutions also earn fees as a secondary source of revenues. These revenues are earned from market-making activities and/or from acting as agents for retail or wholesale customers.

### MISMATCHED FOREIGN ASSET AND LIABILITY POSITIONS

A financial institution can also have foreign exchange exposure due to mismatches between foreign financial asset and liability portfolios. The following example shows the exposure resulting from such a mismatch.

#### Example: Foreign investment returns

**Figure 1: Balance Sheet**

<i>Assets</i>	<i>Liabilities</i>
USD50 million U.S. loans, 1-year maturity, in USD, yielding 8%	USD100 million U.S. CDs, 1-year maturity, in USD, yielding 6%
USD50 million equivalent Swiss loans, 1-year maturity, made in CHF, yielding 13%	

This firm has matched the duration of its assets and liabilities ( $D_A = D_L = 1$  year) but has mismatched the currency composition of its portfolio. Note that the firm would earn a positive spread of 2% ( $8\% - 6\%$ ) from investing domestically. In order to invest in Switzerland, this firm decides to take 50% of its \$100 million and make 1-year Swiss loans while keeping 50% to make U.S. dollar loans. What transactions must the firm undertake to make the CHF-denominated loan (assuming the FX position is not hedged)?

#### Answer:

1. Sell USD50 million for CHF on the spot currency markets at the beginning of the year. If the exchange rate is USD1.70 to 1 CHF, this yields  $\text{USD}50,000,000 / 1.7 = \text{CHF}29,411,765$ .
2. Use the CHF29,411,765 to make 1-year Swiss loans at a 13% interest rate.
3. At the end of the one year, CHF revenue from these loans will be  $\text{CHF}29,411,765(1.13) = \text{CHF}33,235,294$  (assuming no default).
4. At the end of the year, repatriate these funds back to the United States. In other words, the U.S. bank will sell CHF33,235,294 in the FX market at the spot exchange rate that exists at the end of the year.

In this example, we assume the spot FX rate has not changed over the 1-year period and remains at USD1.70/CHF. The dollar proceeds from the Swiss investment would be:

$\text{CHF}33,235,294 \times \text{USD}1.70 / \text{CHF} = \text{USD}56,500,000$ , for a return of:

$$\frac{\text{USD}56,500,000 - \text{USD}50,000,000}{\text{USD}50,000,000} = 13.0\%$$

Thus, the weighted return on this portfolio will be:

$$(0.5)(0.08) + (0.5)(0.13) = 0.105 \text{ or } 10.5\%$$

This exceeds the cost of the bank CDs by 4.5% (=10.5% – 6.0%).

**Example, continued:**

Now, suppose that at the end of the year, the Swiss franc has *fallen* in value relative to the U.S. dollar. If the exchange rate is now USD1.55/CHF, **compute** what the Swiss loan revenues would be at the end of Year 1.

**Answer:**

The Swiss loan revenues at the end of one year equal:

$\text{CHF}33,235,294 \times \text{USD}1.55 / \text{CHF} = \text{USD}51,514,706$ , for a return of:

$$\frac{\text{USD}51,514,706 - \text{USD}50,000,000}{\text{USD}50,000,000} = 3.03\%$$

Thus, the weighted return on this portfolio will be:

$$(0.5)(0.08) + (0.5)(0.0303) = 0.0552 \text{ or } 5.52\%$$

Under this scenario, the bank would actually have a negative interest margin on its balance sheet investments of –0.48% since its cost of funds (COFs) is 6.0%.

**Example, continued:**

If the Swiss franc had *appreciated* against the dollar over the year, the bank would have generated a double benefit: (1) from the appreciation of the franc, and (2) from the higher yield on the domestic Swiss loans. If the exchange rate is now USD1.82/CHF, **compute** what the Swiss loan revenues would be at the end of Year 1.

**Answer:**

$\text{CHF}33,235,294 \times \text{USD}1.82 / \text{USD} = \text{USD}60,488,235$ , for a return of:

$$\frac{\text{USD}60,488,235 - \text{USD}50,000,000}{\text{USD}50,000,000} = 20.98\%$$

The previous example illustrates an important concept. As with any investment, returns for the bank's portfolio are derived from differences between income and costs. However, foreign investing provides the additional dynamic of having profits or losses affected by changes in foreign exchange rates. There are two principle methods available to control the scale of FX exposure: on-balance-sheet hedging and off-balance-sheet hedging.

## BALANCE SHEET HEDGING

### LO 49.7: Explain balance-sheet hedging with forwards.

#### On-Balance-Sheet Hedging

On-balance-sheet hedging is achieved when a financial institution has a matched maturity and currency foreign asset-liability book. Figure 2 is an illustration.

Figure 2: Balance Sheet

<i>Assets</i>	<i>Liabilities</i>
USD50 million U.S. loans, 1-year maturity, in USD, yield 8%	USD50 million U.S. CDs, 1-year maturity, in USD, yielding 6%
USD50 million equivalent Swiss loans, 1-year maturity, made in CHF, yielding 13%	USD50 million Swiss CDs, 1-year maturity, raised in CHF, yielding 10%

Using the data in Figure 2, we can examine the effects of the franc depreciating by the same amount as in the previous example:

1. The bank borrows USD50 million equivalent in Swiss francs for one year at an interest rate of 10%. At the exchange rate of USD1.70/CHF, this equates to  $\text{USD}50,000,000 / 1.70 = \text{CHF}29,411,765$ .
2. At the end of one year, the bank must pay back the Swiss franc CD holders their principal and interest:  $\text{CHF}29,411,765 \times (1.10) = \text{CHF}32,352,941$ .
3. If the franc *depreciated* to USD1.55/CHF in the period, repayment in dollar terms would be  $\text{CHF}32,352,941 \times \text{USD}1.55/\text{CHF} = \text{USD } 50,147,059$ , or a dollar cost of funds of 0.3%.
4. The bank makes CHF29,411,765 in loans at 13% for one year.
5. At the end of one year, the loans are repaid with interest.  $\text{CHF}29,411,765 (1.13) = \text{CHF}33,235,294$ , but at USD1.55/CHF, this equals only USD51,514,706 for a return of 3.03%.

At the end of the year, we would have the following.

Average return on assets:

$$(0.5)(0.08) + (0.5)(0.0303) = 0.0552 \text{ or } 5.52\%$$

U.S. asset return + CHF asset return = overall return

Average cost of funds:

$$(0.5)(0.06) + (0.5)(0.003) = 0.0315 \text{ or } 3.15\%$$

U.S. cost of funds + CHF cost of funds = overall cost

Net return:

$$5.52\% - 3.15\% = 2.37\%$$

average return on assets – average cost of funds

By directly matching foreign assets and liabilities, we can lock in a positive return or profit spread if exchange rates move in either direction over the investment period.

### Off-Balance-Sheet Hedging

Rather than matching foreign assets with foreign liabilities, we may choose to remain unhedged on the balance sheet. If we do, we could hedge off-balance-sheet by taking a position in the forward market. This hedge would appear as a contingent off-balance-sheet claim as an item below the net income line.

Referring to the previous example, the function of the forward FX contract is to offset the uncertainty of the future spot rate on the CHF at the end of the investment horizon. A forward foreign exchange agreement involves the exchange of a foreign currency at some point in the future at an exchange rate that is determined today. Rather than repatriating CHF and exchanging them for USD at the end of the period at an unknown rate, the bank can enter into a contract to sell forward the *expected* principal and interest on the loan at the current known **forward exchange rate** for USD/CHF, with the delivery of Swiss francs to the buyer of the forward contract taking place at the end of the investment horizon. This method effectively removes the future spot exchange rate uncertainty that is related to investment returns on the Swiss loan. By using the data in Figure 2, we can illustrate how this technique would work.

**Example: Hedging with forward contracts**

**Outline** the transactions necessary for the financial institution to use an off-balance-sheet hedge for the asset-liability position described in Figure 2.

**Answer:**

The following transactions create the off-balance-sheet hedge.

1. The U.S. bank sells USD50 million for Swiss francs at the *spot* exchange rate *today* and receives  $\text{USD}50,000,000 / \text{USD}1.7/\text{CHF} = \text{CHF}29,411,765$ .
2. Immediately after the sale, the bank lends the CHF29,411,765 to a Swiss customer at 13% for one year.
3. In addition, the bank sells the expected principal and interest proceeds from the franc loan forward for U.S. dollars at today's forward rate (say, USD1.65/CHF) for 1-year delivery:  $(\text{USD}1.65 - \text{USD}1.70) / \text{USD}1.70 = -2.94\%$ .

The forward buyer of the francs will pay USD54,838,235 to the seller when the bank delivers the CHF33,235,294 proceeds of the loan to the financial institution seller.

$$\text{CHF}29,411,765(1.13) \times \text{USD}1.65/\text{CHF} = \text{CHF}33,235,294 \times \text{USD}1.65/\text{CHF} \\ = \text{USD}54,838,235$$

4. At the end of one year, the Swiss borrower repays the loan to the bank plus interest in Swiss francs (CHF33,235,294).
5. The bank gives the CHF33,235,294 to the buyer of the 1-year forward contract and receives USD54,838,235.

By using this method, the bank knows it has locked in a guaranteed return of 9.68% on the Swiss franc (assuming, of course, the loan will not default and the forward buyer does not renege on the forward contract).

$$\frac{\text{USD}54,838,235 - \text{USD}50,000,000}{\text{USD}50,000,000} = 0.0968 = 9.68\%$$

The overall expected return on the bank's asset portfolio would then be:

$$(0.5)(0.08) + (0.5)(0.0968) = 8.84\%$$

Regardless of spot exchange rate fluctuations over the year, the bank has locked in a risk-free return spread of 2.84% (8.84% return – 6% cost of funds) over the cost of funds for the bank's CDs.



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**LO 49.8: Describe how a non-arbitrage assumption in the foreign exchange markets leads to the interest rate parity theorem, and use this theorem to calculate forward foreign exchange rates.**

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Because the hedged Swiss loans offer a higher return than the U.S. loans, it makes sense for the bank to focus its activities on making hedged Swiss loans. However, as more is invested in Swiss loans, the bank must buy more Swiss francs. This will continually reduce the forward rate spread until no additional profits could be made by making the forward contract-hedged investments.

As the bank moves into more Swiss loans, the spot exchange rate for buying francs will rise. In equilibrium, the forward exchange rate would have to fall to completely eliminate the attractiveness of the Swiss investments.

This relationship is called **interest rate parity** (IRP) since the discounted spread between domestic and foreign interest rates equals the percentage spread between forward and spot exchange rates. In other words, the hedged dollar return on foreign investments should be equal to the return on domestic investments. IRP implies that in a competitive market, a firm should not be able to make excess profits from foreign investments (i.e., a higher domestic currency return from lending in a foreign currency and locking in the forward rate of exchange).

For the exam, you should know that the exact IRP equation using direct quotes is:

$$\text{forward} = \text{spot} \left[ \frac{(1 + r_{\text{DC}})}{(1 + r_{\text{FC}})} \right]^T$$

where:

$r_{\text{DC}}$  = domestic currency rate

$r_{\text{FC}}$  = foreign currency rate

If this equality does not hold, an arbitrage opportunity exists. To remember this formula, note that when the forward and spot rates are expressed as direct quotes (DC/FC), right-hand side of the equation also has the domestic (interest rate) in the numerator and the foreign (interest rate) in the denominator.

If we expressed the forward and spot rates as indirect quotes (FC/DC), then the right-hand side of the equation would have the foreign (interest rate) in the numerator and the domestic (interest rate) in the denominator. So it's either domestic over foreign for everything, or foreign over domestic for everything.

IRP can also be stated using continuously compounded rates as follows:

$$\text{forward} = \text{spot} \times e^{(r_{\text{DC}} - r_{\text{FC}})T}$$



**Example: Interest rate parity**

Suppose you can invest in NZD at 5.127%, or you can invest in Swiss francs at 5.5%. You are a resident of New Zealand, and the current spot rate is 0.79005 NZD/CHF. Calculate the 1-year forward rate expressed in NZD/CHF.

**Answer:**

$$\text{forward}(\text{DC} / \text{FC}) = \text{spot}(\text{DC} / \text{FC}) \left[ \frac{(1 + r_{\text{DC}})}{(1 + r_{\text{FC}})} \right] = 0.79005 \left( \frac{1.05127}{1.055} \right) = 0.78726$$



*Professor's Note: Notice here that the NZD/CHF rate fell from 0.79005 to 0.78726. This implies that it now takes fewer NZD to buy one CHF. So, in other words, the New Zealand dollar has appreciated relative to the Swiss franc. Consequently, the Swiss franc has depreciated relative to the New Zealand dollar.*

## DIVERSIFICATION IN MULTICURRENCY FOREIGN ASSET-LIABILITY POSITIONS

**LO 49.9: Explain why diversification in multicurrency asset-liability positions could reduce portfolio risk.**

**LO 49.10: Describe the relationship between nominal and real interest rates.**

Our previous examples have used matched and mismatched asset-liability portfolios that involve only one foreign currency. In reality, most financial institutions hold positions in many different currencies in their asset-liability portfolios. Since currencies may be less than perfectly correlated, diversification across several asset and liability markets can potentially reduce portfolio risk as well as the cost of funds. Domestic and foreign interest rates and stock returns generally do not move together perfectly over time. This means that the risks from mismatching one-currency positions may be offset by potential gains from asset-liability portfolio diversification.

Each domestic and foreign nominal interest rate consists of two components. The first component is the **real interest rate**, which reflects a given currency's real demand and supply for its funds. Differences in real interest rates will cause a flow of capital into those countries with the highest available *real* rates of interest. Therefore, there will be an increased demand for those currencies, and they will appreciate relative to the currencies of countries whose available real rate of return is low.

The second component is the **expected inflation rate**, which reflects the amount of compensation required by investors to offset the expected erosion of real value over time due to inflation. Differences in inflation rates will cause the residents of the country with the highest inflation rate to demand more imported (cheaper) goods. For example, if prices in the United States are rising twice as fast as in Australia, U.S. citizens will increase their

demand for Australian goods (because Australian goods are now cheaper relative to domestic goods). If a country's inflation rate is higher than its trading partners', the demand for the country's currency will be low, and the currency will depreciate.

The **nominal interest rate**,  $r$ , is the compounded sum of the real interest rate, *real*  $r$ , and the expected rate of inflation,  $E(i)$ , over an estimation horizon.

$$\text{exact methodology: } (1 + r) = (1 + \text{real } r)[1 + E(i)]$$

$$\text{linear approximation: } r \approx \text{real } r + E(i)$$

## KEY CONCEPTS

### LO 49.1

Net exposure in a foreign currency measures the extent to which a bank is net long or net short a foreign currency. A financial institution's net currency exposure is calculated as:

$$\begin{aligned}\text{net currency exposure} &= (\text{currency assets} - \text{currency liabilities}) \\ &\quad + (\text{currency bought} - \text{currency sold})\end{aligned}$$

### LO 49.2

A net long (short) position in a currency means that a bank faces the risk that the FX rate will fall (rise) in value versus the domestic currency.

### LO 49.3

If a financial institution fails to maintain a balanced position, the institution will be exposed to variations in the FX rate. The more volatile the FX rate, the more potential impact a net exposure (either long or short) will have on the value of a bank's foreign currency portfolio.

### LO 49.4

A financial institution's buying and selling of foreign currencies, and hence the institution's position in the FX market, reflects four key trading activities:

- Enabling customers to participate in international commercial business transactions.
- Enabling customers (or the financial institution itself) to take positions in real and financial foreign investments.
- Offsetting exposure to gain currency for hedging purposes.
- Speculating on future FX rate movements.

### LO 49.5

Most of the profits and losses on FX come from speculation or open position taking. A secondary source of revenue comes from market-making activities and/or agency fees. Mismatches between foreign financial assets and liabilities can create FX risk exposure.

### LO 49.6

Returns for the bank's portfolio are derived from differences between income and costs. However, there is an extra dimension of return and risk from adding foreign currency assets and liabilities to a portfolio.

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**LO 49.7**

There are two principle methods of better controlling the impact of FX exposure:

- On-balance-sheet hedging is achieved when a financial institution has a matched maturity and foreign currency balance sheet.
- Off-balance-sheet hedging occurs through the purchase of forwards for institutions that choose to remain unhedged on the balance sheet.

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**LO 49.8**

Interest rate parity (IRP) suggests that the discounted spread between domestic and foreign interest rates equals the percentage spread between forward and spot exchange rates. IRP can be stated using continuously compounded rates as follows:

$$\text{forward rate} = \text{spot rate} \times e^{(r_{\text{DC}} - r_{\text{FC}})T}$$

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**LO 49.9**

Since domestic and foreign interest rates and stock returns do not usually move in perfect correlation, opportunities for potential gains from asset-liability portfolio diversification can offset currency risk.

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**LO 49.10**

The real interest rate reflects a given currency's real demand and supply for its funds. The nominal interest rate is the compounded sum of the real interest rate and the expected rate of inflation over an estimation horizon.

## CONCEPT CHECKERS

1. Ion National Bank issues a 6-month, USD1 million CD at 4.0% and funds a loan in Argentine pesos (ARS) at 6.50%. The spot rate for the ARS was ARS2.27 per USD at the time of the transaction. In 6 months, the ARS will have depreciated to ARS2.30 per USD. What is the realized nominal annual spread on the loan?
  - A. -1.07%.
  - B. -0.19%.
  - C. 0.11%.
  - D. 0.13%.
2. With respect to Japanese yen (JPY), a U.S. firm has exchange-rate risk:
  - A. that depends only on its net asset-liability position.
  - B. if its JPY-denominated bonds have greater value than its JPY-denominated loans.
  - C. only if its net JPY position is positive.
  - D. whenever its total JPY assets are not equal to its total JPY liabilities.

Use the following data to answer Questions 3 through 5.

Century Bank issues USD20 million in U.S. CDs to fund its loan portfolio. The following characteristics pertain to the asset-liability position of the bank:

- A promised 1-year rate on the CDs of 7%.
  - It invests 50% of its USD20 million in 1-year U.K. loans at 12% (loans made in GBP).
  - The bank invests the other 50% in U.S. loans at 8% for one year.
  - At the beginning of the year, the bank sells USD10 million for GBP in the spot currency markets at an exchange rate of USD1.42/GBP.
  - The 1-year forward exchange rate is USD1.40/GBP.
3. If the spot foreign exchange rate does not change over the year, the USD proceeds from the U.K. investment will be:
    - A. USD7,040,000.
    - B. USD7,890,000.
    - C. USD11,200,000.
    - D. USD12,000,000.
  4. If the exchange rate falls to USD1.38/GBP, what is the weighted return on the bank's asset portfolio?
    - A. 1.41%.
    - B. 2.82%.
    - C. 5.41%.
    - D. 8.42%.
  5. If the bank hedges its GBP loan in the forward market, what is the return on the bank's loan portfolio?
    - A. 8.37%.
    - B. 9.21%.
    - C. 9.79%.
    - D. 10.11%.

## CONCEPT CHECKER ANSWERS

1. B  $\text{USD1 M} \times \text{ARS2.27} \times 1.0325 = \text{ARS2,343,775} / 2.30 = \text{USD1,019,033} - (\text{USD1 M} \times 1.02) = -\$967.40$ ;  $-967.40 / 1 \text{ M} = -0.0009674 \times 2 = -0.19\%$
2. D A firm's exchange-rate risk depends on its net asset-liability exposure and on the volatility of the exchange rate with the JPY. Bonds and loans are only part of the whole JPY-denominated portfolio; forward contracts and currency holdings must be included to calculate the net asset-liability exposure. Either a positive or negative imbalance between JPY-denominated assets and liabilities will expose the firm to exchange rate risk.
3. C  $\text{USD1.42/GBP} = \text{USD10,000,000} / 1.42 = \text{GBP7,042,254}$   $(1.12) = \text{GBP7,887,324} \times 1.42 = \text{USD11,200,000}$
4. D  $\text{USD10,000,000} \times 1 / 1.42 = \text{GBP7,042,254}$   
 $\text{GBP7,042,254} \times 1.12 \times \text{USD1.38/GBP} = \text{USD10,884,507}$   
 $(\text{USD10,884,507} - \text{USD10,000,000}) / 10,000,000 = 0.08845 = 8.845\%$   
 $(0.5)(0.08) + (0.5)(0.08845) = 8.42\%$
5. B  $\text{USD 10,000,000} \times 1 / 1.42 = \text{GBP7,042,254} \times 1.12 \times \text{USD 1.40/GBP} = \text{USD 11,042,254}$ ;  
 $(11,042,254 - 10,000,000) / 10,000,000 = 10.42\%$ ;  $(0.5)(0.08) + (0.5)(0.1042) = 9.21\%$

# CORPORATE BONDS

Topic 50

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## EXAM FOCUS

The term “bond” refers to a variety of assets which offer a wide range of interest rate payments from fixed cash payments, to accruals without cash, to payments in the form of additional securities. In this topic, we will provide an overview of major fixed-income instruments and their payment structures. We will also address the impact of credit risk and event risk on bond ratings and features. For the exam, be familiar with the types of bonds discussed and the methods for retiring bonds. Also, know the terminology associated with high-yield issues.

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## BOND INDENTURE AND ROLE OF CORPORATE TRUSTEE

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### LO 50.1: Describe a bond indenture and explain the role of the corporate trustee in a bond indenture.

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The **bond indenture** is a document that sets forth the obligation of the issuer and the rights of the investors in the bonds (i.e., the bondholders). It is usually a detailed document filled with legal language. One of the roles of the **corporate trustee** is to interpret this language and represent the interests of the bondholders. Banks or trust companies most often serve as corporate trustees, and the position requires that they act in a fiduciary capacity on behalf of the bondholders. The trustee would authenticate the issue, which includes keeping track of the amount of bonds issued and making sure the number does not exceed the limit specified in the indenture. The trustee would monitor the corporation's activities to make sure the issuer abides by the indenture's covenants (e.g., maintaining key ratios below a given number).

All corporate bond offerings over \$5 million and sold in interstate commerce must have a corporate trustee as set forth in the **Trust Indenture Act**. The corporate trustees must be competent and financially responsible and should also not have any conflicts of interest, (e.g., being a creditor of the issuer). The indenture would specify how the trustee would make reports to bondholders and what to do if the issuer fails to pay interest or principal. As mentioned earlier, the basic goal of the trustee is to protect the rights of bondholders.

## MATURITY DATE

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### LO 50.2: Explain a bond's maturity date and how it impacts bond retirements.

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The maturity date of a bond is when the bond issuer's obligations are fulfilled. At maturity, the issuer pays the principal and any accrued interest or premium. The contract, as set forth by the indenture, may terminate prior to the maturity date if, for example, the corporation chooses to retire the bonds early. The longer the maturity of the bond, the more time a company has to retire the bond issue.



## INTEREST PAYMENT CLASSIFICATIONS

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**LO 50.3: Describe the main types of interest payment classifications.**

**LO 50.4: Describe zero-coupon bonds and explain the relationship between original-issue discount and reinvestment risk.**

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The main types of bond interest payment classifications are: straight-coupon bonds, zero-coupon bonds, and floating-rate bonds. The interest rate on a bond is often called the **coupon**. However, bonds today technically no longer have coupons attached directly to them. Now, bonds are registered and represented by a certificate, or they are kept in book-entry form where one master or global certificate is issued and held by a central securities depository that issues receipts. This method is considered a safer way to make payments.

**Straight-coupon bonds**, also called fixed-rate bonds, have a fixed interest rate set for the entire life of the issue. In the United States, fixed-rate bonds typically pay interest every six months. In Europe and some other countries, bonds make annual interest rate payments. A bond issued in the United States with an 8% interest rate and a \$1,000 par value on March 1, 2009 will pay \$40 of interest each September 1 and each March 1 until its maturity date or until the bond is retired, at which time the issuer would pay both the final interest payment and the \$1,000 principal back to the bondholder.

In addition to just paying a fixed dollar interest, bonds in the United States have been issued that pay in foreign currency. Two other variations are a participating bond and an income bond. **Participating bonds** pay at least the specified interest rate but may pay more if the company's profits increase. **Income bonds** pay at most the specified interest, but they may pay less if the company's income is not sufficient. In both cases, the conditions for paying more or less than the specified coupon would be set forth in the indenture.

**Floating-rate bonds** are also known as variable rate bonds. The interest paid is generally linked to some widely used reference rate such as LIBOR or the Federal Funds rate.

**Zero-coupon bonds** pay the face value or principal at maturity. There is not a cash interest payment; instead, the bondholder earns a return by purchasing the bond at a discount to face value and receiving the full face value at maturity. Variations of the zero-coupon bond include the **deferred-interest bond (DIB)** and the **payment-in-kind bond (PIK)**. The DIB will not pay cash interest for some number of years early in the life of the bond. That period is the deferred-interest period. During this period, cash interest accrues and is then paid semiannually until maturity or when redeemed. PIK bonds pay interest with additional bonds for the initial period, and then cash interest after that period ends.

Most zero-coupons issued today share a host of other features such as being convertible, callable, and puttable. A zero-coupon bond's interest rate is determined by the original-issue discount (OID):

$$\text{original-issue discount (OID)} = \text{face value} - \text{offering price}$$

The value of the bond grows each year and thus pays implicit interest, which is a function of the OID and the term-to-maturity. In other words, the rate of return depends on the amount of the discount and the period over which it grows.

One advantage of zero-coupon bonds is zero **reinvestment risk**. The bondholder does not have to make an effort to reinvest cash interest payments or worry about the available rates at which to reinvest them. A disadvantage is that the bondholder must pay taxes each year on the accrued interest even though no cash is received from the bond issuer.

If the issuer goes into bankruptcy prior to the maturity of a zero-coupon bond, the bondholders are only entitled to the accrued interest up to that date and not the full face value of the bond. In other words, the zero-coupon bond creditor can only claim the original offering price plus accrued and unpaid interest up to the date of the bankruptcy filing. The bond issuer faces a huge liability with a zero-coupon bond because of the large balloon payment at maturity.

## BOND TYPES

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**LO 50.5: Distinguish among the following security types relevant for corporate bonds: mortgage bonds, collateral trust bonds, equipment trust certificates, subordinated and convertible debenture bonds, and guaranteed bonds.**

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Corporate bonds can have a security, such as real property, underlying the issue. Those who own mortgage bonds have a first-mortgage lien on the properties of the issuer. This security allows the issuer to pay a lower rate of return than it would have to pay on unsecured bonds, which are known as debentures. The lien gives the bondholders the right to sell the mortgaged property to satisfy unpaid obligations to bondholders. In practice, this right is usually used for bargaining purposes only, and the bankruptcy takes the form of reorganization as opposed to liquidation.

**Mortgage bonds** can be issued in a series in a blanket arrangement. In this case, one group of bonds is issued under the mortgage, and then others are issued later. When earlier issues mature, additional bonds are then issued in their place.

**Collateral trust bonds** are backed by stocks, notes, bonds, or other similar obligations that the company owns. The underlying assets are called the collateral or personal property. The issuers are holding companies, and the collateral consists of claims on their subsidiaries.

The trustee holds the collateral for the benefit of the bondholders; however, the issuer retains voting rights for stock used as collateral, so they retain control over their subsidiaries. The indenture may have provisions covering what to do if the value of the collateral falls below the value of the loan. If the collateral falls in value, the issuer may have to contribute additional securities to back the bonds. The issuer may be able to withdraw collateral if the value rises in order to exceed the loan value. Like mortgage bonds, collateral trust bonds may be issued in series.

**Equipment trust certificates (ETCs)** are a variation of a mortgage bond where a particular piece of equipment underlies the bond. The usual arrangement is that the borrower does not actually purchase the equipment. Instead, the trustee purchases the equipment and leases it to the user of the equipment (the effective borrower), who pays rent on the equipment, and that rent is passed through to the holders of the ETCs. The payments to the creditors are called dividends. The trustee pays for the equipment with the money raised from the issuance of the ETCs, usually about 80% of the value of the equipment, and what is effectively a down payment from the user of the equipment. This provides more security to the creditors than that of a mortgage bond. It is especially attractive if the equipment is standardized, as in the case of railroad cars, which provides for easy sale or lease of the equipment in the case the user of the equipment defaults. ETCs are generally considered the most secure type of bond since the underlying assets are actually owned by the trustee and rented to the borrower.

As noted earlier, **debentures** are unsecured bonds (i.e., they do not have any assets underlying the issue). Most corporate bonds are debentures and usually pay a higher interest rate for that reason. However, if the company is highly rated and has not issued any secured bonds, then debentures are almost the equivalent of mortgage bonds in that they have a claim on all the assets of the issuer along with the general creditors. If the issuer has issued secured debt along with debentures, the debenture holders have a claim on the assets that are not backing the secured debt. Typically the issuer is restricted to one issue of debentures if there is already secured debt. If there is no secured debt, and the company issues debentures, there is often a negative-pledge clause that says that the debentures will be secured equally with any secured bonds that may be issued in the future.

**Subordinated debenture bonds** have a claim that is at the bottom of the list of creditors if the issuer goes into default. They are bonds that are unsecured and have another unsecured bond with a higher claim above them. This means that the issuer has to offer a higher interest rate on the subordinated debentures.

Issuers may choose to issue **convertible debentures**, which give the bondholder the right to convert the bond into common stock. This feature will lower the interest rate paid. The cost to the issuer, however, is the possibility of increased dilution of the stock. A variation of convertible debentures is **exchangeable debentures** that are convertible into the common stock of a corporation other than that of the issuer.

Bonds issued by one company may also be guaranteed by other companies. These bonds are known as **guaranteed bonds**. A guarantee does not ensure that the issue will be free of default risk since the risk will depend on the ability of the guarantor(s) to satisfy all obligations.

## METHODS FOR RETIRING BONDS

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### LO 50.6: Describe the mechanisms by which corporate bonds can be retired before maturity.

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There are a variety of methods for retiring debt, and some are included in the bond's indenture while others are not included. The indenture would include the call and refunding provisions, sinking funds, maintenance and replacement funds, and redemption through sale of assets. The indenture would not include fixed-spread tender offers.

Call and refunding provisions are essentially call options on the bonds that the issuer owns and give the issuer the right to purchase at a fixed price either in whole or in part prior to maturity. These provisions allow a firm to call back debt that has a high coupon and reissue debt with a lower coupon. Other reasons for exercising these options are to eliminate restrictive covenants, alter capital structure, increase shareholder value, or improve financial/managerial flexibility. A **call provision** can either be a fixed-price call or a make-whole call.

- **Fixed-price call.** The firm can call back the bonds at specific prices that can vary over the life of the bonds as specified in the indenture. They generally start out high and decline toward par. Also, for most bonds, the bonds are not callable during the first few years of the issue's life.
- **Make-whole call.** In this case, market rates determine the call price, which is the present value of the bond's remaining cash flows subject to a floor price equal to par value. A discount rate based on the yield of comparable-maturity Treasury securities (usually the rate plus a premium) determines the present value and the bond's price. The redemption price is the greater of that present value or the par value plus accrued interest.

A **sinking fund provision** generally means the issuing firm retires a specified portion of the debt each year as outlined in the indenture. The bonds can either be retired by use of a lottery where the owners of the selected bonds must redeem them, or the bonds are purchased in the open market. The purchase of some sufficient amount of equipment in excess of the value of the amount of the bonds to be retired is another action that may satisfy a sinking fund provision.

The lottery approach to satisfying the sinking fund is very similar to a call provision in that the bondholders must sell back their bonds at a specified price. Unlike the call provision, there may be advantages to the bondholders. First, the retirement of bonds improves the financial health of the firm. Second, the redemption price may exceed the market price. However, the indenture may give the issuer some flexibility as to how much of the bonds to call back each period, which would give the firm some latitude to call back more bonds when the market conditions are favorable to do so. One example is an accelerated sinking-fund provision, which allows the firm to call back more bonds in early years, which the firm would do if interest rates fall in those early years.

A **maintenance and replacement fund (M&R)** has the same goal as a sinking fund provision, which is to maintain the credibility of the property backing the bonds. The provisions differ in that the M&R provision is more complex since it requires valuation formulas for the underlying assets. The main point is that the provision specifies that the fund must keep up the value of the underlying assets much like a home mortgage specifies the home buyer must keep up the value of the home. One way to satisfy the provision is



to acquire sufficient cash to maintain the health of the firm. That cash can then be used to retire debt.

**Tender offers** are usually a means for retiring debt for most firms. The firm openly indicates an interest in buying back a certain dollar amount of bonds or, more often, all of the bonds at a set price. The goal is to eliminate restrictive covenants or to use excess cash. If the first tender offer price does not get sufficient interest, the firm can increase its offer price. Firms can also announce that they will buy back bonds based on the price as determined by a certain market interest rate (e.g., the yield to maturity on a comparable-maturity Treasury plus a spread). This lowers interest rate risk for both the bondholders and the bond issuer.

As a final note, the issuing firm may be able to call back bonds if it is necessary to sell assets associated with the bond issue. For example, if the government requires a firm to sell property, but that property is being used as collateral for the bonds, the firm would sell the property and call back the bonds.

## CREDIT RISK

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### LO 50.7: Differentiate between credit default risk and credit spread risk.

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Credit risk includes credit default risk and credit spread risk. **Credit default risk** is the uncertainty concerning the issuer making timely payments of interest and principal as prescribed by the bond's indenture. The most widely-used indicators of this risk are bond ratings that major rating agencies assign when those agencies perform credit analysis of a firm. Fitch Ratings, Moody's, and Standard & Poor's are the main rating agencies in the United States. The agencies assign a symbol associated with the rating (e.g., AAA or Aaa for the corporate debt with the least credit default risk). The rating can be interpreted as a probability of default within some time period.

**Credit spread risk** focuses on the difference between a corporate bond's yield and the yield on a comparable-maturity benchmark Treasury security. This difference is known as the **credit spread**. It should be noted that other factors such as embedded options and liquidity factors can affect this spread; therefore, it is not only a function of credit risk.

The risk here is from possible changes in this spread from changes in investor risk aversion, which will change the value of the associated bond. Other factors affecting credit spreads are macroeconomic forces such as the level and slope of the Treasury yield curve, the business cycle, and issue-specific factors such as the corporation's financial position and the future prospects of the firm and its industry.

A method commonly used to evaluate credit spread risk is **spread duration**. The duration of the spread is the approximate percentage change in a bond's price for a 100 basis point change in the credit spread assuming that the Treasury rate is constant. If a bond has a spread duration of 4, for example, a 50 basis point change in the spread will change the value of the bond by 2%.

## EVENT RISK

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### LO 50.8: Describe event risk and explain what may cause it in corporate bonds.

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Event risk addresses the adverse consequences from possible events such as mergers, recapitalizations, restructurings, acquisitions, leveraged buyouts, and share repurchases, which may escape being included in the indenture. Such events can drastically change the firm's capital structure and reduce the creditworthiness of the bonds and their value. In order to protect shareholders, a company may include in the indenture a **poison put**, which can require the company to repurchase the debt at or above par value in the event of a takeover not approved by the board of directors (i.e., a hostile takeover). The purpose of this feature is to protect bondholders, but its effectiveness toward this goal can be misleading in that the acquiring firm may offer a sufficiently high price for the stock so that the hostile takeover becomes friendly. As a result, the poison puts would not be exercised.

Investors can lobby for clauses in the indenture to activate a put option for a variety of reasons including a change in the bond's rating. Some of the debt rating services issue commentary on the indenture's protective features, which could include the possibility of the firm being able to circumvent the features through careful legal moves (e.g., turning a hostile takeover into a friendly takeover). It should be noted that event risk can change on a market level. During times of increased merger activity, for example, the event risk increases for most bonds.

## HIGH-YIELD BONDS

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### LO 50.9: Define high-yield bonds, and describe types of high-yield bond issuers and some of the payment features unique to high yield bonds.

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High-yield bonds (a.k.a. junk bonds) are those bonds rated below investment grade by ratings agencies. This includes a broad range of ratings below the cutoff, (e.g., BB to default). **Businessman's risk** refers to bonds with a rating at the bottom rung of the investment-grade category (Baa and BBB) or at the top end of the speculative-grade category (Ba and BB). Over long periods of time, high-yield bonds should offer higher average returns. However, over shorter periods, the returns will be volatile where large losses are possible.

There are many types of high-yield bonds. One type includes companies who issue bonds with a non-investment grade rating. Such issuers include young and growing companies that do not have strong financial statements but who have promising prospects. Firms may issue such bonds to raise venture capital, and their prospects are tied to a particular project or story, which gives them the name "story bonds."

Established firms that have had a deteriorating financial situation may need to raise debt capital as well, and they would issue bonds that reflect their situation. Also, an established firm who already has unsecured debt issued with an investment-grade credit rating may be able to issue subordinated debt, but that debt would be non-investment grade.

**Fallen angels** are another type of high-yield bond. They are bonds that were issued with an investment-grade rating, but then events led to the ratings agencies lowering the rating to below investment grade. If the issuers are in or near bankruptcy, they are often called “special situations,” which could either pay off if the company recovers or lead to big losses.

**Restructurings** and **leveraged buyouts** may increase the credit risk of a company to the point where the bonds become non-investment grade. The new management may pay high dividends, deplete the acquired firm’s cash, and lower the rating of the existing bonds. In this process, the firm may issue non-investment grade debt to pay off the bridge loans taken to finance the acquisition.

High-yield bonds can have several types of coupon structures. There are **reset bonds**, where designated investment banks periodically reset the coupon to reflect market rates and the creditworthiness of the issuer. There are also **deferred-coupon structures**, which include three types: (1) deferred-interest bonds, (2) step-up bonds, and (3) payment-in-kind bonds. Deferred-interest bonds sell at a deep discount and do not pay interest in the early years of the issue, say, for three to seven years. Step-up bonds pay a low coupon in the early years and then a higher coupon in later years. Payment-in-kind bonds allow the issuer to pay interest in the form of additional bonds over the initial period.

## DEFAULT RATE

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### LO 50.10: Define and differentiate between an issuer default rate and a dollar default rate.

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A default occurs if there are any missed or delayed disbursements of interest and/or principal. It has been proven that lower credit ratings indicate a higher probability of default, but there are two ways to measure default: by the raw number of issuers that defaulted or the dollar amount of issues that defaulted. For each approach in measuring default rates, there are different formulas, which can lead to researchers reporting different default rates for the same data set.

The **issuer default rate** is the number of issuers that defaulted over a year divided by the total number of issuers at the beginning of the year. It is only a proportion of the number of issuers who do fulfill their obligations and does not include a measure of the dollar amount involved.

The **dollar default rate** is the par value of all bonds that defaulted in a given calendar year divided by the total par value of all bonds outstanding during the year. Over a multi-year period, often-used measures are ratios of cumulative dollar value of all defaulted bonds divided by some weighted-average measure of all bonds issued. One such measure attempts to weight the bonds outstanding by the number of years they are in the market:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{(\text{cumulative dollar value of all issuance}) \times (\text{weighted average \# of years outstanding})}$$



Another measure simply takes a raw total as shown in the following equation:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{\text{cumulative dollar value of all issuance}}$$

## RECOVERY RATE

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### LO 50.11: Define recovery rates and describe the relationship between recovery rates and seniority.

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The recovery rate is the amount received as a proportion of the total obligation after a bond defaults. Measuring this can be complicated because the value of the total obligation requires computing the present value of the remaining cash flows at the time of the default. Furthermore, some of the amount that the investor recovers may be in the form of securities (e.g., stock in the company). A study by Moody's estimated that the recovery rate for bonds has been about 38%. Bonds with higher seniority will obviously have higher recovery rates.

## KEY CONCEPTS

### LO 50.1

A bond indenture sets forth the obligations of the issuer. The trustee interprets the legal language of the indenture and works to make sure the issuer fulfills obligations to bondholders.

### LO 50.2

The bond issuer's obligations are fulfilled on the maturity date or before. Bonds can be retired before that date.

### LO 50.3

The main types of interest payment classifications are straight-coupon bonds, zero-coupon bonds, and floating-rate bonds. Straight-coupon bonds pay a fixed cash coupon periodically. Floating-rate bonds pay a cash amount that varies with market rates. Zero-coupon bonds increase in value over the life of the issue.

There are many variations of the main types of bond structures. For example, deferred-interest bonds are a mix of zero-coupon and coupon bonds in that they do not pay cash interest in early years and pay a cash coupon in later years. Some bonds have principal in one currency and pay coupons in another currency.

### LO 50.4

Zero-coupon bonds have low reinvestment risk. The interest is based on the time-to-maturity at issuance and the original-issue discount, which is the difference between the face value and the offering price. In the case of bankruptcy, the bondholder has a claim only equal to the issue price plus accrued interest to that date, and not the full face value.

### LO 50.5

The holder of a mortgage bond has the first lien on real property owned by the issuer.

Collateral trust bonds are backed by stocks and bonds that represent claims against the subsidiaries of the issuer. The collateral is also called personal property.

Equipment trust certificates are a form of mortgage bond where the trustee actually owns the property and rents it to the bond issuer. The property is often in the form of standardized equipment (e.g., rail cars) that is easily sold.

Debentures are unsecured debt. Owners of debentures have a claim on the company's assets not backing outstanding secured debt.

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### LO 50.6

Call provisions allow the firm to retire debt early at a given price. Sinking-fund provisions require the firm to buy back portions of debt each year. Call provisions are generally considered detrimental to bondholders, but sinking-fund provisions may be beneficial to bondholders.

A maintenance and replacement fund helps maintain the financial health of the firm. Cash in the fund can be used to retire debt.

Bond issuers can retire debt through a tender offer. The offer price may either be a fixed price or a price that varies with a market rate such as that on comparable Treasury securities.

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### LO 50.7

Credit default risk is the possibility that the issuer does not make the payments specified in the indenture. Credit spread risk is the price risk from changes in the spread of a bond's interest rate over the corresponding Treasury rate.

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### LO 50.8

Event risk is the possibility that a merger, restructuring, acquisition, et cetera, increases the risk of the bond by changing the ability of the firm to pay off the bonds. The indenture can try to address some of these events, but some can be omitted and lawyers can find loopholes around those included.

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### LO 50.9

High-yield bonds can either be issued by growing, risky firms or established firms with senior debt outstanding. High-yield bonds may also be fallen angels (i.e., one-time investment grade bonds).

High-yield bonds may have coupon structures which allow the firm to conserve cash in early years, such as: (1) deferred-interest bonds, (2) step-up bonds, and (3) payment-in-kind bonds.

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### LO 50.10

The issuer default rate is a proportion based on the number of issues that default as a proportion of all issues. The dollar default rate estimates the dollar amount of defaulted bonds compared to the dollar amount of the corresponding population of bonds outstanding.

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### LO 50.11

In the event of default, the recovery rate refers to the amount a bondholder receives as a proportion of the amount owed. Bonds with higher seniority usually have higher recovery rates.

**CONCEPT CHECKERS**

1. Which of the following responsibilities is least likely to be part of the role of a corporate trustee in a bond issue?
  - A. Interpret the language of the indenture.
  - B. Determine the interest rate on a reset bond.
  - C. Keep track of the amount of bonds issued by the corporation.
  - D. Monitor the corporation's activities to make sure the corporation abides by the indenture's covenants.
2. In bankruptcy, the holder of a zero-coupon bond obligation of the bankrupt corporation would have a claim equal to:
  - A. the face value of the bond.
  - B. the issuing price of the bond only.
  - C. the issuing price plus accrued interest.
  - D. nothing, since zeros are always unsecured.
3. All other things being equal, which of the following types of bond instruments would have the lowest interest rate?
  - A. Equipment trust certificates.
  - B. Mortgage bonds.
  - C. Junior debentures.
  - D. Senior debentures.
4. Which of the following methods for retiring bonds before maturity is generally considered the most detrimental for the bondholders?
  - A. Tender offers.
  - B. Call provision.
  - C. Sinking fund provision.
  - D. Maintenance and replacement funds.
5. With respect to default risk and credit spread risk, the ratings of bond-rating agencies such as Moody's provide information concerning:
  - A. default risk only.
  - B. credit spread risk only.
  - C. both default risk and credit spread risk.
  - D. neither default risk nor credit spread risk.

## CONCEPT CHECKER ANSWERS

1. **B** Investment banks other than the trustee set the rate on a reset bond.
2. **C** The claim equals the value at that point in time as implied by the issuing price, the original-issue discount, and accrued interest.
3. **A** ETCs, or equipment trust certificates, are generally the most secure because the underlying assets are actually owned by the trustee and rented to the borrower. Also, the assets are usually standardized for easy resale.
4. **B** The call provision gives the issuer the right to purchase the bonds at a given price, which the issuer would not do unless that price was below the market price. Sinking fund provisions can benefit bondholders because the issuer is obligated to purchase bonds, which improves the creditworthiness of the issue, and the issuer may have to do so at a price higher than the market price. There are no features in M&R funds or tender offers that would be detrimental to bondholders.
5. **A** Bond rating agencies issue ratings based on the default risk of the issue. Credit spread risk is determined by spread duration.

# MORTGAGES AND MORTGAGE-BACKED SECURITIES

Topic 51

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## EXAM FOCUS

Mortgage-backed securities (MBSs) are debt securities backed by a pool of residential loans, which serve to transform mortgages from an illiquid asset into a liquid asset. Because the underlying mortgages can be prepaid, prepayment risk is a major concern for MBS investors. Monte Carlo simulation is the most common methodology used for valuing MBSs because it is able to account for prepayment risk. Alternate interest rate paths are assumed in the model to generate an option-adjusted spread (OAS). For the exam, be able to calculate the payments for a fixed-rate, level paying mortgage. Also, be familiar with the factors that affect prepayment rates and how to measure prepayment speeds with a conditional prepayment rate (CPR). Finally, be prepared to discuss the steps involved in valuing an MBS using the Monte Carlo methodology and understand the advantages and disadvantages of using an OAS.

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## RESIDENTIAL MORTGAGE PRODUCTS

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### LO 51.1: Describe the various types of residential mortgage products.

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A **mortgage** is a loan that is collateralized with a specific piece of real property. Before the 1970s, mortgages existed solely in the **primary market** where banks that issued the mortgage loans collected all interest and principal payments from the borrower. Within the past few decades, it is more common for mortgage lenders to sell the loans in the **secondary market** through a process known as **securitization**. The secondary market has allowed more banks to issue mortgage loans.

In the secondary market, mortgages are pooled together and packaged to investors in the form of a **mortgage-backed security (MBS)**. The payments of an MBS can follow a **pass-through structure** where the interest and principal collected from the borrower pass through the banks and ultimately end up with the MBS investor. Because default risk is present in mortgage lending, banks will often guarantee the borrower's payments when mortgages are securitized.

### Lien Status

Whether the mortgage is a first lien, a second lien, or a subsequent lien will greatly impact the lender's ability to recover the balance owed in the event of default. For example, a first lien would give the lender the first right to receive proceeds on liquidation, so from a seniority perspective, a first lien is more desirable than a second lien.

## Original Loan Term

Mortgage terms of 10 to 30 years are common, with the most popular being 30 years (long term). However, medium terms in the 10- to 20-year range are starting to become more common, given the desire of many individuals to pay off their mortgages as soon as possible.

## Credit Classification

Classifying loans between prime and subprime is determined mainly by credit score (i.e., Fair Isaac Corporation or FICO model).

**Prime (A-grade) loans** constitute most of the outstanding loans. They have low rates of delinquency and default as a result of low **loan-to-value (LTV) ratios** (i.e., far less than 95%), borrowers with stable and sufficient income (i.e., *front income ratio* of no more than 28% of monthly income to service payments relating to the home and *back income ratio* of no more than 36% for those payments plus other debt payments), and a strong history of repayments (e.g., FICO score of 660 or greater). Home payments include interest, principal, property taxes, and homeowners insurance.

**Subprime (B-grade) loans** have higher rates of delinquency and default compared to prime loans. They could be associated with high LTV ratios (i.e., 95% or above), borrowers with lower income levels, and borrowers with marginal or poor credit histories (e.g., FICO score below 660). High LTV ratios suggest a higher risk of default. Upon issuance, subprime loans are carefully scrutinized by the servicer to ensure timely payments.

**Alternative-A loans** are the loans in between prime and subprime. Although they are essentially prime loans, certain characteristics of Alternative-A loans make them riskier than prime loans. For example, the loan value may be unusually high, the LTV ratio may be high, or there may be less documentation available (e.g., income verification, down payment source).

## Interest Rate Type

**Fixed-rate mortgages** have a set rate of interest for the term of the mortgage. Payments are constant for the term and consist of blended amounts of interest and principal.

**Adjustable-rate mortgages (ARMs)** have rate changes throughout the term of the mortgage. The rate is usually based on a base rate (e.g., prime rate, LIBOR) plus a spread. Rates can usually change on a monthly, semiannual, or annual basis. The risk of default is high, especially if there are large rate increases after the first year, thereby significantly increasing the total payment amount (due to the increase in interest).



## Prepayments and Prepayment Penalties

Prepayments reduce the mortgage balance and amortization period. They can occur because of the following reasons:

- Home is sold, which requires the mortgage balance to be paid off.
- Refinancing due to lower rates or more attractive loan features elsewhere.
- Partial prepayments by the borrower during the term.

To counteract the negative effects of prepayments, many loans contain prepayment penalties. They are amounts payable to the servicer for prepayments within a certain time and/or over a certain amount. Soft penalties are those that may be waived on the sale of the home; hard penalties may not be waived.

## Credit Guarantees

The ability to create mortgage-backed securities requires loans that have credit guarantees.

**Government loans** are those that are backed by federal government agencies (e.g., Government National Mortgage Association or GNMA).

**Conventional loans** could be securitized by either government-sponsored enterprises (GSEs): Federal Home Loan Mortgage Corporation (FHLMC) or Federal National Mortgage Association (FNMA). For a guarantee fee, these GSEs will guarantee payment of principal and interest to the investors.

Agency (or conforming) MBSs are those that are guaranteed by any of three government-sponsored entities (GSEs): GNMA, FNMA, and FHLMC. Most of the MBSs are issued by these GSEs.

Also known as **private label** securitizations, non-agency (or non-conforming) MBSs grew along with U.S. home prices over time up to the 2007 credit crisis. The GSEs have restrictions on what mortgages they can guarantee/securitize [e.g., dollar value limit, loan-to-value (LTV) ratio limit], which opened up the private label market for those participants willing to take on the risks inherent in nonconventional loans—**jumbo loans** (mortgage principal balance over the limit) and/or loans with high LTVs. The rising prices of the underlying homes held as collateral provided some risk mitigation. Unfortunately, the falling prices of homes and the credit crisis beginning in 2007 caused a significant drop in MBS issuances in the non-agency segment because they did not have government guarantees.

With agency MBSs, the investor bears no credit risk because the GSEs have been paid a fee to guarantee the underlying mortgages. If there is a default with a mortgage, the GSE will pay the outstanding balance to the investors. With a non-agency MBS, there is some credit risk but that is mitigated through the process of subordination.

## FIXED-RATE, LEVEL-PAYMENT MORTGAGES

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### LO 51.2: Calculate a fixed rate mortgage payment, and its principal and interest components.

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A **conventional mortgage** is the most common residential mortgage. The loan is based on the creditworthiness of the borrower and is collateralized by the residential real estate that it is used to purchase. If a borrower's credit quality is questionable or the borrower is lacking a sufficient down payment, the mortgage lender may require mortgage insurance to guarantee the loan. Mortgage insurance is made available by both government agencies and private insurers. The cost of the insurance is borne by the borrower and effectively raises the interest rate on the mortgage loan.

There are a wide variety of mortgage designs that specify the rates, terms, amortization, and repayment methods. All of the concepts associated with risk analysis and valuation, however, can be understood through an examination of **fixed-rate, level payment, fully amortized mortgage loans**. This common type of mortgage loan requires equal payments (usually monthly) over the life of the mortgage. Each of these payments consists of an interest component and a principal component.

There are four important features of fixed-rate, level payment, fully amortized mortgage loans to remember when we move on to mortgage-backed securities (MBS):

1. The amount of the principal payment increases as time passes.
2. The amount of interest decreases as time passes.
3. The servicing fee also declines as time passes.
4. The ability of the borrower to repay results in **prepayment risk**. Prepayments and curtailments reduce the amount of interest the lender receives over the life of the mortgage and cause the principal to be repaid sooner.

#### Example: Calculating a mortgage payment

Consider a 30-year, \$500,000 level payment, fully amortized mortgage with a fixed rate of 12%. Calculate the monthly payment and prepare an amortization schedule for the first three months.

**Answer:**

The monthly payment is \$5,143.06:

$$N = 360; I/Y = 1.0 (12/12); PV = -500,000; FV = 0; CPT \rightarrow PMT = 5,143.06$$

With reference to the partial amortization schedule in the figure below, the portion of the first payment that represents interest is \$5,000.00 ( $0.01 \times \$500,000$ ). The remainder of the payment, \$143.06 ( $\$5,143.06 - \$5,000.00$ ), goes toward the reduction of principal. The portion of the second payment that represents interest is \$4,998.57 ( $0.01 \times \$499,856.94$ ). The remaining \$144.49 ( $\$5,143.06 - \$4,998.57$ ) goes toward the further reduction of principal.

**Monthly Amortization Schedule for a 30-Year, \$500,000 Mortgage Loan at 12%**

<i>Payment Number</i>	<i>Initial Principal</i>	<i>Monthly Payment</i>	<i>Interest Component</i>	<i>Reduction of Principal</i>	<i>Outstanding Principal</i>
1	\$500,000.00	\$5,143.06	\$5,000.00	\$143.06	\$499,856.94
2	499,856.94	5,143.06	4,998.57	144.49	499,712.45
3	499,712.45	5,143.06	4,997.12	145.94	499,566.51

Notice that the monthly interest charge is based on the beginning-of-period outstanding principal. As time passes, the proportion of the monthly payment that represents interest decreases, and, because the payment is level, the proportion that goes toward the repayment of principal increases. This process continues until the outstanding principal reaches zero and the loan is paid in full.

The incremental reduction of outstanding principal is referred to as scheduled amortization (or scheduled principal repayment). The previous figure is a portion of what is commonly called an **amortization schedule**. Amortization schedules are easily constructed using an electronic spreadsheet.

The collection of payments and all of the other administrative activities associated with mortgage loans are paid for via a servicing fee, also known as the servicing spread, because it is usually built into the mortgage rate.

For example, if the mortgage rate is 10.5% and the servicing fee is 35 basis points, the provider of the mortgage funds will receive 10.15%. This amount is called the net interest or net coupon. The dollar amount of the servicing fee is based on the outstanding loan balance; thus, it declines as the mortgage is amortized. Keep in mind that the reduction in principal associated with each payment is based on the mortgage rate and is unaffected by the servicing fee.

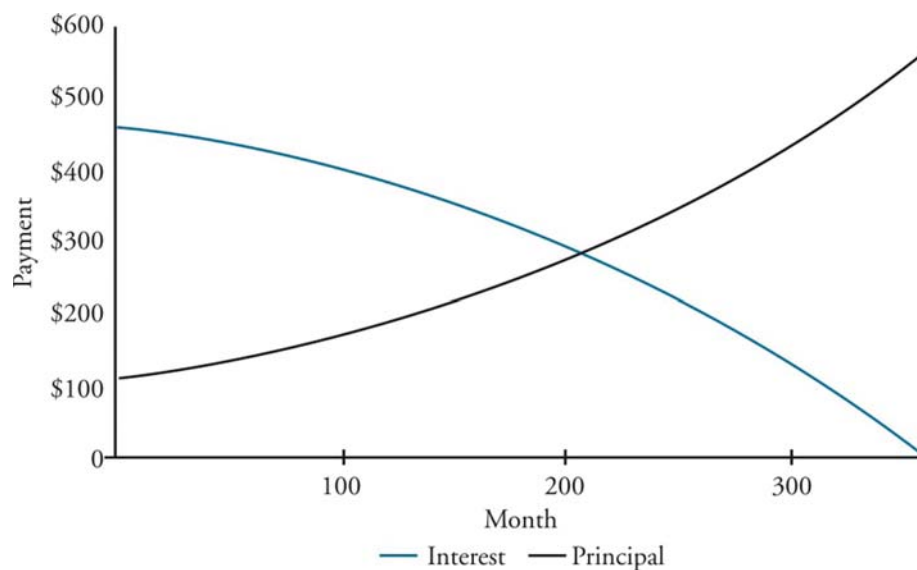
## Allocation Between Principal and Interest

Fully amortizing fixed-rate mortgage:

- The mortgage payment consists primarily of interest in the early years.
- Interest is calculated on a declining principal balance so the interest payable will gradually decrease over time. As a result, more of the fixed mortgage payment will be applied toward reducing the principal amount.
- The crossover point is the point in the mortgage where principal and interest allocation amounts are the same. After that point, relatively more amounts will be allocated to principal.
- Mortgages with shorter amortization periods result in less interest paid and more of the payment applied toward reducing the principal balance sooner. In other words, equity buildup occurs at a quicker rate when the amortization period is shorter.

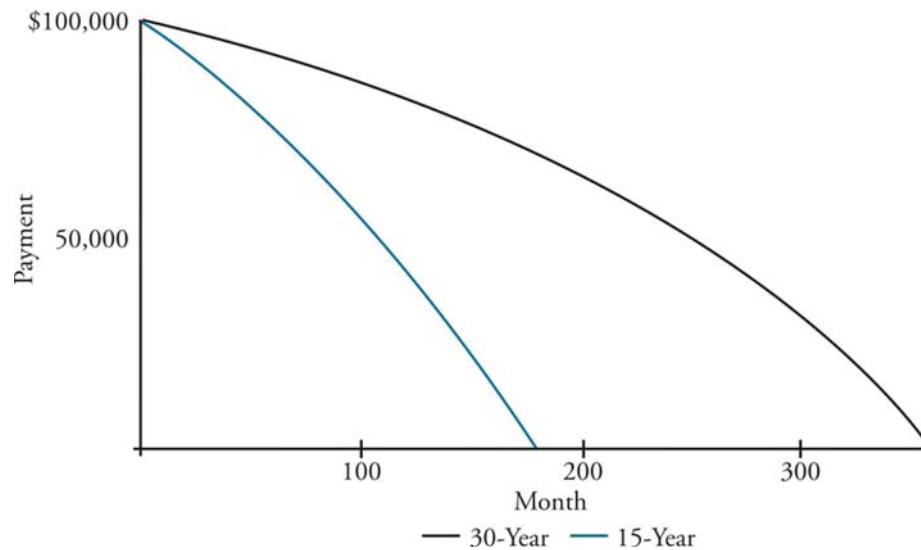
Figure 1 illustrates the relationship of interest and principal over the term of the loan.

Figure 1: Interest and Principal Over Time



As indicated previously, the reason principal payments will increase over time is due to the reduction in the outstanding loan balance. Figure 2 illustrates the relationship between loan balance and time for a \$100,000 loan.

Figure 2: Loan Balance Over Time



## PREPAYMENT

### LO 51.3: Describe the mortgage prepayment option and the factors that influence prepayments.

In the previous example, it was assumed that the borrower paid the exact amount of the monthly payment, and the interest and principal followed the amortization schedule. However, it is possible for a borrower to pay an amount in excess of the required payment or even to pay off the loan entirely. The option to prepay a mortgage is essentially a call option for the borrower. The borrower is in a position that is very similar to the issuer of a callable bond. A prepayment will effectively free the borrower of the mortgage obligation.

### Mortgage Prepayment Option

Mortgage prepayments come in two forms: (1) increasing the frequency or amount of payments (where permitted) and (2) repaying/refinancing the entire outstanding balance. Prepayments are much more likely to occur when market interest rates fall and borrowers wish to refinance their existing mortgages at a new and lower rate. For the lender, prepayments represent a loss for two reasons: (1) they stop receiving interest income at the high rate and (2) they have to reinvest the proceeds received from prepayment at the prevailing lower market rates. Therefore, the pricing of the initial mortgage rate should be somewhat higher to take into account the possibility of prepayment. With agency MBSs, prepayments and defaults have the same impact on investors. Prepayments result in the investors actually receiving cash from the borrowers, whereas with defaults, the borrower does not pay the outstanding mortgage balances, but the GSE does, thereby causing a prepayment.

## Other Factors That Influence Prepayments

**Seasonality.** The summertime is a popular time for individuals to move (and mortgages must be paid out prior to the sale of a home), so it is the period of time with the greatest prepayment risk. Given some time lags, the prepayments often start to appear in the late summer and early fall.

**Age of mortgage pool.** Refinancing often involves penalties and administrative charges, so borrowers tend not to do so until several years into the mortgage. Also, it takes some time for borrowers to build up equity and savings to make prepayments and/or attempt to refinance. As a result, the lower the age of the mortgage pool, the less likely the risk of prepayment.

**Personal.** Marital breakdown, loss of employment, family emergencies, and destruction of property are commonly cited reasons for prepayments based on personal reasons. It is difficult to assess this type of prepayment risk.

**Housing prices.** Property value increases may spur an increase in prepayments caused by borrowers wanting to take out some of the increased equity for personal use. Property value decreases reduce the value of collateral, reduce the ability to refinance, and, therefore, decrease the risk of prepayment. The increasingly popular use of home equity lines of credit where the mortgage balance is revolving (i.e., mortgage balance can be drawn up to a certain limit and paid down to zero at any time) reduces refinancing and prepayment risk due to the nature of the loan.

**Refinancing burnout.** To the extent that there has been a significant amount of prepayment or refinancing activity in the mortgage pool in the past, the risk of prepayment in the future decreases. That is because presumably the only borrowers remaining in the pool are those who were unable to refinance earlier (e.g., due to poor credit history or insufficient property value), and those who did refinance have been removed from the pool already. Also, those who made only large prepayments (instead of fully refinancing) in the past would have exhausted their savings to make the prepayment and would require quite some time to do so again in the future.

## SECURITIZATION

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**LO 51.4: Summarize the securitization process of mortgage backed securities (MBS), particularly formation of mortgage pools including specific pools and TBAs.**

**LO 51.5: Calculate weighted average coupon, weighted average maturity, and conditional prepayment rate (CPR) for a mortgage pool.**

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To reduce the risk from holding a potentially undiversified portfolio of mortgage loans, a number of financial institutions (i.e., originators) will work together to pool residential mortgage loans with similar characteristics into a more diversified portfolio. They will then sell the loans to a separate entity, called a **special purpose vehicle (SPV)**, in exchange for



cash. An issuer will purchase those mortgage assets in the SPV and then use the SPV to issue MBSs to investors; the securities are backed by the mortgage loans as collateral.

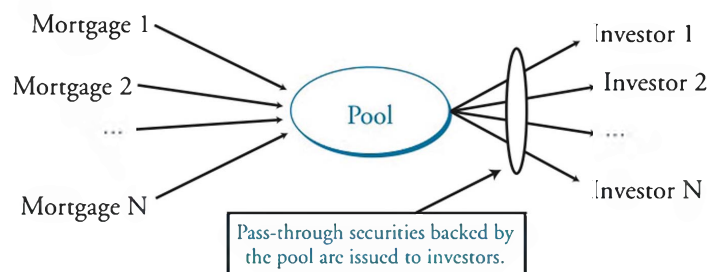
As of now, the securitization process has become a crucial part of the U.S. credit system. Financial institutions expect to originate mortgage loans and sell them through securitization. The lack of a securitization market for mortgages would lead to the downfall of mortgage lending because financial institutions would not want to retain the risks.

## PASS-THROUGH SECURITIES

A **mortgage pass-through security** represents a claim against a pool of mortgages. Any number of mortgages may be used to form the pool and any mortgage included in the pool is referred to as a **securitized mortgage**. The mortgages in the pool have different maturities and different mortgage rates. The **weighted average maturity (WAM)** of the pool is equal to the weighted average of all mortgage ages in the pool, each weighted by the relative outstanding mortgage balance to the value of the entire pool. The **weighted average coupon (WAC)** of the pool is the weighted average of the mortgage rates in the pool. The investment characteristics of a mortgage pass-through are a function of its cash flow features and the strength of its government guarantee.

As illustrated in Figure 3, pass-through security investors receive the monthly cash flows generated by the underlying pool of mortgages, less any servicing and guarantee/insurance fees. The fees account for the fact that **pass-through rates** (i.e., the coupon rate on the pass-through) are less than the average coupon rate of the underlying mortgages in the pool.

**Figure 3: Mortgage Pass-through Cash Flow**



Because pass-through securities may be traded in the secondary market, they effectively convert illiquid mortgages into liquid securities (as mentioned, this process is called **securitization**). More than one class of pass-through securities may be issued against a single mortgage pool.

The timing of the cash flows to pass-through security holders does not exactly coincide with the cash flows generated by the pool. This is due to the delay between the time the mortgage service provider receives the mortgage payments and the time the cash flows are “passed through” to the security holders.



The most important characteristic of pass-through securities is their prepayment risk; because the mortgages used as collateral for the pass-through can be prepaid, the pass-throughs themselves have significant prepayment risk.

### Measuring Prepayment Speeds

Prepayments cause the timing and amount of cash flows from mortgage loans and MBSs to be uncertain; they speed up principal repayments and reduce the amount of interest paid over the life of the mortgage. Thus, it is necessary to make specific assumptions about the rate at which prepayment of the pooled mortgages occurs when valuing pass-through securities. Two industry conventions have been adopted as benchmarks for prepayment rates: the **conditional prepayment rate (CPR)** and the **Public Securities Association (PSA)** prepayment benchmark.

The *CPR* is the annual rate at which a mortgage pool balance is assumed to be prepaid during the life of the pool. A mortgage pool's CPR is a function of past prepayment rates and expected future economic conditions.

We can convert the CPR into a monthly prepayment rate called the **single monthly mortality rate (SMM)** (also referred to as constant maturity mortality) using the following formula:

$$\text{SMM} = 1 - (1 - \text{CPR})^{1/12}$$

If given the SMM rate, you can annualize the rate to solve for the CPR using the following formula:

$$\text{CPR} = 1 - (1 - \text{SMM})^{12}$$

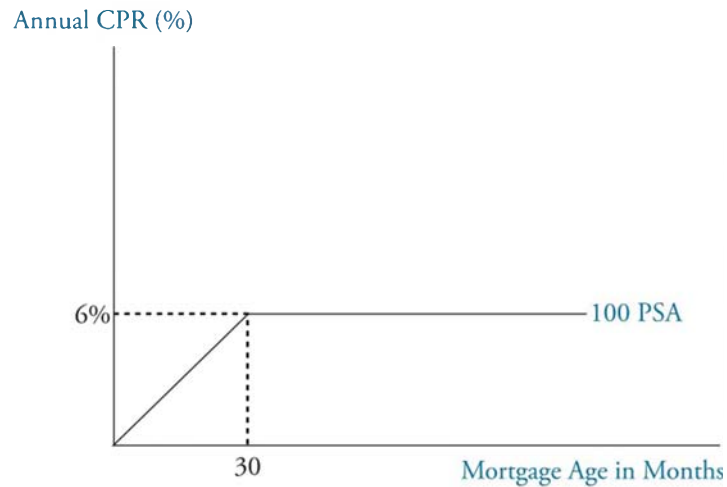
An SMM of 10% implies that 10% of a pool's beginning-of-month outstanding balance, less scheduled payments, will be prepaid during the month.

The *PSA prepayment benchmark* assumes that the monthly prepayment rate for a mortgage pool increases as it ages or becomes seasoned. The PSA benchmark is expressed as a monthly series of CPRs.

The PSA standard benchmark is referred to as 100% PSA (or just 100 PSA). 100 PSA (see Figure 4) assumes the following graduated CPRs for 30-year mortgages:

- CPR = 0.2% for the first month after origination, increasing by 0.2% per month up to 30 months. For example, the CPR in month 14 is  $14(0.2\%) = 2.8\%$ .
- CPR = 6% for months 30 to 360.

Figure 4: 100 PSA



Remember that the CPRs are expressed as annual rates.

A particular pool of mortgages may exhibit prepayment rates faster or slower than 100% PSA, depending on the current level of interest rates and the coupon rate of the issue. A 50% PSA refers to one-half of the CPR prescribed by 100% PSA, and 200% PSA refers to two times the CPR called for by 100% PSA.

#### Example: Computing the SMM

Compute the CPR and SMM for the 5th and 25th months, assuming 100 PSA and 150 PSA.

**Answer:**

Assuming 100 PSA:

$$\begin{aligned}\text{CPR}(\text{month } 5) &= 5 \times 0.2\% = 1\% \\ 100 \text{ PSA} &= 1 \times 0.01 = 0.01 \\ \text{SMM} &= 1 - (1 - 0.01)^{1/12} = 0.000837\end{aligned}$$

$$\begin{aligned}\text{CPR}(\text{month } 25) &= 25 \times 0.2\% = 5\% \\ 100 \text{ PSA} &= 1 \times 0.05 = 0.05 \\ \text{SMM} &= 1 - (1 - 0.05)^{1/12} = 0.004265\end{aligned}$$

Assuming 150 PSA:

$$\text{CPR}(\text{month } 5) = 5 \times 0.2\% = 1\%$$

$$150 \text{ PSA} = 1.5 \times 0.01 = 0.015$$

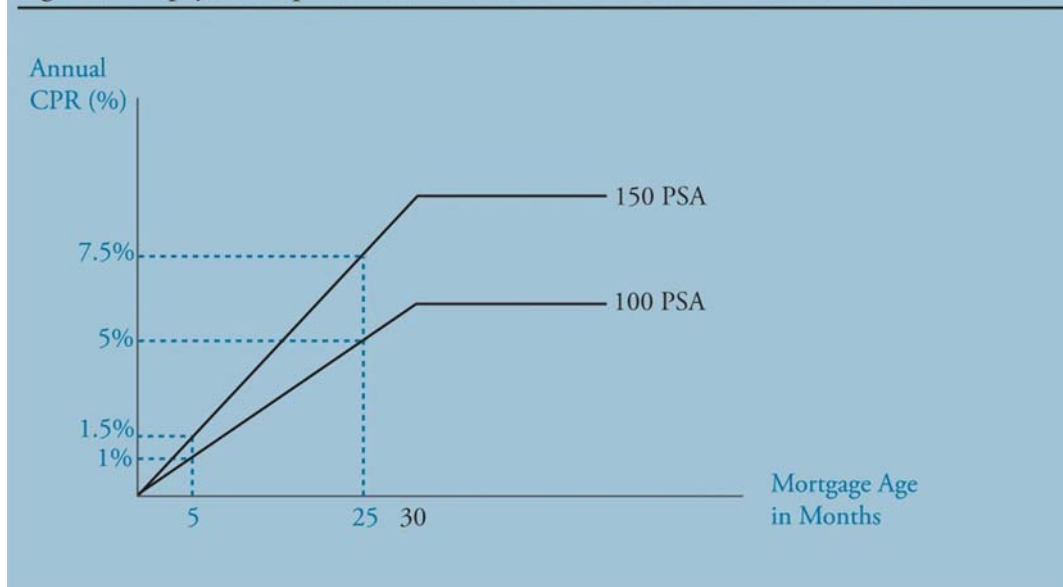
$$\text{SMM} = 1 - (1 - 0.015)^{1/12} = 0.001259$$

$$\text{CPR}(\text{month } 25) = 25 \times 0.2\% = 5\%$$

$$150 \text{ PSA} = 1.5 \times 0.05 = 0.075$$

$$\text{SMM} = 1 - (1 - 0.075)^{1/12} = 0.006476$$

Figure 5: Prepayment Speeds for 5th and 25th Months at 100 and 150 PSA



It is important for you to recognize that the nonlinear relationship between CPR and SMM implies that the SMM for 150% PSA does *not* equal 1.5 times the SMM for 100% PSA. Also, keep in mind that the PSA standard benchmark is nothing more than a market convention. It is not a model for predicting prepayment rates for MBS. In fact, empirical studies have shown that actual CPRs differ substantially from those assumed by the PSA benchmark.

### Trading Pass-Through Securities

Trade settlements occur every month on a predetermined basis; delivery dates during a month are specified. In addition, prices are usually quoted for three settlements months, however, trades could be done for a longer period into the future.

Fixed rate pass-through securities (i.e., agency mortgage pools) trade in one of the following ways:

- Specified pools.
- To Be Announced (TBA).

The **specified pools** market identifies the number and balances of the pools prior to a trade. As a result, the characteristics of a given pool will influence the price of a trade. For example, high loan-balance pools, which make better use of prepayment options, trade for relatively lower prices.

The **TBA** market, which is more liquid than specified pools, involves identifying the security and establishing the price in a forward market. However, there is a pool allocation process whereby the actual pools are not revealed to the seller until immediately before settlement. The characteristics of the pools that can be used for TBA trades are regulated to ensure reasonable consistency.

## DOLLAR ROLL TRANSACTION

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### LO 51.6: Describe a dollar roll transaction and how to value a dollar roll.

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MBS trading requires the same securities to be priced for different settlement dates. A **dollar roll transaction** occurs when an MBS market maker buys positions for one settlement month and, at the same time, sells those same positions for another month.

#### How to Value a Dollar Roll

The process involves assessing the income and the expenses related over the holding period. Income is determined by coupon payments, reinvested interest, and principal payments. Expenses are determined by financing costs [i.e., repurchase (repo) market]. One could purchase the security in the earlier (front) month, hold it, and then dispose of it in the later (back) month at settlement.

The back month price of a dollar roll should take both income and expenses into account so the net cash flows are equivalent to simply purchasing the security in the back month for settlement at that time. However, empirical evidence suggests that the most likely outcome is that a price drop between the two settlement dates makes purchasing the security in the back month more attractive. In other words, purchasing a position for back month settlement results in financing at an implied repo rate lower than that of the repurchase market.

Factors that impact dollar roll valuations:

- The security's coupon, age, and WAC.
- Holding period (period between the two settlement dates).
- Assumed prepayment speed.
- Funding cost in the repo market.

#### Factors Causing a Dollar Roll to Trade Special

When the price difference/drop is large enough to result in financing at less than the implied cost of funds, then the dollar roll is trading *special*. It could be caused by:

- A decrease in the back month price (due to an increased number of sale/settlement transactions on the back month date by originators).

- An increase in the front month price (due to an increased demand in the front month for deal collateral).
- Shortages of certain securities in the market that require the dealer to suddenly purchase the security for delivery in the front month, thereby increasing the front month price.

## OTHER PRODUCTS

### Collateralized Mortgage Obligations

All investors have varying degrees of concern about exposure to prepayment risk. Some are primarily concerned with extension risk (the increase in the expected life of a mortgage pool due to rising interest rates and lower prepayment rates), while others want to minimize exposure to contraction risk (the decrease in the expected life of a mortgage pool due to falling interest rates and higher prepayment rates). Fortunately, all of the pass-through securities issued on a pool of mortgages do not have to be the same. The ability to partition and distribute the cash flows generated by a mortgage pool into different risk packages has led to the creation of **collateralized mortgage obligations** (CMOs).

CMOs are securities issued against pass-through securities (securities secured by other securities) for which the cash flows have been reallocated to different bond classes called *tranches*. Each tranche has a different claim against the cash flows of the mortgage pass-throughs or pool from which it was derived. Each CMO tranche represents a different mixture of contraction and extension risk. Hence, CMO securities can be more closely matched to the unique asset/liability needs of institutional investors and investment managers.

### Planned Amortization Class Tranches

The most common type of CMO today is the **planned amortization class** (PAC). A PAC is a tranche that is amortized based on a sinking fund schedule that is established within a range of prepayment speeds called the *initial PAC collar* or *initial PAC bond*.

What makes a PAC bond work is that it is packaged with a *support*, or *companion*, tranche created from the original mortgage pool. Support tranches are included in a structure with PAC tranches specifically to provide prepayment protection for the PAC tranches (each tranche is, of course, priced according to the timing risk of the cash flows). If prepayment rates are faster than the upper repayment rate, the PAC tranche receives principal according to the PAC schedule, and the support tranche absorbs (i.e., receives) the excess. If prepayment speeds are below the lower repayment rate, the funds needed to keep the PAC on schedule come from the cash flows scheduled for the support tranche(s). It should be pointed out that the extent of prepayment risk protection provided by a support tranche increases as its par value increases relative to its associated PAC tranche.

There is an *inverse* relationship between the prepayment risk of PAC tranches and the prepayment risk associated with the support tranches. In other words, *the certainty of PAC bond cash flow comes at the expense of increased risk to the support tranches*.

To understand the relatively high prepayment risk for support tranches, consider the situation in which prepayments are slower than planned. Because the PAC tranches have



priority claim against the cash flows, principal payments to the support tranches must be deferred until the PAC repayment schedule is satisfied. Thus, the average life of the support tranche is extended. Similarly, when actual prepayments come at a rate that is faster than expected, the support tranches must absorb the amount that is in excess of that required to maintain the repayment schedule for the PAC. In this case, the average life of the support tranche is contracted. If these excesses continue to occur, the support tranches will eventually be paid off and the principal will then go to the PAC holders. When this happens, the PAC is referred to as a *broken* or *busted* PAC, and any further prepayments go directly to the PAC tranche. Essentially, the PAC tranche becomes an ordinary sequential-pay structure.

Notice that the prepayment risk protection provided by the support tranches causes their average lives to extend and contract. This relationship is such that as the prepayment risk protection for a PAC tranche increases, its average life variability decreases, and the average life variability of the support tranche increases.

## Strips

A distinguishing characteristic of a traditional pass-through security is that the interest and principal payments generated by the underlying mortgage pool are allocated to the bondholders on a pro rata basis. This means that each pass-through certificate holder receives the same amount of interest and the same amount of principal. *Stripped MBSs* differ in that principal and interest are not allocated on a pro rata basis. The unequal allocation of principal and interest results in a price/yield relationship that is different from that of the underlying pass-through.

The two most common types of stripped MBSs are **principal-only strips** (PO strips) and **interest-only strips** (IO strips). PO strips are a class of securities that receive only the principal payment portion of each mortgage payment, while IO strips are a class that receive only the interest component of each payment.

PO strips are sold at a considerable discount to par. The PO cash flow stream starts out small and increases with the passage of time as the principal component of the mortgage payments grows. The investment performance of a PO is extremely sensitive to prepayment rates. Higher prepayment rates result in a faster-than-expected return of principal and, thus, a higher yield. Since prepayment rates increase as mortgage rates decline, PO prices increase when interest rates fall. The entire par value of a PO is ultimately paid to the PO investor. The only question is whether realized prepayment rates will cause it to be paid sooner or later than expected.

In contrast to PO strips, an IO strip cash flow starts out big and gets smaller over time. Thus, IOs have shorter effective lives than POs.

The major risk associated with IO strips is that the value of the cash flow investors receive over the life of the mortgage pool may be less than initially expected and possibly less than the amount originally invested. Why? The amount of interest produced by the pool depends on its beginning-of-month balance. If market rates fall, the mortgage pool will be paid off sooner than expected, leaving IO investors with no interest cash flow. Therefore, IO investors want prepayments to be slow.

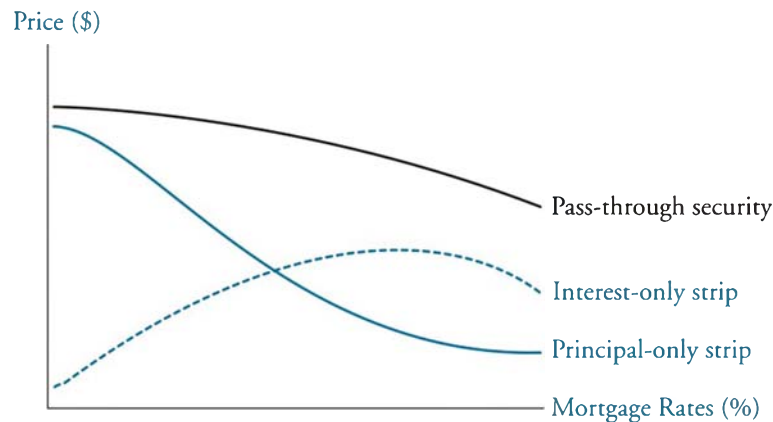
An interesting property of an IO is that its price has a tendency to move in the same direction as market rates. When market rates decline below the contract rate and prepayment rates increase, the diminished cash flow usually causes the IO price to decline, despite the fact that the cash flows are discounted at a lower rate. As interest rates rise above the contract rate, the expected cash flows improve. Even though the higher rate must be used to discount these improved cash flows, there is usually a range above the contract rate for which the price increases.

Both IOs and POs exhibit greater *price volatility* than the pass-through from which they were derived. This occurs because IO and PO returns are negatively correlated (their prices respond in opposite directions to changes in interest rates), but the combined price volatility of the two strips equals the price volatility of the pass-through.

The price/yield relationships for IO and PO securities are shown in Figure 6. Notice the following:

- The underlying pass-through security exhibits significant negative convexity.
- The PO exhibits some negative convexity at low rates.
- The IO price is positively related to mortgage rates at low current rates.
- The PO and IO prices are more volatile than the underlying pass-through.

**Figure 6: Investment Characteristics of IOs and POs**



## PREPAYMENT MODELING

**LO 51.7: Explain prepayment modeling and its four components: refinancing, turnover, defaults, and curtailments.**

Borrowers may prepay their mortgages due to the sale or destruction of the property or a desire to refinance at lower prevailing rates. In addition, prepayments may occur because the borrower has defaulted on the mortgage and the lender is forced to sell the property to cover the mortgage. Finally, many mortgages have partial prepayment privileges (curtailments) that may be used, especially when the borrower has excess cash available to do so.



**Refinancing** a mortgage involves using the proceeds of a new mortgage to pay off the principal from an existing mortgage. If a homeowner is holding a high interest rate mortgage and the current mortgage rates fall, the incentive to refinance is large (given that rates decline enough to cover the transaction costs of refinancing). Historically, if mortgage rates fall by more than 2%, refinancing activity increases dramatically. This is known as the *media effect* because large declines in rates will likely gain the attention of the media.

Extracting home equity is another motive for refinancing a mortgage. Given a substantial increase in property value, a borrower may take out a new mortgage with a higher balance that not only pays off the existing mortgage but also has extra cash for other purposes. Extracting home equity is also known as *cash-out refinancing*.

*Incentive functions* are used to model refinancing activity and are based on the term structure of mortgage rates. Past rates, also called *lagged rates*, can be included in the model to help explain refinancing behavior. Incentive functions essentially forecast the present value of any dollar gains given that a borrower will refinance.

The path that mortgage rates follow on their way to the current level affects prepayments through *refinancing burnout*. To better understand this phenomenon, consider a mortgage pool that was formed when rates were 12%, then interest rates dropped to 9%, rose to 12%, and then dropped again to 9%. Many homeowners will have refinanced when interest rates dipped the first time. On the second occurrence of 9% interest rates, most homeowners in the pool who were able to refinance would have already done so.

It is typically the case that the mortgage is due once the property is sold. This is referred to as *due on sale*. Because most borrowers sell their homes without regard for the path of mortgage rates, MBS investors will be subjected to a degree of **housing turnover** that does not correlate with the behavior of rates. One factor that slows the degree of housing turnover is known as the *lock-in effect*. This essentially means that borrowers may wish to avoid the costs of a new mortgage, which likely consists of a higher mortgage rate.

Modeling turnover typically starts with a base rate and then adjusts for seasonality (turnover is higher in the summer and lower in the winter). The turnover model may also include a *seasoning ramp*, which is partially based on improvements to creditworthiness over time, and, thus, the homeowner's increased ability to prepay the mortgage. As a group, housing turnover only accounts for 10% of overall prepayments.

When a borrower **defaults**, mortgage guarantors pay the interest and principal outstanding. These payments act as a source of prepayment. Modeling prepayments from default requires an analysis of loan-to-value (LTV) ratios and FICO scores, as well as an overall analysis of the housing market.

Partial payments by the borrower are referred to as **curtailments**. These partial payments tend to occur when a mortgage is older or has a relatively low balance. Thus, prepayment modeling due to curtailment typically takes into account the age of the mortgage.

## DYNAMIC VALUATION

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### LO 51.8: Describe the steps in valuing an MBS using Monte Carlo Simulation.

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Any discussed earlier, mortgage borrowers have an option to prepay the underlying securities. The value of MBSs with embedded options to prepay cannot be determined using traditional option valuation techniques. Therefore, the Monte Carlo valuation methodology is used to value MBSs and other fixed-income securities with embedded options.

The **binomial model** is only applicable for securities where the decision to exercise a call option is not dependent on how interest rates evolve over time. While the binomial model is useful for callable agency debentures and corporate bonds, it is not applicable to valuing an MBS. The historical evolution of interest rates over time impacts prepayments and makes the binomial model inappropriate for MBSs.

Prepayments on mortgage pass-through securities are interest rate path-dependent. This means that a given month's prepayment rate depends on whether there were prior opportunities to refinance since the origination of the underlying mortgages. For example, if mortgage rates trend downward over a period of time, prepayment rates will increase at the beginning of the trend as homeowners refinance their mortgages, but prepayments will slow as the trend continues because many of the homeowners who can refinance will have already done so. As mentioned earlier, this prepayment pattern is called refinancing burnout. Another problem of the path-dependency of MBSs is related to the nature of structured securities, such as collateralized mortgage obligations (CMOs). The amount a CMO tranche receives in the form of cash flows for a specific month depends on the outstanding balances of other tranches in the deal. These outstanding balances are impacted by earlier principal and interest prepayments.

The **Monte Carlo methodology** is a simulation approach for valuing MBSs. Monte Carlo is actually a process of steps rather than a specific model. It is extremely useful when there are numerous variables with multiple outcomes. Monte Carlo is used to provide a probability distribution of the value of an MBS. The valuation of an MBS is influenced by future interest rates, the shape of the yield curve, future interest rate volatility, prepayment rates, default rates, and recovery rates.

Each of these variables or parameters of the Monte Carlo model could have multiple outcomes with different probabilities associated for each outcome. One valuation approach in these circumstances is the **best guess approach** where the expected value of each variable is used to estimate the value of the MBS. Unfortunately, this method is highly inaccurate. For example, suppose the probability of the best guess occurring for each variable is 70%. Then, with six different variables, the probability that the best guess MBS value will occur is only 11.8% ( $= 0.70^6$ ).

The Monte Carlo approach provides a range of possible outcomes with a probability distribution for the value of a mortgage security. The mean or average value of this range of outcomes is then taken as the estimated value of the MBS. The other information, such as the range of possible outcomes and percentile information, is useful in gauging the value of the security.

The following steps are required to value a mortgage security using the Monte Carlo methodology:

*Step 1:* Simulate the interest rate path and refinancing path.

*Step 2:* Project cash flows for each interest rate path.

*Step 3:* Calculate the present value of cash flows for each interest rate path.

*Step 4:* Calculate the theoretical value of the mortgage security.

*Step 1:* Simulate the interest rate path and refinancing path.

The first step in applying the Monte Carlo approach is to estimate monthly interest rates for the entire life of the mortgage security. For example, a 30-year mortgage security would require 360 monthly interest rates. In equations to follow, the total number of months on an interest rate path will be denoted by  $T$ . Also, the total number of interest rate paths or *trials* that are simulated will be denoted by  $N$ . Random interest rate paths are generated using the term structure of interest rates and a volatility assumption. The term structure of interest rates is created using the theoretical spot rate (zero-coupon) curve for the market on the pricing date. The simulations are adjusted to ensure the average simulated price of a zero-coupon Treasury bond is equal to the actual price corresponding to the pricing date. Some models use LIBOR or swap rates instead of Treasury rates.

The dispersion of future interest rates in the simulation is determined by the volatility assumption. It is common practice to use more than one level of volatility. For example, with a short/long yield volatility approach, the volatility is specified based on maturities. One volatility number is used for shorter maturities (short yield volatility) and a second yield volatility is specified for longer maturities (long yield volatility). Short yield volatility is typically assumed to be greater than long yield volatility. When yield volatility is assumed for each maturity, it is referred to as **term structure yield volatility**.

The derivatives market is used to construct an arbitrage-free term structure of future interest rates. Short-term interest rate paths are used to discount the cash flows in Step 3 of the Monte Carlo process. These interest rate paths are also used to create the prepayment paths or *vectors*, which are cash flows for each interest rate path. The prepayment vector is computed based on refinancing rates that are available each month. The mortgagor has an incentive to refinance if the refinancing rate is low relative to the mortgagor's original coupon rate. The relationship between refinancing rates and short-term interest rates is an important assumption of the model.

*Step 2:* Project cash flows for each interest rate path.

Cash flows for each month on each interest rate path are equal to the scheduled principal for the mortgage pool, the net interest, and prepayments. Scheduled principal payments are simply calculated based on the projected mortgage balance from the prior month. A prepayment model is used rather than a simple prepayment rate. A prepayment rate is specified for each month on a given interest rate path, and rates for a given month across all interest rate paths are not the same. In fact, there could actually be  $T \times N$  different prepayment rates.

CMO deal structures dictate how principal and interest is to be paid. Therefore, it is necessary to reverse engineer the deal to determine the cash flows for a senior CMO. The cash flows for each month on an interest rate path are calculated using the scheduled principal, net interest, and prepayments for the collateral (i.e., the pool of agency pass-

throughs). The tranche's cash flows for each path are determined by the total principal and interest paid to the tranche, the interaction of the cash flow rules, and the prepayment model.

*Step 3:* Calculate the present value of cash flows for each interest rate path.

The present values of cash flows for each interest rate path are calculated by discounting the cash flows for each path by a discount rate. The discount rate is estimated using the simulated spot rates for each month on the interest rate path plus an appropriate spread. The simulated spot rates are determined from the simulated future monthly rates. The following equation quantifies the relationship that holds between the simulated spot rate,  $z_T(n)$ , for month  $T$  on path  $n$ , and the simulated future monthly rates,  $f_j(n)$ :

$$z_T(n) = \{[1 + f_1(n)][1 + f_2(n)] \dots [1 + f_T(n)]\}^{1/T} - 1$$

where:

$z_T(n)$  = simulated spot rate for month  $T$  on path  $n$

$f_j(n)$  = simulated future one-month rate for month  $j$  on path  $n$

The interest rate paths for the simulated future one-month rates are converted to the interest rate paths for the simulated monthly spot rates. The present value of the cash flows for month  $T$  on interest rate path  $n$  discounted at the simulated spot rate for month  $T$ ,  $z_T(n)$ , plus a spread,  $K$ , is:

$$PV[C_T(n)] = \frac{C_T(n)}{[1 + z_T(n) + K]^T}$$

where:

$PV[C_T(n)]$  = present value of cash flows for month  $T$  on path  $n$

$C_T(n)$  = cash flow for month  $T$  on path  $n$

$z_T(n)$  = spot rate for month  $T$  on path  $n$

$K$  = spread

The present value for path  $n$  is determined as the sum of the present values of the cash flows for each month on path  $n$  as follows:

$$PV[\text{path}(n)] = PV[C_1(n)] + PV[C_2(n)] + \dots + PV[C_T(n)]$$

where:

$PV[\text{path}(n)]$  = present value of interest rate path  $n$

*Step 4:* Calculate the theoretical value of the mortgage security.

The theoretical value for a specific interest rate path is thought of as the present value of all cash flows in that path, assuming that path was actually realized. The theoretical value of the mortgage security is calculated as the average present value of all theoretical values for each interest rate path as follows:

$$\text{theoretical value} = \frac{PV[\text{path}(1)] + PV[\text{path}(2)] + \dots + PV[\text{path}(N)]}{N}$$

where:

$N$  = number of interest rate paths



This average theoretical value is typically the only measurement that is evaluated when Monte Carlo simulations are used to value MBSs. It is unfortunate that other potentially valuable information, such as the distribution of the path present values, is usually ignored.

## OPTION-ADJUSTED SPREAD

### LO 51.9: Define Option Adjusted Spread (OAS), and explain its challenges and its uses.

The **option-adjusted spread (OAS)** is defined as the spread,  $K$ , that, when added to all the spot rates of all the interest rate paths, will make the average present value of the paths equal to the actual observed market price plus accrued interest. The OAS is mathematically determined by the following relationship:

$$\text{market price} = \frac{\text{PV}[\text{path}(1)] + \text{PV}[\text{path}(2)] + \dots + \text{PV}[\text{path}(N)]}{N}$$

where:

$N$  = number of interest rate paths

The left-hand side of the equation is the current market price of the MBS. The right-hand side of the equation is the Monte Carlo model's output of the average theoretical value of the MBS. The OAS is determined with an iterative process. If the average theoretical value determined by the model is higher (lower) than the MBS market value, the spread is increased (decreased).

The OAS can be interpreted as a measure of MBS returns that indicates the potential compensation after adjusting for prepayment risk. In other words, the OAS is *option adjusted* because the cash flows on the interest rate paths take into account the borrowers' option to prepay. An investor could estimate the value of a security using the OAS for comparable bonds to determine whether or not to invest in the security. A second approach is to compare the OAS generated at the market price to those available for comparable securities or an investment benchmark (such as a cost of funds).

Cash flows for MBSs are monthly annuity payments, while Treasury securities pay semiannual interest-only payments and a large bullet payment. The **zero-volatility spread (z-spread)** is a spread measure that an investor realizes over the entire Treasury spot rate curve, assuming the mortgage security is held to maturity. It is a more accurate measure because it compares an MBS to a portfolio of Treasury securities. The zero-volatility spread is the yield that equates the present value of the cash flows from the MBS to the price of the MBS discounted at the Treasury spot rate plus the spread. Thus, an iterative process is required to determine the zero-volatility spread.

The zero-volatility spread accounts for variations in MBS principal payments at a given prepayment rate or speed. However, it does not consider the impact that prepayment risk or changing prepayment rates have on the value of the MBS.

The option cost measures the prepayment (or option) risk. It is the implied cost of the option embedded in the MBS. The option cost is calculated as the difference between the OAS at the assumed volatility of interest rates and the zero-volatility spread as follows:

$$\text{option cost} = \text{zero-volatility spread} - \text{OAS}$$

Therefore, the option cost is a by-product of the Monte Carlo analysis and is not determined using traditional option value approaches. As volatility declines, the option cost decreases, and the previously described relationship suggests that OAS increases as volatility declines, all other things equal.

### OAS Challenges

There are four important limitations to consider when using OAS:

- Modeling risk associated with Monte Carlo simulations.
- Required adjustments to interest rate paths.
- An underlying assumption of a constant OAS over time in the model.
- The dependency of the underlying prepayment model.

The OAS is generated through Monte Carlo simulations. Therefore, the OAS is subject to all modeling risks associated with the simulation. Interest rate paths must be adjusted to ensure securities or rates making up the benchmark curve are properly valued when using Monte Carlo methods. This process of adjusting interest rate paths is subject to modeling error. If there is a term structure to the OAS, then this is not reflected in the Monte Carlo process because the OAS methodology assumes a constant OAS.

The prepayment model is very complex, given the amount of uncertainty regarding important variables. The behavior of both borrowers and lenders changes over time. Thus, the greatest weakness of using OAS valuation estimates generated from the Monte Carlo simulation is the dependence on the prepayment model.

Additionally, both  $z$ -spreads and OAS measures assume the securities are held to maturity. Some investors may hold a security to maturity, but many investors will only hold a security over a finite horizon. Thus, the investor should analyze the securities in a manner that is consistent with the investor's asset management horizon.

## KEY CONCEPTS

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### LO 51.1

Key attributes that define mortgages are lien status, original loan term, credit classification, interest rate type, prepayments/prepayment penalties, and credit guarantees.

Agency MBSs are those that are guaranteed by government-sponsored enterprises (GSEs). Most of the MBSs are issued by these GSEs.

The GSEs have restrictions on which mortgages they can guarantee/securitize, which opened up the private label market (non-agency MBSs) for those participants willing to take on the risks inherent in nonconventional loans—jumbo loans and/or loans with high loan-to-value ratios.

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### LO 51.2

A mortgage is a loan that is collateralized with a specific piece of real property, either residential or commercial. A level-payment, fixed-rate conventional mortgage has a fixed term, a fixed interest rate, and a fixed monthly payment. Even though the term, rate, and payment are fixed, the cash flows are not known with certainty because the borrower has the right to repay all or any part of the mortgage balance at any time.

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### LO 51.3

Mortgage prepayments come in two forms: (1) increasing the frequency or amount of payments and (2) repaying/refinancing the entire outstanding balance. Prepayments are much more likely to occur when market interest rates fall and borrowers wish to refinance their existing mortgages at a new and lower rate.

Other factors that influence prepayments include seasonality, age of mortgage pool, personal, housing prices, and refinancing burnout.

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### LO 51.4

To reduce the risk from holding a potentially undiversified portfolio of mortgage loans, a number of financial institutions (originators) will work together to pool residential mortgage loans with similar characteristics into a more diversified portfolio. They will then sell the loans to a separate entity, called a special purpose vehicle (SPV), in exchange for cash. An issuer will purchase those mortgage assets in the SPV and then use the SPV to issue mortgage-backed securities (MBSs) to investors; the securities are backed by the mortgage loans as collateral.

Fixed-rate pass-through securities trade in one of the following ways:

- The specified pools market.
- The To Be Announced (TBA) market.



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**LO 51.5**

The value of an MBS is a function of:

- Weighted average maturity (WAM).
- Weighted average coupon (WAC).
- Speed of prepayments.

Regarding prepayment speeds, the single monthly mortality (SMM) rate is derived from the conditional prepayment rate and is used to estimate monthly prepayments for a mortgage pool:

$$\text{SMM} = 1 - (1 - \text{CPR})^{1/12}$$

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**LO 51.6**

A dollar roll transaction occurs when an MBS market maker is buying positions for one settlement month and, at the same time, selling those same positions for another month.

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**LO 51.7**

Borrowers may prepay a mortgage due to the sale of the property or a desire to refinance at lower prevailing rates. In addition, prepayments may occur when the borrower has defaulted on the mortgage or when the borrower has cash available to make partial prepayments (curtailment).

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**LO 51.8**

The Monte Carlo methodology is a simulation approach for valuing MBSs. The binomial model is not appropriate for valuing MBSs because MBSs have embedded prepayment options and the historical evolution of interest rates over time impacts prepayments.

A mortgage security is valued using the Monte Carlo methodology by simulating the interest rate path and refinancing path, projecting cash flows for each interest rate path, calculating the present value of cash flows for each interest rate path, and calculating the theoretical value of the mortgage security.

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**LO 51.9**

The option-adjusted spread (OAS) is the spread that, when added to all the spot rates of all the interest rate paths, will make the average present value of the paths equal to the actual observed market price plus accrued interest. The zero-volatility spread (*z*-spread) is the spread that an investor realizes over the entire Treasury spot rate curve, assuming the mortgage security is held to maturity. The option cost is the implied cost of the embedded prepayment option and is calculated as the difference between the *z*-spread and OAS.

Four major limitations of OASs are related to: (1) modeling risk associated with Monte Carlo simulations, (2) required adjustments to interest rate paths, (3) model assumption of a constant OAS over time, and (4) dependency on the underlying prepayment model.

**CONCEPT CHECKERS**

1. Which of the following factors is least likely to influence the level of residential mortgage prepayments?
  - A. Seasonality.
  - B. Inflation.
  - C. Housing prices.
  - D. Age of mortgage pool.
2. If the conditional prepayment rate (CPR) for a pool of mortgages is assumed to be 5% on an annual basis and the weighted average maturity of the underlying mortgages is 15 years, which of the following amounts is closest to the constant maturity mortality?
  - A. 0.333%.
  - B. 0.405%.
  - C. 0.427%.
  - D. 0.5%.
3. Which of the following factors would not cause a dollar roll to trade special?
  - A. Decrease in the back month price.
  - B. Increase in the front month price.
  - C. Surplus of securities in the market used for settlement.
  - D. Shortage of securities in the market used for settlement.
4. When using the Monte Carlo approach to estimate the value of mortgage-backed securities (MBSs), the model should:
  - A. use one consistent volatility measure for all interest rate paths.
  - B. use a short/long yield volatility approach.
  - C. use annual interest rates over the entire life of the mortgage security.
  - D. ignore the distribution of the interest rate paths used to determine the theoretical value.
5. All of the following describe limitations of using option-adjusted spreads (OASs) for valuing mortgage-backed securities (MBSs) except:
  - A. modeling risk is associated with Monte Carlo simulations.
  - B. model requires making adjustments to interest rate paths.
  - C. model assumes a dynamic OAS over time.
  - D. prepayment model influences the model valuation.

## CONCEPT CHECKER ANSWERS

1. **B** Seasonality does impact the level of prepayments—they are noticeably higher in the summertime. Increases in housing prices may spur an increase in prepayments caused by refinancing mortgages stemming from borrowers wanting to take out some of the increased equity for personal use. The lower the age of the mortgage pool, the less likely the risk of prepayment.
2. **C** The constant maturity mortality (or single monthly mortality rate) is a monthly measure. Its relationship to CPR is as follows:  
  

$$\text{SMM} = 1 - (1 - \text{CPR})^{1/12} = 1 - (1 - 0.05)^{1/12} = 1 - 0.95^{1/12} = 0.43\%$$
3. **C** When the drop is large enough to result in financing at less than the implied cost of funds, then the dollar roll is trading special. It could be caused by:
  - A decrease in the back month price (due to an increased number of sale/settlement transactions on the back month date by originators).
  - An increase in the front month price (due to an increased demand in the front month for deal collateral).
  - Shortages of certain securities in the market that require the dealer to suddenly purchase the security for delivery in the front month, which would increase the front month price.
4. **B** When using the Monte Carlo approach to estimate the value of MBSs, the model should use more than one volatility measure for all interest rate paths. It is very common to use a short/long yield volatility approach to estimate monthly rates. Although the information regarding the distributions of interest rate paths is oftentimes ignored, it contains valuable information and should be considered.
5. **C** When using OAS to value MBS, the model assumes a constant OAS over time. This is problematic if there is a term structure to the OAS because this is not reflected in the Monte Carlo process.

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# SELF-TEST: FINANCIAL MARKETS AND PRODUCTS

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15 Questions: 36 Minutes

1. An investor enters a short position in a gold futures contract at \$318.60. Each futures contract controls 100 troy ounces. The initial margin is \$5,000 and the maintenance margin is \$4,000. At the end of the first day the futures price rises to \$329.22. Which of the following is the amount of the variation margin at the end of the first day?
  - A. \$0.
  - B. \$62.
  - C. \$1,000.
  - D. \$1,062.
  
2. A large-cap U.S. equity portfolio manager is concerned about near-term market conditions and wishes to reduce the systematic risk of her portfolio from 1.2 to 0.90. Her portfolio value is \$56 million, and the S&P 500 futures index is currently trading at 1,050 and has a multiplier of 250. How can the portfolio manager's objective be achieved?
  - A. Sell 47 contracts.
  - B. Buy 47 contracts.
  - C. Sell 64 contracts.
  - D. Buy 64 contracts.
  
3. Suppose you observe a 1-year (zero-coupon) Treasury security trading at a yield to maturity of 5% (price of 95.2381% of par). You also observe a 2-year T-Note with a 6% coupon trading at a yield to maturity of 5.5% (price of 100.9232). And, finally, you observe a 3-year T-Note with a 7% coupon trading at a yield to maturity of 6.0% (price of 102.6730). Assume annual coupon payments and discrete compounding. Use the bootstrapping method to determine the 2-year and 3-year spot rates.

	<u>2-year spot rate</u>	<u>3-year spot rate</u>
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4. Former Treasury Secretary Robert Rubin decided to stop issuing 30-year Treasury bonds in 2001 and to replace them by borrowing more with shorter-maturity Treasury bills and notes (although the U.S. Treasury has since resumed issuing 30-year bonds). Which of the following statements concerning this decision is most accurate?
- A. If the expectations theory of the term structure is correct, this decision will reduce the government's borrowing cost.
  - B. If the liquidity theory of the term structure is correct, this decision will reduce the government's borrowing cost.
  - C. If the liquidity theory of the term structure is correct, this decision will not change the government's borrowing cost.
  - D. If the expectations theory of the term structure is correct, this decision will increase the government's borrowing cost.
5. A portfolio manager owns Macrogrow, Inc., which is currently trading at \$35 per share. She plans to sell the stock in 120 days but is concerned about a possible price decline. She decides to take a short position in a 120-day forward contract on the stock. The stock will pay a \$0.50 per share dividend in 35 days and \$0.50 again in 125 days. The risk-free rate is 4%. The value of the trader's position in the forward contract in 45 days, assuming in 45 days the stock price is \$27.50 and the risk-free rate has not changed, is closest to:
- A. \$7.16.
  - B. \$7.50.
  - C. \$7.92.
  - D. \$7.00.
6. A 6-month futures contract on an equity index is currently priced at 1,276. The underlying index stocks are valued at 1,250 and pay dividends at a continuously compounded rate of 1.70%. The current continuously compounded risk-free rate is 5%. The potential arbitrage is closest to:
- A. 5.20.
  - B. 8.32.
  - C. 16.58.
  - D. 26.00.

7. Company J and Company K enter into a 2-year plain vanilla interest rate swap. Company J agrees to pay Company K a periodic fixed rate on a notional principal over the swap's tenor. In exchange, Company K agrees to pay Company J a periodic floating rate on the same notional principal. Assume currency is the same, so the net payment will be exchanged. The exchanges will be made semi-annually. The reference rate is the 6-month LIBOR. The fixed rate of the swap is 1.1%, and the notional principal is \$100 million. 6-month LIBOR rates are as follows:

<i>Beginning of Period</i>	<i>LIBOR</i>
1	0.5%
2	0.75%
3	1.00%
4	1.25%
5	1.50%

What is the net payment for the end of the first period?

- A. Company J pays Company K \$300,000.
- B. Company J pays Company K \$550,000.
- C. Company K pays Company J \$250,000.
- D. Company K pays Company J \$50,000.

Use the following information to answer Questions 8 and 9.

Stock ABC trades for \$60 and has 1-year call and put options written on it with an exercise price of \$60. The annual standard deviation estimate is 10%, and the continuously compounded risk-free rate is 5%. The value of the call is \$4.09.

Chefron, Inc. common stock trades for \$60 and has a 1-year call option written on it with an exercise price of \$60. The annual standard deviation estimate is 10%, the continuous dividend yield is 1.4%, and the continuously compounded risk-free rate is 5%.

8. The value of the put on ABC stock is closest to:
- A. \$1.16.
  - B. \$3.28.
  - C. \$4.09.
  - D. \$1.00.



9. The value of the call on Chevron stock is closest to:
- A. \$3.51.
  - B. \$4.16.
  - C. \$5.61.
  - D. \$6.53.
10. One of your clients, Christopher Stachowski, realizes that the market prices of options must take into account the beliefs of the market participants. He thinks he will be able to make significant profits because he believes that there will be a large movement in the direction of stock prices but is unsure which direction. Such a belief is completely different from the other market participants. As a result, Christopher would like you to implement an options trading strategy to generate him those profits. Which of the following combination option strategies is likely to benefit the least amount from a large positive or negative movement in the price of the underlying?
- A. Strip.
  - B. Strap.
  - C. Collar.
  - D. Long strangle.
11. Consider a bearish option strategy of buying one \$50 put for \$7, selling two \$42 puts for \$4 each, and buying one \$37 put for \$2. All the options have the same maturity. Calculate the final profit per share of the strategy if the underlying is trading at \$33 at expiration.
- A. \$1 per share.
  - B. \$2 per share.
  - C. \$3 per share.
  - D. \$4 per share.
12. You believe that a stock will increase in price and would like to buy a call option. You would like to choose the date during the option's term when the option payoff is determined. However, if the option payoff is greater at the option's maturity, you want to be paid this value. What type of option should you buy?
- A. Chooser option.
  - B. Compound option.
  - C. Shout option.
  - D. Asian option.
13. Suppose the spot rate is 0.7102 USD/CHF. Swiss and U.S. interest rates are 7.6% and 5.2%, respectively. If the 1-year forward rate is 0.7200 USD/CHF, an investor could:
- A. not earn arbitrage profits.
  - B. earn arbitrage profits by investing in USD.
  - C. earn arbitrage profits by investing in CHF.
  - D. only earn arbitrage profits by investing in a third currency.

14. Consider a U.K.-based company that exports goods to the EU. The U.K. company expects to receive payment on a shipment of goods in 60 days. Because the payment will be in euros, the U.K. company wants to hedge against a decline in the value of the euro against the pound over the next 60 days. The U.K. risk-free rate is 3% and the EU risk-free rate is 4%. No change is expected in these rates over the next 60 days. The current spot rate is 0.9230 £ per €. To hedge the currency risk, the U.K. company should take a short position in a Euro contract at a forward price of:
- A. 0.9205.
  - B. 0.9215.
  - C. 0.9244.
  - D. 0.9141.
15. A level-payment, fixed-rate mortgage has the following characteristics:
- Term 30 years.
  - Mortgage rate 9.0%.
  - Servicing fee 0.5%.
  - Original mortgage loan balance \$150,000.
- The monthly mortgage payment is:
- A. \$416.67.
  - B. \$1,125.00.
  - C. \$1,206.93.
  - D. \$1,216.70.

# SELF-TEST ANSWERS: FINANCIAL MARKETS AND PRODUCTS

1. D The short position loses when the price rises.

$$(\$329.22 - \$318.60) \times 100 = 1,062 \text{ loss}$$

Margin account will change as follows:  $\$5,000 - \$1,062 = \$3,938$

Variation margin of \$1,062 is required because the balance has fallen below the maintenance margin level. This variation margin payment is required in order to restore the account back to the initial level.

(See Topic 35)

2. C The portfolio manager wants to reduce exposure to systematic risk so she will want to sell S&P index futures. This will reduce the current beta to her target beta of 0.90.

number of contracts = (target beta – current beta) × (portfolio value / futures value)

$$\text{number of contracts} = (0.9 - 1.2) \times [\$56 \text{ million} / (1,050 \times 250)]$$

number of contracts = –64 (i.e., sell 64 contracts)

(See Topic 36)

3. C Here are the cash flows associated with the three bonds:

	0	1	2	3
1-year	–\$95.2381	+\$100		
2-year	–\$100.9232	+\$6	+\$106	
3-year	–\$102.6730	+\$7	+\$7	+\$107

To find  $Z_2$ , the 2-year spot rate:

$$\$100.9232 = \frac{\$6}{1.05^1} + \frac{\$106}{(1 + Z_2)^2} \Rightarrow Z_2 = 5.51\%$$

To find  $Z_3$ , the 3-year spot rate:

$$\$102.6730 = \frac{\$7}{1.05^1} + \frac{\$7}{1.0551^2} + \frac{\$107}{(1 + Z_3)^3} \Rightarrow Z_3 = 6.05\%$$

(See Topic 37)

4. B If the expectations theory of the term structure is correct, altering the maturity of the government's borrowing will not affect the government's borrowing cost (i.e., borrowing once for 30 years is the same as borrowing 30 times for one year at a time). If the liquidity theory is correct, the government's borrowing cost will go down, as it no longer has to compensate lenders with the liquidity premium for borrowing long term.

(See Topic 37)

5. A The dividend in 125 days is irrelevant because it occurs after the forward contract matures.

$$PVD = \$0.50e^{-0.04 \times (35/365)} = \$0.4981$$

$$FP = (\$35 - \$0.4981) \times 1.04^{120/365} = \$34.95$$

$$V_{45}(\text{short position}) = -(\$27.50 - \$34.95e^{-0.04 \times (75/365)}) = \$7.16$$

(See Topic 38)

6. A  $F = S \times e^{(\text{risk-free rate} - \text{dividend yield}) \times t}$

$$F = 1,250 \times e^{(0.05 - 0.017) \times 0.5}$$

$$F = 1,270.80$$

The actual futures price is 1,276, so selling the futures and buying the underlying index nets a profit of  $1,276 - 1,270.80 = 5.20$ .

(See Topic 38)

7. A Floating =  $\$100 \text{ million} \times 0.005 \times 0.5 = \$250,000$

$$\text{Fixed} = \$100 \text{ million} \times 0.011 \times 0.5 = \$550,000$$

(See Topic 40)

8. A According to put/call parity, the put's value is:

$$p_0 = c_0 - S_0 + \left( X \times e^{-R_c^f \times T} \right) = \$4.09 - \$60.00 + \left[ \$60.00 \times e^{-(0.05 \times 1.0)} \right] = \$1.16$$

(See Topic 42)

9. A ABC and Chevron stock are identical in all respects except Chevron pays a dividend. Therefore, the call option on Chevron stock must be worth less than the call on ABC (i.e., less than \$4.09). \$3.51 is the only possible answer.

(See Topic 42)

10. C A collar is the combination of a protective put and a covered call. Ignoring transaction costs, at levels below the put strike price or above the call strike price, the profit from a collar levels off. Between the put strike price and the call strike price, the profit level is gradually rising.

(See Topic 43)

11. B Consider each option separately:

$$\$50 \text{ long put:} \quad \$50 - \$33 = +\$17$$

$$\$42 \text{ short put:} \quad \$42 - \$33 = -\$9 \times 2 = -\$18$$

$$\$37 \text{ long put:} \quad \$37 - \$33 = +\$4$$

$$\text{Net cost of options:} \quad (-7 + 8 - 2) = -\$1$$

$$\text{Overall profit per share:} \quad \$2 \text{ per share}$$

(See Topic 43)

12. C The shout option allows the buyer to choose the date when he “shouts” to the option seller that the intrinsic value should be determined. At expiration, the option buyer receives the maximum of the shout value or the intrinsic value at expiration.

(See Topic 44)

13. C Note that while the USD has the lower interest rate, it is also trading at a forward discount relative to the CHF. Since the USD will earn less interest *and* depreciate in value, we definitely want to invest in CHF (not in USD), and no calculation is necessary.

As an illustration of covered interest arbitrage, we have:

$$(1 + R_A) < \frac{(1 + R_B)(\text{forward rate})}{\text{spot rate}}$$

$$1.052 < \frac{(1.076)(0.72)}{0.7102} = 1.0908$$

Today:

- (1) Borrow USD1 at 5.2% and purchase CHF at \$0.7102 to get \$1 / 0.7102 = 1.408 CHF at spot rate.
- (2) Lend the purchased CHF at 7.6% and sell forward 1.5150 CHF at the forward rate of 0.7200 USD/CHF.

In one year:

- (1) Use the proceeds of the savings account [(1.408)(1.076) = 1.5150 CHF] to purchase USD1.0908 at the forward rate (1.515 CHF × 0.72 USD/CHF).
- (2) Pay off the loan of USD1 × 1.052 = USD1.052 and earn a riskless profit = USD1.0908 – USD1.052 = USD0.0388.

(See Topic 49)

14. B The U.K. company will be receiving euros in 60 days, so it should short the 60-day forward on the euro as a hedge. The no-arbitrage forward price is:

$$F_T = £0.923 \times \frac{1.03^{60/365}}{1.04^{60/365}} = 0.9215$$

(See Topic 49)

15. C     $N = 360$ ;  $I = 9/12 = 0.75$ ;  $PV = 150,000$ ;  $CPT \rightarrow PMT = \$1,206.93$

(See Topic 51)



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# FORMULAS

## Topic 32

combined ratio: loss ratio + expense ratio

combined ratio after dividends: combined ratio + dividends

operating ratio: combined ratio after dividends – investment income

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## Topic 33

net asset value:  $NAV = \frac{\text{fund assets} - \text{fund liabilities}}{\text{total shares outstanding}}$

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## Topic 34

call option payoff:  $C_T = \max(0, S_T - X)$

put option payoff:  $P_T = \max(0, X - S_T)$

forward contract payoff: payoff =  $S_T - K$

where:

$S_T$  = spot price at maturity

$K$  = delivery price

---

## Topic 36

basis =  $S_t - F_0$

where:

$S_t$  = cash (or spot) price of the underlying asset at time  $t$

$F_0$  = current price of the futures contract

hedge ratio:  $HR = \rho_{S,F} \frac{\sigma_S}{\sigma_F}$

beta:  $\frac{\text{Cov}_{S,F}}{\sigma_F^2} = \beta_{S,F}$

correlation:  $\rho = \frac{\text{Cov}_{S,F}}{\sigma_S \sigma_F}$

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hedging with stock index futures:

$$\begin{aligned}\text{number of contracts} &= \beta_{\text{portfolio}} \times \left( \frac{\text{portfolio value}}{\text{value of futures contract}} \right) \\ &= \beta_{\text{portfolio}} \times \left( \frac{\text{portfolio value}}{\text{futures price} \times \text{contract multiplier}} \right)\end{aligned}$$

adjusting the portfolio beta:  $\text{number of contracts} = (\beta^* - \beta) \frac{P}{A}$

### Topic 37

discrete compounding:  $FV = A \left( 1 + \frac{R}{m} \right)^{m \times n}$

continuous compounding:  $FV = Ae^{R \times n}$

forward rate agreement:  $\text{cash flow (if receiving } R_K) = L \times (R_K - R) \times (T_2 - T_1)$   
 $\text{cash flow (if paying } R_K) = L \times (R - R_K) \times (T_2 - T_1)$

### Topic 38

forward price:  $F_0 = S_0 e^{rT}$

forward price with carrying costs:  $F_0 = (S_0 - I) e^{rT}$

forward price when the underlying asset pays a dividend:  $F_0 = S_0 e^{(r-q)T}$

### Topic 39

accrued interest =  $\text{coupon} \times \frac{\text{\# of days from last coupon to the settlement date}}{\text{\# of days in coupon period}}$

cash price of a bond:  $\text{cash price} = \text{quoted price} + \text{accrued interest}$

annual rate on a T-Bill:  $\text{T-bill discount rate} = \frac{360}{n} (100 - Y)$

cheapest-to-deliver bond:  $\text{quoted bond price} - (QFP \times CF)$

Eurodollar futures price =  $\$10,000 [100 - (0.25)(100 - Z)]$

convexity adjustment:

$\text{actual forward rate} = \text{forward rate implied by futures} - (0.5 \times \sigma^2 \times t_1 \times t_2)$

$$\text{duration-based hedge ratio: } N = -\frac{P \times D_P}{F \times D_F}$$

#### Topic 40

$$\text{forward rate between } T_1 \text{ and } T_2: R_{\text{forward}} = R_2 + (R_2 - R_1) \frac{T_1}{T_2 - T_1}$$

#### Topic 42

put-call parity:

$$S = c - p + Xe^{-rT}$$

$$p = c - S + Xe^{-rT}$$

$$c = S + p - Xe^{-rT}$$

$$Xe^{-rT} = S + p - c$$

lower and upper bounds for options:

Option	Minimum Value	Maximum Value
European call	$c \geq \max(0, S_0 - Xe^{-rT})$	$S_0$
American call	$C \geq \max(0, S_0 - Xe^{-rT})$	$S_0$
European put	$p \geq \max(0, Xe^{-rT} - S_0)$	$Xe^{-rT}$
American put	$P \geq \max(0, X - S_0)$	$X$

#### Topic 43

$$\text{bull call spread: profit} = \max(0, S_T - X_L) - \max(0, S_T - X_H) - C_{L0} + C_{H0}$$

$$\text{bear put spread: profit} = \max(0, X_H - S_T) - \max(0, X_L - S_T) - P_{H0} + P_{L0}$$

butterfly spread with calls:

$$\text{profit} = \max(0, S_T - X_L) - 2\max(0, S_T - X_M) + \max(0, S_T - X_H) - C_{L0} + 2C_{M0} - C_{H0}$$

$$\text{straddle: profit} = \max(0, S_T - X) + \max(0, X - S_T) - C_0 - P_0$$

$$\text{strangle: profit} = \max(0, S_T - X_H) + \max(0, X_L - S_T) - C_0 - P_0$$

#### Topic 45

$$\text{pricing a commodity forward with a lease payment: } F_{0,T} = S_0 e^{(r - \delta_1)T}$$

$$\text{commodity forward pricing with storage costs: } F_{0,T} = S_0 e^{(r + \lambda)T}$$

$$\text{commodity forward pricing with convenience yield: } F_{0,T} = S_0 e^{(r - c)T}$$

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Topic 49

interest rate parity:  $\text{forward} = \text{spot} \left[ \frac{(1 + r_{\text{DC}})}{(1 + r_{\text{FC}})} \right]^T$

$$\text{forward} = \text{spot} \times e^{(r_{\text{DC}} - r_{\text{FC}})T}$$

nominal interest rate:  $\text{exact methodology: } (1 + r) = (1 + \text{real } r)[1 + E(i)]$   
 $\text{linear approximation: } r \approx \text{real} + E(i)$

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Topic 50

original-issue discount (OID) = face value – offering price

dollar default rate:

$$\frac{\text{cumulative dollar value of all defaulted bonds}}{(\text{cumulative dollar value of all issuance}) \times (\text{weighted average \# of years outstanding})}$$

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Topic 51

single monthly mortality rate:  $\text{SMM} = 1 - (1 - \text{CPR})^{1/12}$

option cost = zero-volatility spread – OAS

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# INDEX

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## A

absolute advantage 125  
accrued interest 109  
adjustable-rate mortgages (ARMs) 271  
adverse selection 15, 229  
alternative-A loans 271  
American depository receipts (ADRs) 33  
American options 51, 140  
amortization schedule 274  
arbitrage 31  
arbitrage opportunity 50  
arbitrageurs 50, 51  
Asian options 188  
asset-or-nothing call 187  
auctioning 226

## B

back-end load 26  
backfill bias 34  
backwardation 104, 200  
banking book 4  
barrier options 186  
basis 58, 206  
basis point 89  
basis risk 69, 207  
basket options 188  
bear call spread 169  
bearish calendar spread 172  
bear put spread 169  
Bermudan option 184  
best efforts 3  
best guess approach 287  
bid-ask spread 51  
bid price 51  
bifurcation 229  
binary options 147, 187  
binomial model 287  
bond indenture 257  
bootstrapping 84  
box spread 173  
breakeven price 143  
bull call spread 168  
bullish calendar spread 172  
businessman's risk 263  
butterfly spread 170

## C

calendar spread 172  
call option 41, 51, 140  
call premium 42  
call provision 261  
caplets 176  
cap rate 176  
carry market 201  
cash-and-carry arbitrage 196  
cash-or-nothing call 187  
cash-settlement contract 61  
casualty (liability) insurance 11  
central clearing 228  
central counterparty 225  
cheapest-to-deliver bond 112  
Chinese walls 4  
chooser options 186  
clawback clause 30  
clean price 110  
clearing 215, 225  
clearinghouse 59  
clearing margin 60  
clearing ring 216  
closed-end mutual funds 26  
closed out 61  
collar 176  
collateralization 60  
collateralized mortgage obligations (CMOs) 283  
collateral trust bonds 259  
combined ratio 15  
combined ratio after dividends 15  
commercial banks 1  
Commodity Futures Trading Commission (CFTC) 62  
commodity spread 206  
commodity swap 134  
comparative advantage 125  
complete clearing 216  
compounding 81  
compound options 185  
concentration risk 238  
conditional prepayment rate (CPR) 279  
confirmations 125  
consumption asset 96  
contango 104, 200  
continuous compounding 81  
convenience yield 103, 202  
conventional loans 272  
conventional mortgage 273

conversion factors 112  
convertible arbitrage hedge funds 32  
convertible bonds 151  
convertible debentures 260  
convexity 89  
corporate trustee 257  
coupon 258  
covered call 150, 168  
crack spread 206  
credit default risk 262  
credit derivative product companies 219  
credit risk 1  
credit spread 262  
credit spread risk 262  
cross hedge 70, 208  
crush spread 206  
currency options 145  
currency swap 130  
curtailments 286  
custody risk 238

## D

day count conventions 109  
debentures 260  
dedicated short hedge funds 31  
default management 228  
default risk 236  
defaults 286  
deferred-coupon structures 264  
deferred-interest bond 258  
defined benefit plans 18  
defined contribution plans 18  
delivery 61  
delivery options 103  
deposit insurance 2  
derivative 51  
derivative markets 40  
derivatives product companies 219  
diagonal spread 172  
direct clearing 216  
dirty price 110  
discrete compounding 81  
discretionary order 62  
distressed securities 31  
dollar default rate 264  
dollar duration 89  
dollar roll transaction 282  
DOOM options 147  
duration 88  
duration-based hedge 116  
Dutch auction 3  
dynamic options replication 189

## E

economic capital 2  
electronic trading system 40  
emerging market hedge funds 33  
employee stock options 151  
equipment trust certificates 260  
equity swap 133  
European options 51, 140  
event risk 263  
exchange 215  
exchangeable debentures 260  
exchange for physicals 61  
exchange-traded funds (ETFs) 27  
exotic swap 134  
expectations model 104  
expectations theory 91  
expected inflation rate 251  
expense ratio 15  
expiration 51

## F

fallen angels 264  
FICO model 271  
fiduciary call 158  
fill-or-kill orders 62  
financial intermediaries 125  
firm commitment 3  
fixed income arbitrage hedge funds 33  
fixed lookback call 187  
fixed lookback put 187  
fixed-price call 261  
fixed-rate mortgages 271, 273  
FLEX options 147  
floating lookback call 187  
floating lookback put 187  
floating-rate bonds 258  
floorlets 177  
floor rate 177  
foreign exchange risk 238  
forward contracts 41, 51, 97  
forward curve 195  
forward exchange rate 248  
forward prices 98  
forward rate agreement 87  
forward rates 86  
forward start options 185  
forward strip 195  
front-end load 26  
futures contracts 41, 51, 97  
futures options 146

## G

gap options 185  
global macro hedge funds 33  
good-till-canceled (GTC) orders 62  
government loans 272  
guaranteed bonds 260  
guaranty system 17

## H

health insurance 11  
hedge accounting 63  
hedge effectiveness 71  
hedgers 45, 51, 56  
high-water mark clause 30  
high-yield bonds 263  
housing prices 277  
housing turnover 286  
hurdle rate 30

## I

incentive fees 29  
income bonds 258  
index options 145  
initial margin 58, 216  
initial public offerings 3  
interest-only strips 284  
interest rate cap 176  
interest rate collar 178  
interest rate floor 177  
interest rate parity 100, 250  
inverse floater 80  
inverted market 61  
investment asset 96  
investment banks 1  
investment risk 238  
invoice price 110  
issuer default rate 264

## J

jumbo loans 272

## L

lagged rates 286  
lease rate 194, 199  
legal risk 237  
leverage 47  
leveraged buyouts 264  
LIBOR 80  
lien status 270  
life insurance 10  
limit down 57  
limit orders 62  
limit up 57

liquidity preference theory 91  
liquidity risk 237  
loan-to-value ratio 271  
lock-in effect 286  
lock-up period 29  
longevity risk 16  
long hedge 68  
long position 51, 56  
long/short equity hedge funds 31  
Long-Term Capital Management (LTCM) 33  
long-term equity anticipation securities 146  
lookback options 187  
loss mutualization 226, 228  
loss ratio 15

## M

maintenance and replacement fund 261  
maintenance margin 58  
make-whole call 261  
managed futures hedge funds 33  
margin 58  
margining 229  
market-if-touched (MIT) orders 62  
market maker 51  
market orders 62  
market risk 2  
market segmentation theory 91  
maturity date 257  
measurement bias 34  
media effect 286  
merger arbitrage hedge funds 32  
model risk 236  
modified duration 89  
monolines 219  
Monte Carlo methodology 287  
moral hazard 2, 15, 228  
mortality risk 16  
mortality tables 12  
mortgage 270  
mortgage-backed security (MBS) 270, 278  
mortgage bonds 259  
mortgage pass-through security 278  
multilateral offsetting 229  
mutual funds 24

## N

naked options 150  
net asset value (NAV) 25, 28  
net position exposure 243  
netting 216, 229  
neutral calendar spread 172  
nominal interest rate 252  
nonperforming loan 4  
nonstandard options 184  
normal backwardation 104



normal market 61  
notice of intention to deliver 61  
novation 229

## O

off-balance-sheet hedging 248  
offer price 51  
offsetting 228  
offsetting trade 61  
on-balance-sheet hedging 247  
open-end mutual funds 24  
open interest 56  
open orders 62  
open outcry system 40  
operating ratio 15  
operational risk 2, 237  
optimal hedge ratio 71  
option-adjusted spread (OAS) 290  
option contract 41  
option cost 291  
Options Clearing Corporation 150  
original-issue discount 258  
originate-to-distribute model 5  
OTC derivatives 216, 228  
over-the-counter (OTC) market 40, 60

## P

participating bonds 258  
par yield 84  
pass-through rates 278  
pass-through structure 270  
payment-in-kind bond 258  
pension funds 18  
plain vanilla interest rate swap 123  
planned amortization class 283  
poison put 263  
portfolio insurance 167  
prepayment risk 273  
prepayment speeds 279  
primary market 270  
prime (A-grade) loans 271  
principal-only strips 284  
private label 272  
private placement 3  
procyclicality 229  
product standardization 215  
property and casualty (P&C) insurance 11  
property insurance 11  
protective put 159, 167  
PSA prepayment benchmark 279  
public offering 3  
put-call parity 158  
put option 43, 51, 143  
put premium 43

## Q

quoted price 110

## R

rainbow options 188  
real interest rate 251  
recovery rate 265  
refinancing 286  
refinancing burnout 277, 286  
regulatory capital 2  
reinsurance contracts 16  
reinvestment risk 259  
repo rates 80  
reporting services 215  
reset bonds 264  
residential mortgage products 270  
restructurings 264  
retail banks 1  
reverse calendar spread 172  
reverse cash-and-carry arbitrage 197  
reverse trade 61  
rolling the hedge forward 74  
rollover risk 74

## S

seasonality 277  
secondary market 270  
securitization 270, 277, 278  
securitized mortgage 278  
servicing fee 274  
settlement 225  
settlement and payment risk 238  
settlement price 61  
short hedge 68  
short position 56  
short sales 96  
short squeeze 96  
shout option 187  
single monthly mortality rate (SMM) 279  
sinking fund provision 261  
sovereign risk 238  
special purpose vehicle 218, 277  
specified pools market 282  
speculators 47, 51, 56  
spot contracts 51  
spot rate 82  
spread duration 262  
stack and roll 207  
stack hedge 207  
static options replication 189  
stock options 145  
stock split 147  
stop-limit orders 62

stop-loss orders 62  
storage costs 102, 201  
straddle 173  
straight-coupon bonds 258  
strangle 174  
strap 175  
strike price 51  
strip hedge 207  
subordinated debenture bonds 260  
subprime (B-grade) loans 271  
swaption 134

## T

tailing the hedge 72  
TBA market 282  
T-bond futures 112  
tender offers 262  
term structure yield volatility 288  
term (temporary) life insurance 10  
tick size 57  
time-of-day order 62  
trading book 5  
trading venue 215  
transparency 228  
Treasury rates 80  
Trust Indenture Act 257

## V

variance swap 188  
variation margin 59, 216  
volatility swap 134, 188

## W

warrants 150  
weather derivatives 209  
weekly options 147  
weighted average coupon 278  
weighted average maturity 278  
whole (permanent) life insurance 10  
wholesale banks 1  
wrong-way risk 238

## Z

zero-cost collar 176  
zero-cost product 184  
zero-coupon bonds 258  
zero-volatility spread 290

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